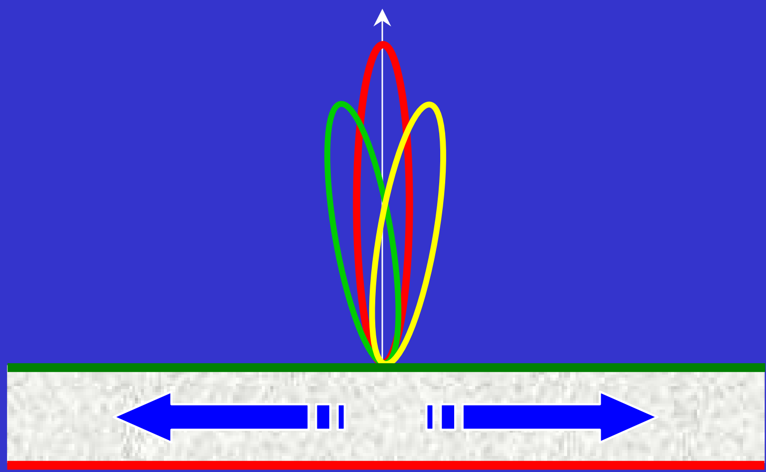
European School of Antennas

"High-frequency techniques and Travelling-wave antennas"

Fundamental Properties and Optimization of Broadside Radiation from Uniform Leaky-Wave Antennas

Giampiero Lovat



"La Sapienza" University of Rome

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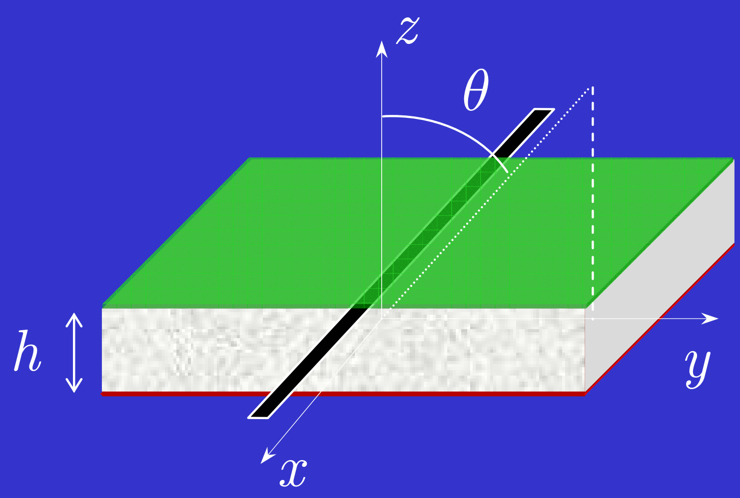
Outline

- Background
  - The Transverse Equivalent Network (TEN) model for a class of uniform Leaky-Wave Antennas (LWAs)

- One-dimensional leaky-wave antennas (line-source excitation)
  - The $\beta = \alpha$ condition for beam splitting at broadside
  - Characterization of the single leaky-wave beam
  - Maximization of the power density radiated at broadside
  - Interpretation of the optimization condition in terms of the involved physical and geometrical parameters

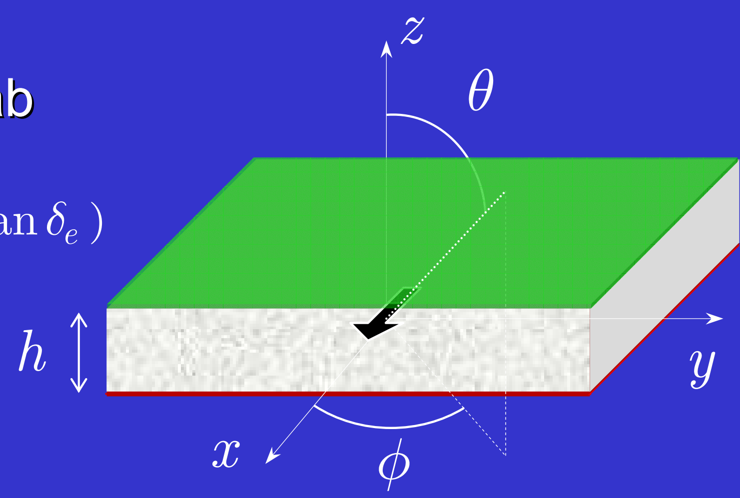
- Two-dimensional leaky-wave antennas (dipole excitation)
  - Far-field behavior in the *E* and *H* planes of a dipole source

