European School of Antennas

Focus on FSS-based EBG surfaces

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Presented at the
European school of Antennas
Antenna Center of Excellence

April 18, 2005

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Focus on FSS-based EBG surfaces

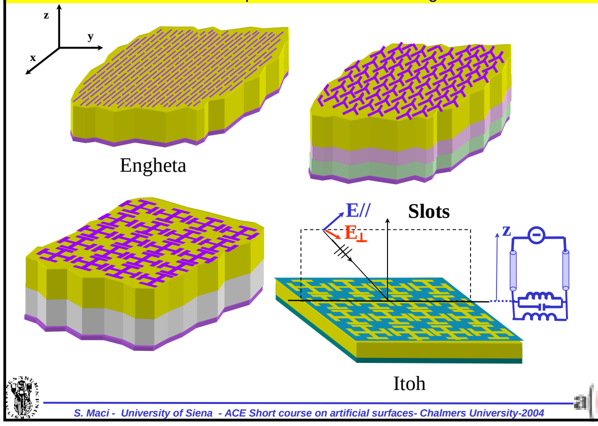
1-Basic physics and simplified analysis models
- **FSS properties**
- **Equivalent circuit analysis**
- **Homogeneization of the boundary conditions**

2- Rigorous integral equation analysis
- **Floquet wave espansion**
- **Spectral domain MoM analysis**
- **Rigorous dispersion equation**
- **Multipole network and accesible mode**
- **“reduced” dispersion equation**

3- Pole-zero matching method

1- Basic physics and simplified analysis models

Periodic structure printed on statified grounded media



Physics and methodology: overview

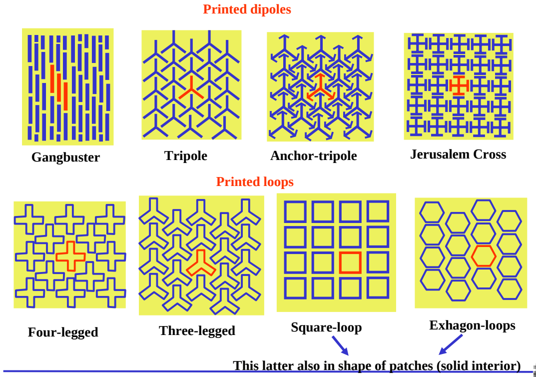
Physical involved mechanisms

1. Reflection and transmission (Floquet wave analysis
2. dispersion (Surface wave and leaky waves, band-gaps)
3. Wave interaction (FW-SW, FW-LW)
4. diffraction and excitation of spurious modes
5. Point source-FSS interaction (Green’s function)

Methods (Integral equation method):

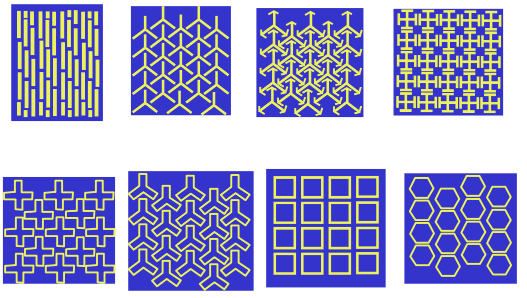
1. Full-wave analysis of reflection and transmission
2. Full-wave analysis of dispersion
3. Full-wave analysis of finite structures
4. Green's function of FSS via spectral synthesis
5. Pole-zero network synthesis

Various type of FSS (from the book of Ben A. Munk)

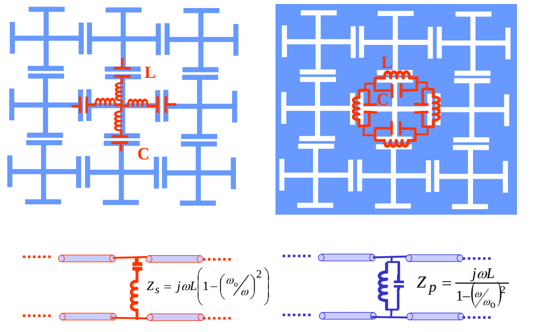


This latter also in shape of patches (solid interior)

Complementary surfaces



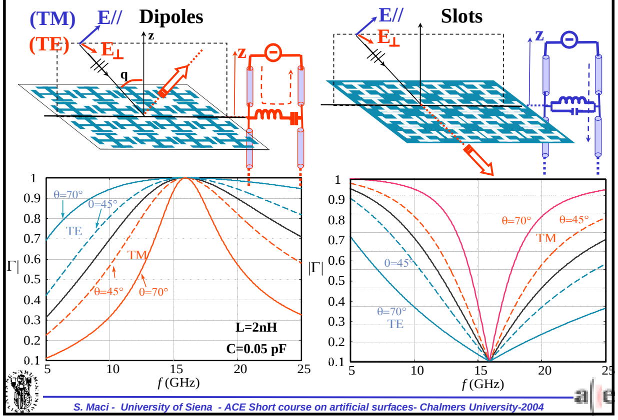
Dipole vs. slot configuration. Es. Jerusalem cross



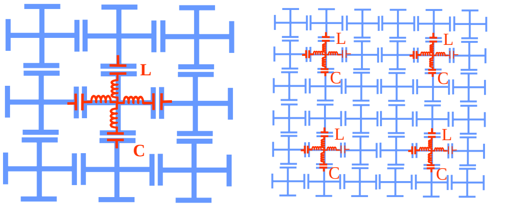
Series resonant circuit (high capacitance)

Parallel resonant circuit (high inductance)

Frequency selective surfcface: slot Vs dipoles

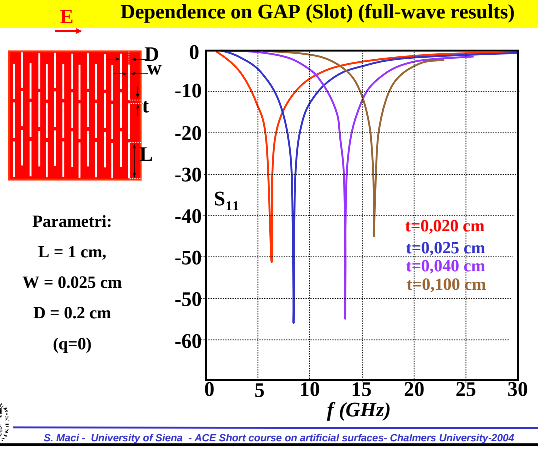


Geometry scaling



Proportional shift of the resonant frequency

Dependence on GAP (Slot) (full-wave results)



Parametri:

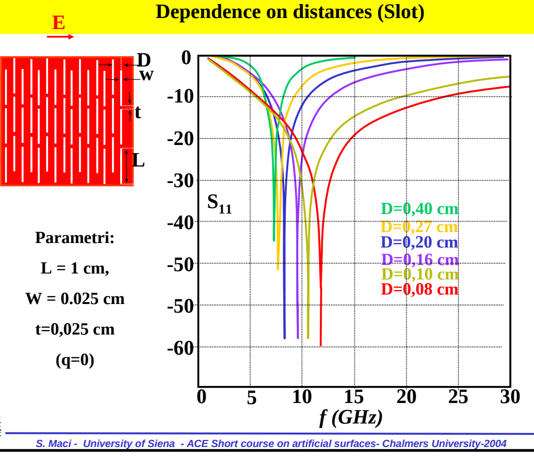
L = 1 cm,

W = 0.025 cm

D = 0.2 cm

(q=0)

Dependence on distances (Slot)



Parametri:

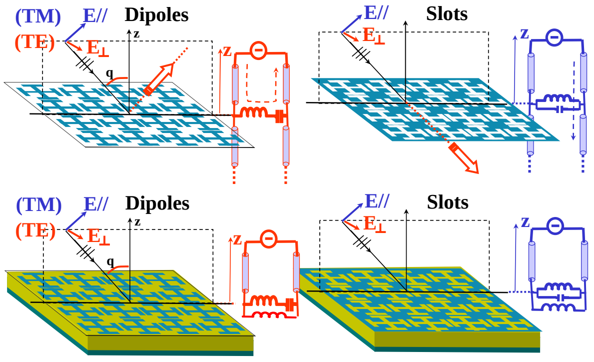
L = 1 cm,

W = 0.025 cm

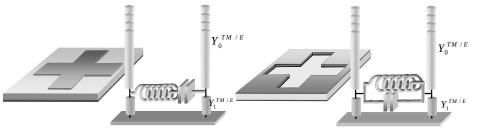
t=0,025 cm

(q=0)

How an FSS can realize a high-impedance surface (dipoles)

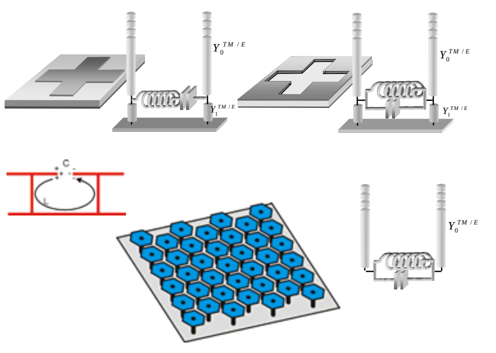


FSS-type Vs Sievenpiper artificial surface

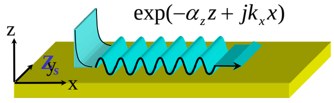


Approximation (quasi static type): the LC circuit is independent on
1-polarisation (TE or TM)
2-angle of incidence (phasing)
3-frequency

FSS-type Vs Sievenpiper artificial surface



Surface waves supported by surface impedances

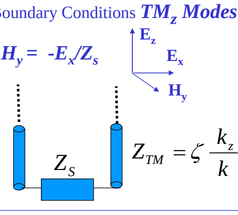


$$\exp(-\alpha_z z + jk_x x)$$

$$k_x > k$$
$$k_z = -j\sqrt{k_x^2 - k^2} = -j\alpha_z$$

Boundary Conditions ***TM$_z$ Modes***

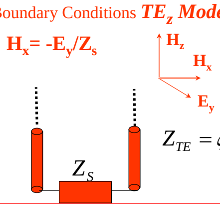
$H_y = -E_x/Z_s$



$$Z_{TM} = \zeta \frac{k_z}{k}$$

Boundary Conditions ***TE$_z$ Modes :***

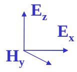
$H_x = -E_y/Z_s$



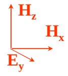
$$Z_{TE} = \zeta \frac{k}{k_z}$$

Modal propagation (absence of excitation)

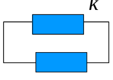
TM$_z$ Modes



TE$_z$ Modes



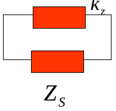
$$Z_{TM} = \zeta \frac{k_z}{k}$$



Z_S

$$Y_{TE/M} + Y_s = 0$$
$$\frac{Z_{TE/M} + Z_s}{Z_{TE/M} Z_s} = 0$$
$$Z_{TE/M} + Z_s = 0$$

$$Z_{TE} = \zeta \frac{k}{k_z}$$



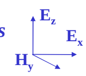
Z_S

$$\boxed{Z_s + \zeta \frac{\sqrt{k^2 - k_x^2}}{k} = 0}$$

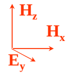
$$\boxed{Z_s + \zeta \frac{k}{\sqrt{k^2 - k_x^2}} = 0}$$