A Class of Uniform LWAs



grounded dielectric slab

$$\mu_r,\ \varepsilon_r = \varepsilon_r' - j\varepsilon_r'' = \varepsilon_r'\,(1 - j\tan\delta_e\,)$$



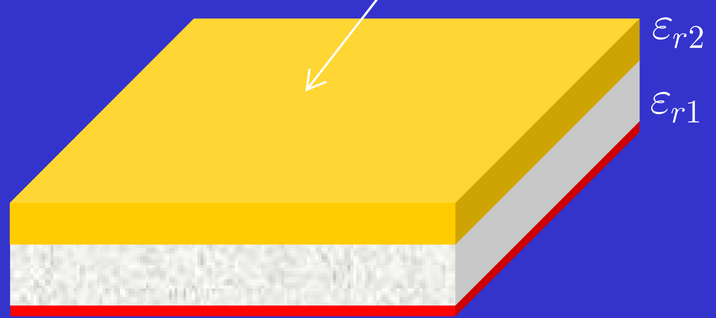
1D LWA
(line source)

At the air-slab interface we assume the presence of some screen (or structure) that partially shields the slab:
Partially-Reflecting Surface (PRS)

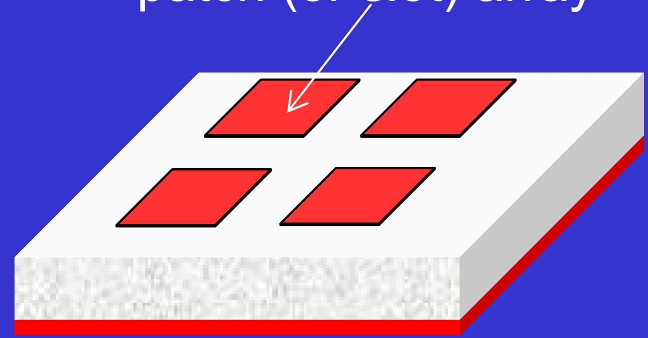
2D LWA
(dipole source)

Examples of PRS:

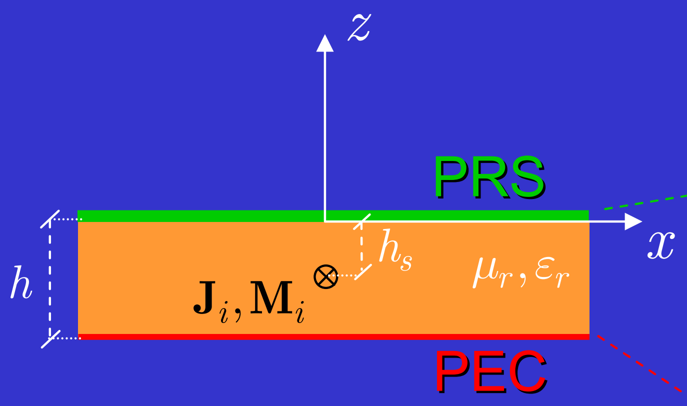
high-permittivity superstrate $\varepsilon_{r2} \gg \varepsilon_{r1}$