1. Dispersion equation for an impedance surface

TM$_z$ Modes



TE$_z$ Modes



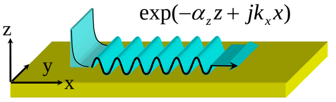
$$\boxed{Z_s - j\zeta \frac{\sqrt{k_x^2 - k^2}}{k} = 0}$$

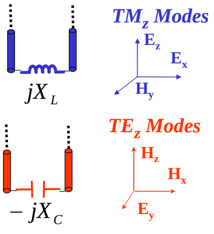
$$\boxed{Z_s + j\zeta \frac{k}{\sqrt{k_x^2 - k^2}} = 0}$$

Z_S	k_x	k_z	TM	TE
R positive	$k_x < k$;	$-j\alpha_z$	No	No
jX (inductive)	$k_x < k$ $k_x > k$ (slow wave)	$k_z > 0$ $-j\alpha_z$	No Yes	No No
-jX (capacitive)	$k_x < k$ $k_x > k$ (slow wave)	$k_z > 0$ $-j\alpha_z$	No No	No Yes

Surface waves on an impedance surface (x propagation)

$$\exp(-\alpha_z z + jk_x x)$$

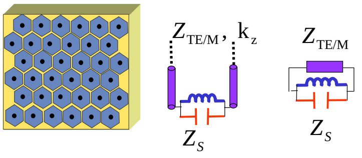




$$k_x = k\sqrt{\left(\frac{X_L}{\zeta}\right)^2 + 1} \qquad \alpha_z = \frac{X_L}{\zeta}k$$

$$k_x = k\sqrt{\left(\frac{\zeta}{X_C}\right)^2 + 1} \qquad \alpha_z = \frac{\zeta}{X_L}k$$

Example: Sievenpiper surface



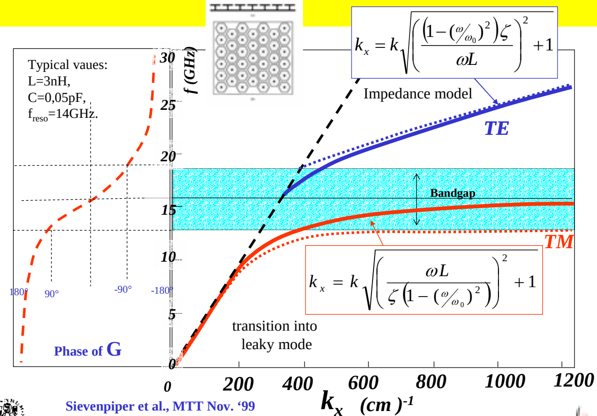
$$Z_S = \frac{j\omega L}{1-(\omega/\omega_0)^2}$$

$\omega < \omega_0 = 1/\sqrt{LC}$ → Inductive ⇨ TM-SW

$$k_x = k\sqrt{\left(\frac{X_L}{\zeta}\right)^2 + 1} \Rightarrow k_x = k\sqrt{\left(\frac{\omega L}{\zeta\left(1-(\omega/\omega_0)^2\right)}\right)^2 + 1}$$

$\omega > \omega_0 = 1/\sqrt{LC}$ → Capacitive TE-SW

$$k_x = k\sqrt{\left(\frac{\zeta}{X_C}\right)^2 + 1} \Rightarrow k_x = k\sqrt{\left(\frac{\left(1-(\omega/\omega_0)^2\right)\zeta}{\omega L}\right)^2 + 1}$$

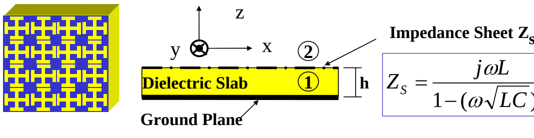


$$k_x = k\sqrt{\left(\frac{\left(1-(\omega/\omega_0)^2\right)\zeta}{\omega L}\right)^2 + 1}$$

$$k_x = k\sqrt{\left(\frac{\omega L}{\zeta\left(1-(\omega/\omega_0)^2\right)}\right)^2 + 1}$$

Sievenpiper et al., MTT Nov. '99

FSS-based surface



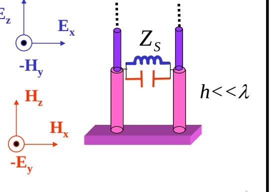
$$Z_S = \frac{j\omega L}{1-(\omega\sqrt{LC})^2}$$

Boundary Conditions TM$_z$ Modes x=d

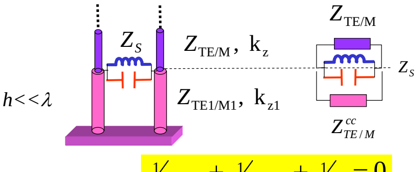
$E_{x1} = E_{x2}$; $H_{y1} - H_{y2} = E_x/Z_s$

Boundary Conditions TE$_z$ Modes x=d :

$E_{y1} = E_{y2}$; $H_{x2} - H_{x1} = -E_y/Z_s$



FSS-based surface



$$Z_S = \frac{j\omega L}{1-(\omega\sqrt{LC})^2}$$

$$1/Z^{cc}_{TE/M} + 1/Z_{TE/M} + 1/Z_s = 0$$

$$Z_{TM1} = \zeta_1 \frac{\sqrt{k_x^2 - k_1^2}}{k_1} \qquad Z_{TM} = \zeta\frac{\sqrt{k_x^2 - k^2}}{k}$$

$$Z_{TE1} = \zeta_1 \frac{k_1}{\sqrt{k_x^2 - k_1^2}} \qquad Z_{TE} = \zeta\frac{k}{\sqrt{k_x^2 - k^2}}$$

$$Z^{cc}_{TE} = j\frac{k\zeta}{k_{z1}\sqrt{\varepsilon_r}}\tan(hk_{z1}) \approx j\frac{k\sqrt{\varepsilon_r}\zeta}{k_{z1}\sqrt{\varepsilon_r}}(hk_{z1}) = jhk\zeta = j\omega L_1$$

$$Z^{cc}_{TM} = j\frac{k_{z1}\zeta}{k\varepsilon_r}\tan(hk_{z1}) \approx j\frac{k_{z1}\zeta}{k\varepsilon_r}(hk_{z1}) = j\frac{(k_{z1})^2 h\zeta}{k\varepsilon_r} = j\frac{h\zeta}{k\varepsilon_r}\left(k_x^2 - \varepsilon_r k^2\right)$$

2- Rigorous integral equation analysis

Floquet theorem

For a given periodic surface, illuminated by a plane (either homogeneous or inhomogeneous) wave, OR NOT ILLUMINATED AT ALL, the (POSSIBLY MODAL) field can be expanded in terms of plane (both omhogeneous or inhomogeneous) wave (Floquet modes, FM) . The wavenumbers of these waves are dictated by the periodicity

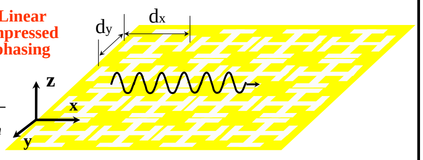
$$k_{ym} = \frac{2\pi m}{d_y} + k_y$$

$$k_{xn} = \frac{2\pi n}{d_x} + k_x$$

$$k_{znm} = \sqrt{k^2 - k_{xn}^2 - k_{ym}^2}$$

$$\mathrm{Im}\,(k_{znm}) < 0$$

Linear impressed phasing



$$H(x,y,z) = \sum_{nm} C_{nm} \exp(-jk_{xn}x - jk_{ym}y - jk_{znm}z)$$

Floquet wave representation of the field

$$H(x,y,z) = \sum_{nm} C_{nm} \exp(-jk_{xn}x - jk_{ym}y - jk_{znm}z)$$

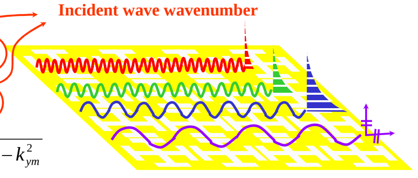
Incident wave wavenumber

$$k_{ym} = \frac{2\pi m}{d_y} + k_{y0}$$

$$k_{xn} = \frac{2\pi n}{d_x} + k_{x0}$$

$$k_{znm} = \sqrt{k^2 - k_{xn}^2 - k_{ym}^2}$$

$$\mathrm{Im}\,(k_{znm}) < 0$$



Impressed phasing

Reflection analysis: **impressed field, phasing known a priori**

Dispersion analysis: **phasing has to be found by boundary condition in absence of excitation**

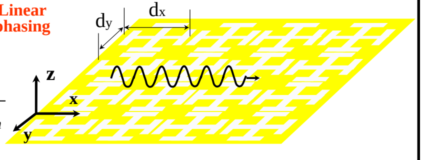
$$k_{ym} = \frac{2\pi m}{d_y} + k_y$$

$$k_{xn} = \frac{2\pi n}{d_x} + k_x$$

$$k_{znm} = \sqrt{k^2 - k_{xn}^2 - k_{ym}^2}$$

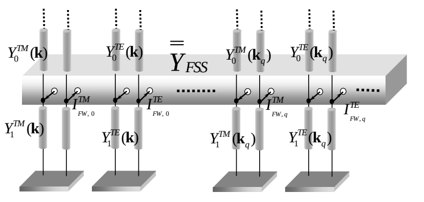
$$\mathrm{Im}\,(k_{znm}) < 0$$

Linear phasing



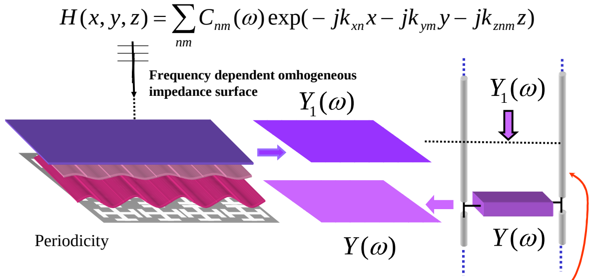
$$H(x,y,z) = \sum_{nm} C_{nm} \exp(-jk_{xn}x - jk_{ym}y - jk_{znm}z)$$

Rigorous multiport network



Definition o the "homogenised" impedance

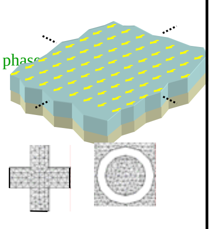
$$H(x,y,z) = \sum_{nm} C_{nm}(\omega) \exp(-jk_{xn}x - jk_{ym}y - jk_{znm}z)$$



The process is legittimated by periodicity less than l/2 **(arbitrary incidence)**

Characteristic impedance of the dominant Floque mode

Integral equation method: Methods and key issues (1)



Reflection and transmission

Method: solving for MoM matrix of single cell with phase shift boundary conditions

Key issues

Periodic Green's function in stratified media

1. Spectral spatial domain representation
2. EFIE/MPIE
3. Entire- vs. small- domain basis functions

Dispersion

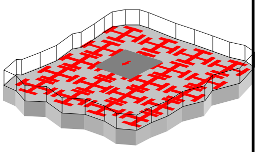
Method: find the zeros of the MoM matrix determinant

Key issues

1. Efficient way to find the zeros
2. Individuation of the poles associated to "physical" leaky waves
3. Individuation of bandgaps and dispersion diagrams

Integral equation method: Methods and key issues (3)