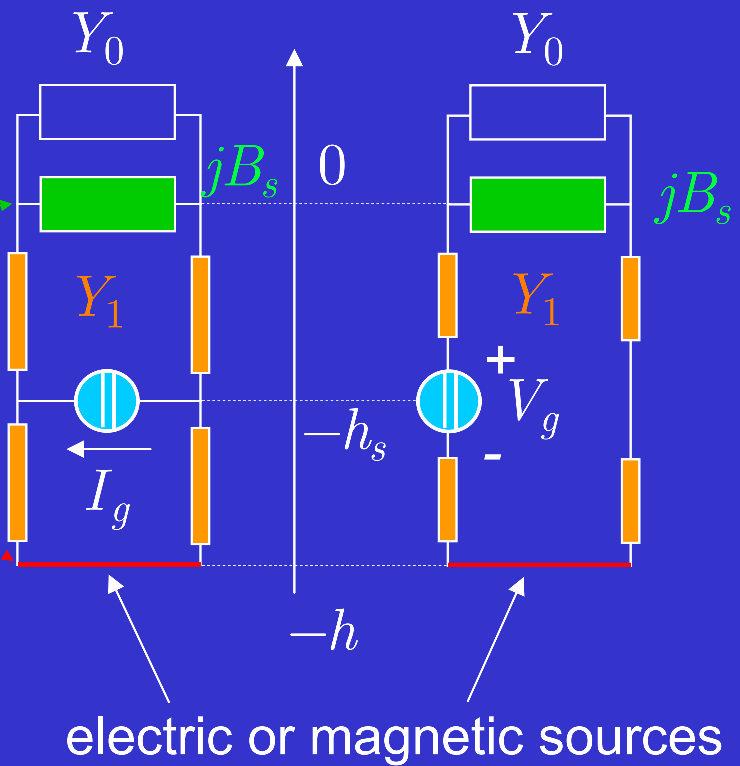
patch (or slot) array



The TEN Model



A transverse equivalent network
is used as model of such an antenna



$$Y_0^{TE} = \frac{k_{z0}}{\omega\mu_0}, \qquad Y_1^{TE} = \frac{k_{z1}}{\omega\mu_0\mu_r} \qquad k_{z0} = \sqrt{k_0^2 - k_t^2}$$

$$Y_1^{TM} = \frac{\omega\varepsilon_0}{k_{z0}}, \qquad Y_1^{TM} = \frac{\omega\varepsilon_0\varepsilon_r}{k_{z1}} \qquad k_{z1} = \sqrt{k_0^2\mu_r\varepsilon_r - k_t^2}$$

Electric-line source $\longrightarrow$ TE_z modes

Magnetic-line source $\longrightarrow$ TM_z modes

The 'Splitting Condition' (1)

Leaky-wave aperture field: $E_y(x) = E_0 \; e^{-jk_{xLW}|x|}$

$$\boxed{k_{xLW} = \beta - j\alpha}$$

Leaky-wave far field: $E_y^{ff}(\theta) \propto \cos\theta \tilde{E}_y(k_0 \sin\theta) = E_0 \dfrac{2jk_{xLW}\cos\theta}{(k_0 \sin\theta)^2 - k_{xLW}^2}$

Power density: $P(\theta) \propto \left|E_y^{ff}(\theta)\right|^2 \propto \dfrac{\cos^2\theta}{\left(k_0^2 \sin^2\theta - \beta^2 + \alpha^2\right)^2 + 4\alpha^2\beta^2}$

near broadside $(\theta \ll 10°)$ $\Rightarrow$ $P(\theta) \propto \dfrac{1}{\left(k_0^2\theta^2 - \beta^2 + \alpha^2\right)^2 + 4\alpha^2\beta^2}$

$$\left(k_0^2\theta^2 - \beta^2 + \alpha^2\right)\theta = 0$$

$\boxed{\beta < \alpha}$ $\rightarrow$ $\theta_0 = 0$

$\boxed{\beta \geq \alpha}$ $\rightarrow$ $\theta_0 = 0 \quad \theta_{1,2} = \pm\sqrt{\hat{\beta}^2 - \hat{\alpha}^2}$

two peaks close to
but off broadside,
with $\theta = 0$ being a local minimum
of the power pattern

The ‘Splitting Condition’ (2)