1. Interaction with point source on FSS **(GF of PBG structure)**



Method

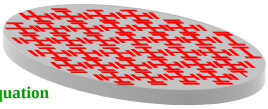
1. Plane wave espansion of the source field
2. FSS response to each spectral plane wave
3. Spectral synthesis

Key issues

1. Speed-up the numerical process for the response of each single plane wave
2. Finding poles
3. Hybrid-asymptotic/numerical process for the GF calculation

Integral equation method: Methods and key issues (2)

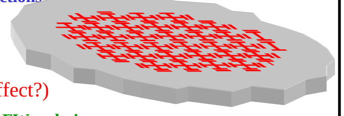
1. Truncation effects (1)
(total truncation)



Method: Surface-surface integral equation

Key issues: matrix compression (SFX), sparsification (FMM), multiscale approach, wavelet basis functions

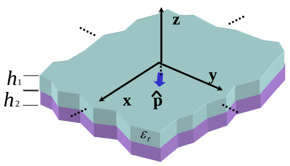
1. Truncation effects (2)
(FSS truncation, dominant effect?)

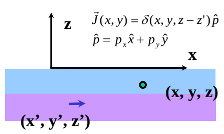


Methods: windowing, truncated FW technique

Key issues: Truncated periodic Green's function, global domain basis functions (supercompression of the MoM matrix)

Single element GF (Geometry)





$\vec{J}(x,y)=\delta(x,y,z-z')\hat{p}$

$\hat{p}=p_x\hat{x}+p_y\hat{y}$

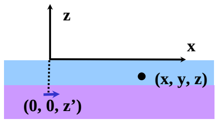
Observation point ➡ $\underline{r}=\underline{\rho}+\hat{z}z$ $\quad\underline{\rho}=\hat{x}x+\hat{y}y$

Source position ➡ $\underline{r}'=\underline{\rho}'+\hat{z}z'$ $\quad\underline{\rho}'=\hat{x}x'+\hat{y}y'$

$\vec{E}(x,y,z;x',y',z')$

⬇

$\vec{E}(x-x',y-y',z,z')$



Single element Green's function (spectral domain)

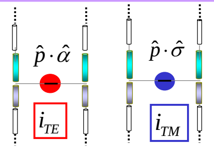
$$\bar{\bar{g}}(x,y,z,z')=\left(\frac{1}{2\pi}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{\bar{G}}^{E,J}(k_x,k_y,z,z')\,e^{-j\vec{k}_\rho\cdot\vec{\rho}}\,d\vec{k}_\rho$$

$$\bar{\bar{G}}^E(k_x,k_y,z,z')=\left(-\hat{\alpha}\;\hat{\alpha}\;v_{TE}-\hat{\sigma}\hat{\sigma}v_{TM}+\zeta\frac{k_\rho}{k}i_{TM}\hat{z}\hat{\sigma}\right)$$

$$\bar{\bar{G}}^H(k_x,k_y,z,z')=\left(-\hat{\alpha}\;\hat{\sigma}i_{TM}+\hat{\sigma}\hat{\alpha}\;i_{TE}-\frac{k_\rho}{\zeta\,k}v_{TE}\hat{z}\hat{\alpha}\right)$$

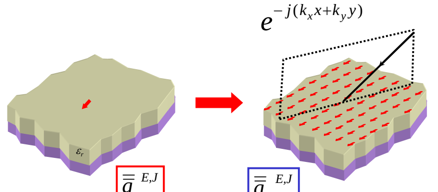
$\hat{\alpha}=\hat{z}\times\hat{k}_\rho=\frac{1}{k_\rho}(k_x\hat{y}-k_y\hat{x})$

$\vec{k}_\rho=\hat{\sigma}k_\rho \quad k_\rho=\sqrt{k_x^2+k_y^2}$



From single element to periodic GF

$e^{-j(k_x x+k_y y)}$



$\bar{\bar{g}}^{E,J}$ $\quad\bar{\bar{g}}_p^{E,J}$

$$\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\bar{\bar{g}}^{E,J}(x-x_n,y-y_m,z,z')e^{-j(k_x x+k_y y)}=\bar{\bar{g}}_P^{E,J}(x,y,z,z')$$

Space domain (slow convergent) summation

Poisson sum

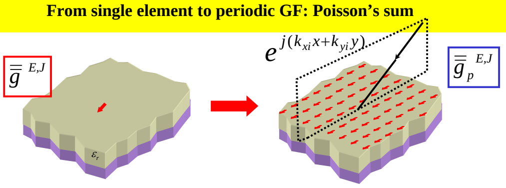
$$\sum_{n=\infty}^{\infty}\sum_{n=\infty}^{\infty}f\left(na,mb\right)=\frac{1}{ab}\sum_{p=\infty}^{\infty}\sum_{q=\infty}^{\infty}F\left(\frac{2\pi p}{a},\;\frac{2\pi q}{b}\right)$$

⬇

FT(f)

$$\sum_{n=\infty}^{\infty}\sum_{m=\infty}^{\infty}f\left(na,mb\right)e^{j(k_x x+k_y y)}=\frac{1}{ab}\sum_{p=\infty}^{\infty}\sum_{q=\infty}^{\infty}F\left(\frac{2\pi p}{a}+k_x,\;\frac{2\pi q}{b}+k_y\right)$$

From single element to periodic GF: Poisson's sum



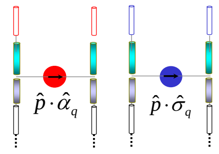
$$\bar{\bar{g}}^{E,J}(x,y,z,z')=\left(\tfrac{1}{2\pi}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{\bar{G}}^{E,J}(k_x,k_y,z,z')e^{-j(k_x x+k_y y)}dk_x dk_y$$

$$\bar{\bar{g}}_P^{E,J}(x,y,z,z')=\sum_{p=-\infty}^{+\infty}\sum_{q=-\infty}^{+\infty}\bar{\bar{G}}^{E,J}(k_{xp},k_{yq},z,z')\ e^{j(k_{xp}x+k_{yq}y)}$$

$$k_{xp}=\frac{2\pi p}{a}+k_x \qquad k_{yq}=\frac{2\pi n}{b}+k_y$$

Interpretation in terms of z-tx line

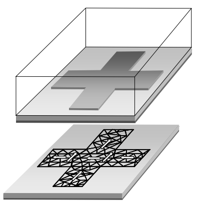
$$\bar{\bar{g}}_P^{E,J}(x,y,z,z')=\sum_{p=-\infty}^{+\infty}\sum_{q=-\infty}^{+\infty}\bar{\bar{G}}^{E,J}(k_{xp},k_{yq},z,z')\ e^{j(k_{xm}x+k_{yn}y)}$$



$$\hat{\sigma}_q=\frac{\mathbf{k}_q}{\sqrt{\mathbf{k}_q\cdot\mathbf{k}_q}}$$

$$\hat{\alpha}_q=\hat{z}\times\hat{\sigma}_q$$

Dipole type FSS (single substrate)



$$\mathbf{E}_s(\mathbf{J})+\mathbf{E}_{imp}=0$$

$$\mathbf{E}_{imp}=\mathbf{E}_{inc}+\mathbf{E}_{ref}$$

$$\mathbf{J}(\mathbf{r}_t)=\sum_{n=1}^{N}I_n\ \mathbf{f}_n(\mathbf{r}_t)$$

$$\mathbf{r}_t=x\hat{x}+y\hat{y}$$

$$\mathbf{k}=k_x\hat{x}+k_y\hat{y}$$

$$\mathbf{k}_q=k_{xp}\hat{x}+k_{yq}\hat{y}$$

$$k_{xp}=k_x+2\pi p/d_x$$

$$k_{yq}=k_y+2\pi q/d_y$$

$$\hat{\sigma}_q=\frac{\mathbf{k}_q}{\sqrt{\mathbf{k}_q\cdot\mathbf{k}_q}}$$

$$\hat{\alpha}_q=\hat{z}\times\hat{\sigma}_q$$

MoM matrix entries

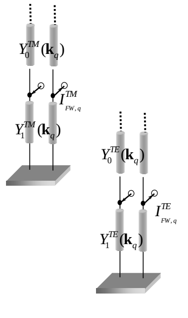
$$\bar{\bar{Z}}_{MOM}\ \bar{I}=\bar{V}$$

$$V_m=-\tilde{\mathbf{F}}_m^*(\mathbf{k})\cdot\mathbf{E}_{imp}(\mathbf{k})$$

$$Z_{mn}^{MoM}=\sum_{q=0}^{M-1}\tilde{\mathbf{F}}_m^*(\mathbf{k}_q)\cdot[Z_{GF}^{TM}(\mathbf{k}_q)\hat{\sigma}_q\hat{\sigma}_q+Z_{GF}^{TE}(\mathbf{k}_q)\hat{\alpha}_q\hat{\alpha}_q]\cdot\tilde{\mathbf{F}}_n(\mathbf{k}_q)$$

$\tilde{\mathbf{F}}_n(\mathbf{k})$ $[\tilde{\mathbf{F}}_m(\mathbf{k})]$ is the Fourier transform of the basis [test] function $\mathbf{f}_n(\mathbf{r}_t)$ $[\mathbf{f}_m(\mathbf{r}_t)]$

TE-TM Z-transmission lines



$$Z_{GF}^{TM/E}(\mathbf{k})=\left[Y_0^{TM/E}(\mathbf{k})-jY_1^{TM/E}(\mathbf{k})\cot(k_{z1}h)\right]^{-1}$$

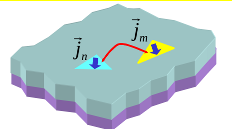
$$Y_0^{TM}(\mathbf{k})=\frac{\omega\varepsilon_0}{k_z},\ Y_0^{TE}(\mathbf{k})\frac{k_z}{\omega\mu_0};$$

$$Y_1^{TE}(\mathbf{k})\frac{k_{z1}}{\omega\mu_0},\ Y_1^{TM}(\mathbf{k})=\frac{\omega\varepsilon_r\varepsilon_0}{k_{z1}}$$

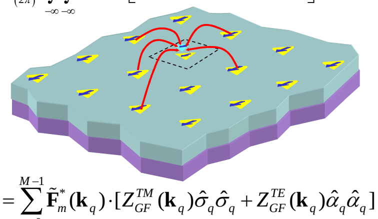
$$k_z=\sqrt{k^2-k_x^2-k_y^2}$$

$$k_{z1}=\sqrt{\varepsilon_r k^2-k_x^2-k_y^2}$$

Impedance: single element vs. array



$$Z_{mn}^{MoM}=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{\mathbf{F}}_m^*(\mathrm{k})\cdot\left[Z_{GF}^{TM}(\mathbf{k})\hat{\sigma}\hat{\sigma}+Z_{GF}^{TE}(\mathbf{k})\hat{\alpha}\hat{\alpha}\right]\cdot\tilde{\mathbf{F}}_m^*(\mathrm{k})dk_xdk_y$$



$$Z_{mn}^{MoM}=\sum_{q=0}^{M-1}\tilde{\mathbf{F}}_m^*(\mathbf{k}_q)\cdot[Z_{GF}^{TM}(\mathbf{k}_q)\hat{\sigma}_q\hat{\sigma}_q+Z_{GF}^{TE}(\mathbf{k}_q)\hat{\alpha}_q\hat{\alpha}_q]\cdot\tilde{\mathbf{F}}_n(\mathbf{k}_q)$$

MoM matrix in terms of the diagonal GF matrix

$$\bar{\bar{Z}}_{MoM} = \bar{\bar{Q}}^{H}\,\bar{\bar{Z}}_{GF}\,\bar{\bar{Q}}$$

$$\bar{\bar{Z}}_{GF} = diag\left\{Z_{GF}^{TM}(\mathbf{k}_q), Z_{GF}^{TE}(\mathbf{k}_q)\right\}_{q=0,M-1} \qquad 2M \times 2M$$

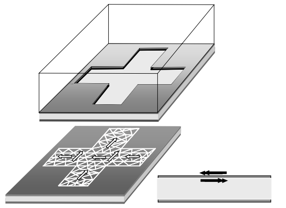
$$\bar{\bar{Q}} = \left\{Q_{q,n}^{TM},\ Q_{q,n}^{TE}\right\}_{\substack{q=0,M-1\\ n=1,N}} \qquad \bar{\bar{Q}}^{H} = \left\{Q_{m,q}^{TM*},\ Q_{m,q}^{TE*}\right\}^{T}_{\substack{m=1,N\\ q=0,M-1}}$$

$$Q_{i,q}^{TM} = \tilde{\mathbf{F}}_i(\mathbf{k}_q)\cdot\hat{\sigma}_q \qquad 2M \times N$$

$$Q_{i,q}^{TE} = \tilde{\mathbf{F}}_i(\mathbf{k}_q)\cdot\hat{\alpha}_q \qquad N \times 2M$$

Slot type FSS (single substrate)

$$\mathbf{M}(\mathbf{r_t}) = \sum_{n=1}^{N} V_n \mathbf{g}_n(\mathbf{r_t}) \times \hat{z}$$



Imposing the continuity of the magnetic field leads to the following representation

$$\bar{\bar{Y}}_{MoM}\,\bar{V} = \bar{I}$$

$$\bar{I} = \left\{I_m\right\}^{T}_{m=1,N},\ I_m = -\tilde{\mathrm{G}}_m^{*}(\mathbf{k})\cdot\mathrm{H}_{imp}(\mathbf{k})$$

$$\bar{V} = \left\{V_n\right\}^{T}_{n=1,N}$$

$$\bar{\bar{Y}}_{MoM} = \bar{\bar{P}}^{H}\,\bar{\bar{Y}}_{GF}\,\bar{\bar{P}}$$

$$\bar{\bar{Y}}_{GF} = diag\left\{Y_{GF}^{TM}(\mathbf{k}_q), Y_{GF}^{TE}(\mathbf{k}_q)\right\}_{q=1,M}$$

$$Y_{GF}^{TM-E}(\mathbf{k}) = Y_0^{TM-E}(\mathbf{k}) - jY_1^{TM-E}(\mathbf{k})\cot\left(k_{z1}h\right)$$

$$\bar{\bar{P}} = \left\{P_{q,n}^{TM},\ P_{q,n}^{TE}\right\}_{\substack{q=0,M-1\\ n=1,N}}$$

Dispersion equation

dipole-type FSS

$$\bar{\bar{Z}}_{MOM}\,\bar{I} = 0$$

$$\det\left[\bar{\bar{Z}}_{MOM}(k_x, k_y, \omega)\right] = 0$$

$$\xi_1(k_x,k_y,\omega)\xi_2(k_x,k_y,\omega)\cdot\cdot\xi_N(k_x,k_y,\omega) = 0$$

eigenvalues of $\bar{\bar{Z}}_{MOM}$

aperture-type FSS

$$\bar{\bar{Y}}_{MOM}\,\bar{V} = 0$$

$$\det\left[\bar{\bar{Y}}_{MOM}(k_x, k_y, \omega)\right] = 0$$

$$\eta_1(k_x,k_y,\omega)\eta_2(k_x,k_y,\omega)\cdot\cdot\eta_N(k_x,k_y,\omega) = 0$$

are the eigenvalues of $\bar{\bar{Y}}_{MOM}$

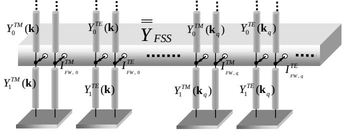
and

Accessible mode network

Let us assume that we are observing the field at a certain distance *z* from the artificial surface. In this case, the FW modes that are completely attenuated do not contribute to the field at *z*. In a multimode network description, this implies that the relevant modal ports can be considered as not “accessible” to the observer, and therefore neglected.

$$\bar{I}_{FW}^{FSS} = \bar{\bar{Y}}_{FSS}\,\bar{V}_{FW} \qquad \bar{V}_{FW} = \bar{\bar{Z}}_{FSS}\,\bar{I}_{FW}^{FSS}$$

For dipoles — For slots



$$\bar{V}_{FW} = [V_{FW,q}^{TM}, V_{FW,q}^{TE}]^{T}_{q=0,M_A-1} \qquad \bar{I}_{FW}^{FSS} = [I_{FW,q}^{TM}, I_{FW,q}^{TE}]^{T}_{q=0,M_A-1}$$

Derivation of the accessible mode Y matrix network for dipoles

$$\bar{I}_{FW}^{FSS} = \bar{\bar{Y}}_{FW}\,\bar{V}_{FW}^{imp}$$ **impressed**

$$\bar{\bar{Y}}_{FW} = \bar{\bar{q}}\left(\bar{\bar{Q}}^{H}\,\bar{\bar{Z}}_{GF}\,\bar{\bar{Q}}\right)^{-1}\bar{\bar{q}}^{H}$$

$$\bar{\bar{Y}}_{FW} = \bar{\bar{q}}\,\bar{\bar{Z}}_{MoM}^{-1}\,\bar{\bar{q}}^{H}$$

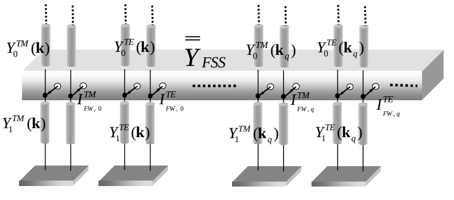
$\bar{\bar{q}} = \left\{Q_{q,n}^{TM},\ Q_{q,n}^{TE}\right\}_{\substack{q=0,M_A-1\\ n=1,N}}$ is the $2M_A \times N$ matrix obtained by the first $2M_A$ rows of $\bar{\bar{Q}}$

$\bar{\bar{q}}^{H} = \left\{Q_{m,q}^{TM*},\ Q_{m,q}^{TE*}\right\}_{\substack{m=1,N\\ q=0,M_A-1}}$ is the $N \times 2M_A$ matrix consisting of the first $2M_A$ column of $\bar{\bar{Q}}^{H}$

$$\bar{V}_{FW} = \bar{V}_{FW}^{imp} - \bar{\bar{Z}}_{GF}\,\bar{I}_{FW}^{FSS} = \bar{V}_{FW}^{imp} - \bar{\bar{Z}}_{GF}\,\bar{\bar{Y}}_{FW}\,\bar{V}_{FW}^{imp} = \left[\bar{\bar{I}} - \bar{\bar{Z}}_{GF}\,\bar{\bar{Y}}_{FW}\right]\bar{V}_{FW}^{imp}$$

$$\bar{\bar{Y}}_{FSS} = \bar{\bar{Y}}_{FW}\left[\bar{\bar{Y}}_{GF} - \bar{\bar{Y}}_{FW}\right]^{-1}\bar{\bar{Y}}_{GF}$$

Accessible mode Y and Z matrix network for dipoles and slots



$$\bar{\bar{Y}}_{FSS} = \bar{\bar{Y}}_{FW}\left[\bar{\bar{Y}}_{GF} - \bar{\bar{Y}}_{FW}\right]^{-1}\bar{\bar{Y}}_{GF}$$ **Dipole type FSS**

$$\bar{\bar{Z}}_{FSS} = \bar{\bar{Z}}_{FW}\left(\bar{\bar{Z}}_{GF} - \bar{\bar{Z}}_{FW}\right)^{-1}\bar{\bar{Z}}_{GF}$$ **Patch type FSS**

"reduced" dispersion equation

dipole-type FSS

$$\det\left[\bar{\bar{Y}}^{-1}_{FW}(k_x,k_y,\omega)\right]=0$$

$$\left\{\delta_0(k_x,k_y,\omega)\delta_1(k_x,k_y,\omega)\cdot\cdot\delta_{M_A-1}(k_x,k_y,\omega)\right\}^{-1}=0$$

eigenvalues of $\bar{\bar{Y}}_{FSS}$

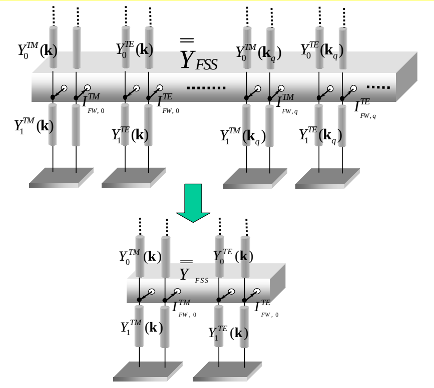
aperture-type FSS

$$\det\left[\bar{\bar{Z}}_{FSS}(k_x,k_y,\omega)\right]=0$$

$$\left\{\gamma_0(k_x,k_y,\omega)\gamma_1(k_x,k_y,\omega)\cdot\cdot\gamma_{M_A-1}(k_x,k_y,\omega)\right\}^{-1}=0$$

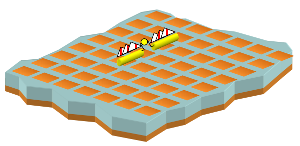
are the eigenvalues of $\bar{\bar{Z}}_{FSS}$

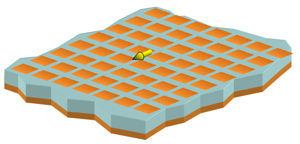
Rigorous multiport network



Motivation

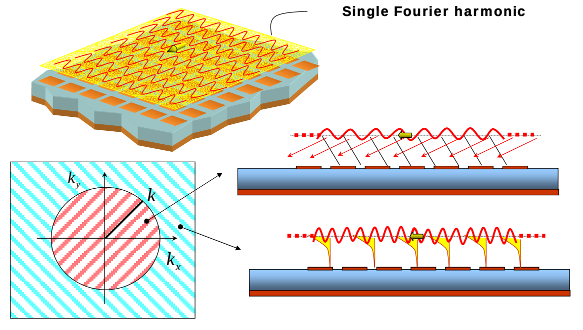
sources radiating in presence of a truncated periodic structures

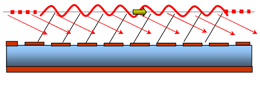


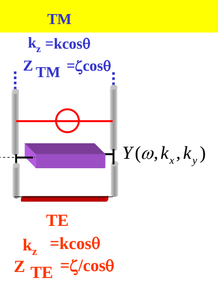


Dividing problem in elementary (GF) problems with increasing the capability of efficient mathematical rapresentation

Motivation: Elementary (GF) problem







Admittance determined by the solution of an infinite sequence of Periodicity full-wave problems.

Problem: extracting few dominant parameters which allows for the determination of the admittance with few