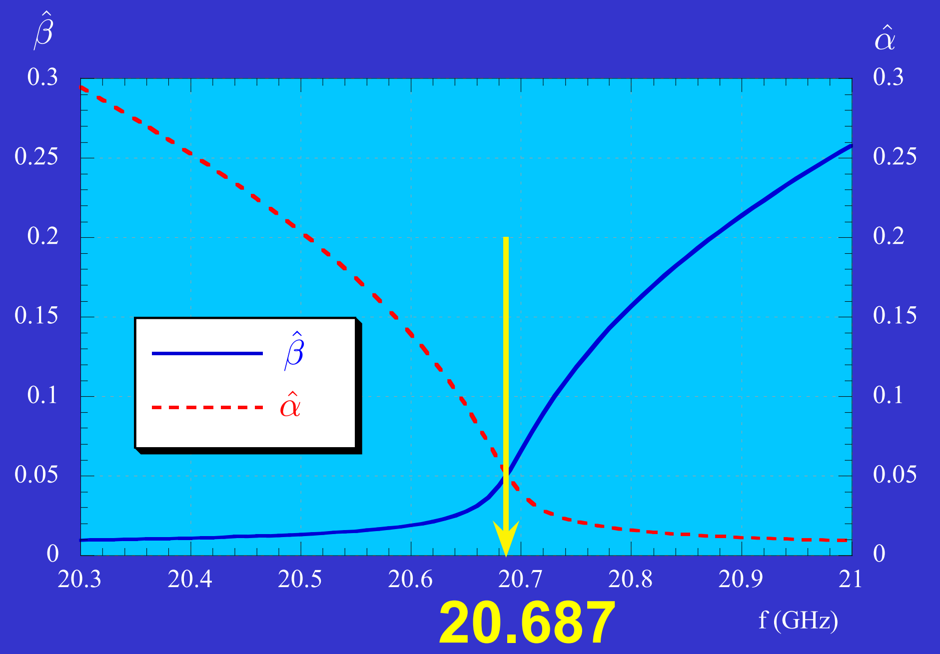
$$\beta = \alpha$$

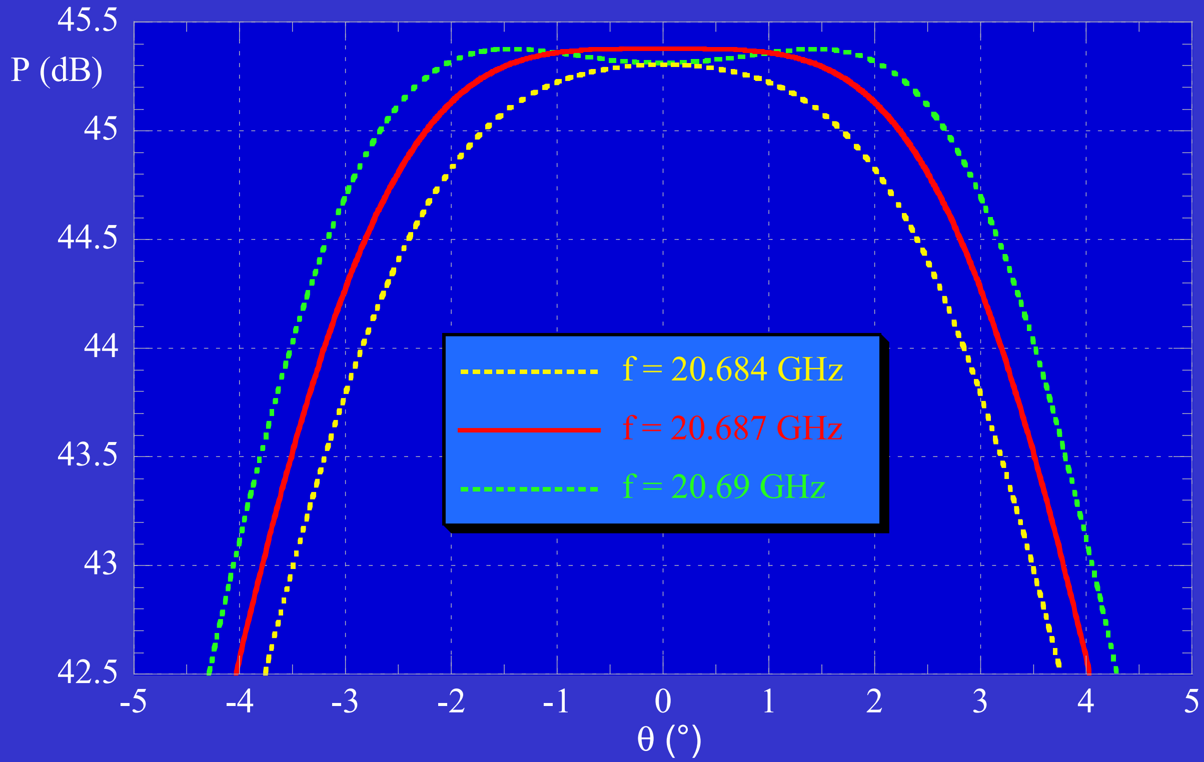
Parameters of the structure:

$\varepsilon_r = 2.2,\ \mu_r = 1$

$h = 5\ \text{mm},\ h_s = h/2$

$\bar{B}_S = B_S \eta_0 = 20$





- f_{sp} = 20.687 GHz
- At f = 20.684 GHz (just below f_{sp}) only one beam is present pointing at broadside
- At f = 20.69 GHz (just above f_{sp}) two separate peaks exist, pointing close to, but off ($\theta_p = \pm 1.44°$), broadside

Characterization of the Single Leaky-Wave Beam (1)

Consider the leaky-wave field launched *on one side only* of the line source

$$E_y^{LW+}(x) = E_0 e^{-jk_{xLW}x} u_{-1}(x)$$

$$E_y^{ff+}(\theta) = E_0 \frac{\cos\theta}{k_0 \sin\theta - k_{xLW}}$$

Maximum at
$$\theta_M = \sin^{-1}(\beta / k_0)$$
(regardless of the value of α)

near broadside $(\theta_M \ll 10°)$

$$P^+(\theta_M) \propto \left| E_y^{ff+}(\theta_M) \right|^2 = \left| E_0 \frac{\cos\theta_M}{k_0 \sin\theta_M - k_{xLW}} \right|^2 \simeq \frac{|E_0|^2}{\alpha^2}$$

$$P^+(0) \propto \frac{|E_0|^2}{\beta^2 + \alpha^2}$$

$$\beta = \alpha$$

$$P^+(0) = \frac{1}{2} P^+(\theta_M)$$

corresponds to having the 3 dB direction of the *single* leaky-wave beam at broadside

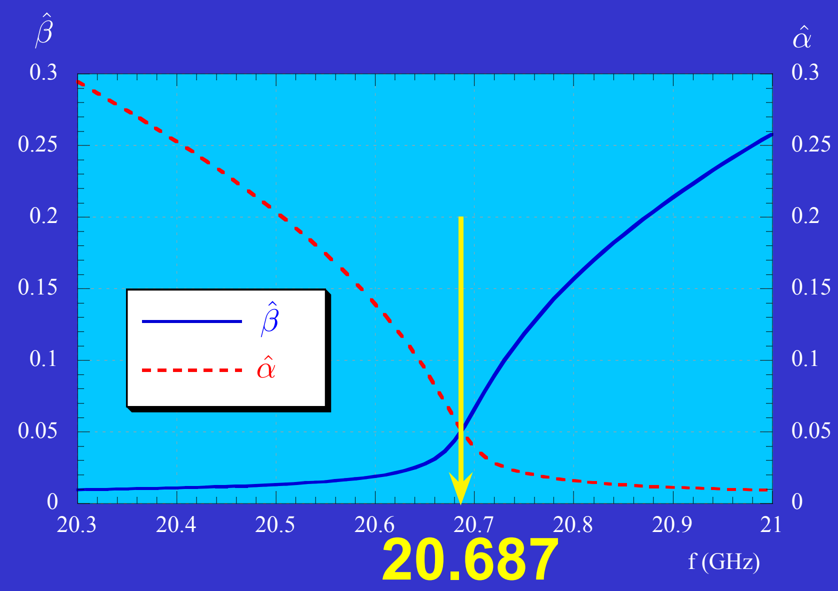
Characterization of the Single Leaky-Wave Beam (2)

Parameters of the structure:

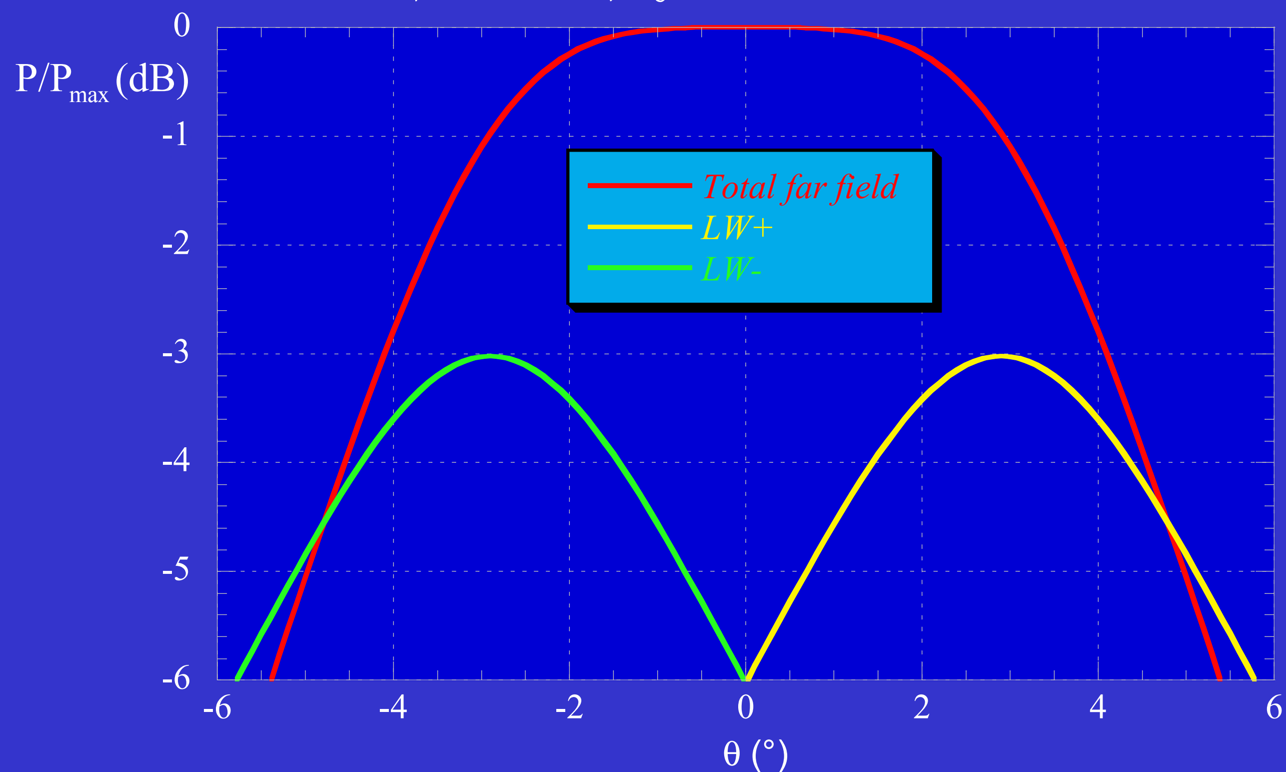
$\varepsilon_r = 2.2,\ \mu_r = 1$

$h = 5\ \mathrm{mm},\ h_s = h/2$

$\bar{B}_S = 20$



$\beta = \alpha,\ f = 20.687\ \mathrm{GHz}$



The power density
of the single leaky-wave field
at broadside
is 3 dB lower than
its power density
at the pointing angle

Maximum Power Radiated at Broadside (1)

$$E_y(x) = \underbrace{E_0(k_{xLW})}_{} e^{-jk_{xLW}|x|} \quad \Longrightarrow \quad E_y^{ff}(\theta) \propto E_0(k_{xLW}) \frac{2jk_{xLW}\cos\theta}{(k_0 \sin\theta)^2 - k_{xLW}^2}$$

depends on the residue of the spectral Green's function (SGF) of the problem at the leaky-wave pole

$$P(0) \propto \frac{|E_0(k_{xLW})|^2}{|k_{xLW}|^2} = \frac{|E_0(k_{xLW})|^2}{\beta^2 + \alpha^2} = \frac{\left|\hat{E}_0\left(\hat{k}_{xLW}\right)\right|^2}{\hat{\beta}^2 + \hat{\alpha}^2}$$

We wish to find the *frequency* at which $P(0)$ is maximum

How $\hat{E}_0\left(\hat{k}_{xLW}\right)$ depends on its argument $\hat{k}_{xLW}$

How to model the dependence of $\hat{\beta}$ and $\hat{\alpha}$ on frequency

Maximum Power Radiated at Broadside (2)

for small values of $\hat{k}_{xLW}$

By studying the SGF: $\hat{E}_0\left(\hat{k}_{xLW}\right) \simeq \dfrac{\hat{R}_0}{\hat{k}_{xLW}}$ $\Rightarrow$

$$P(0) \propto \frac{\left|\hat{R}_0\right|^2}{\left(\hat{\beta}^2 + \hat{\alpha}^2\right)^2}$$

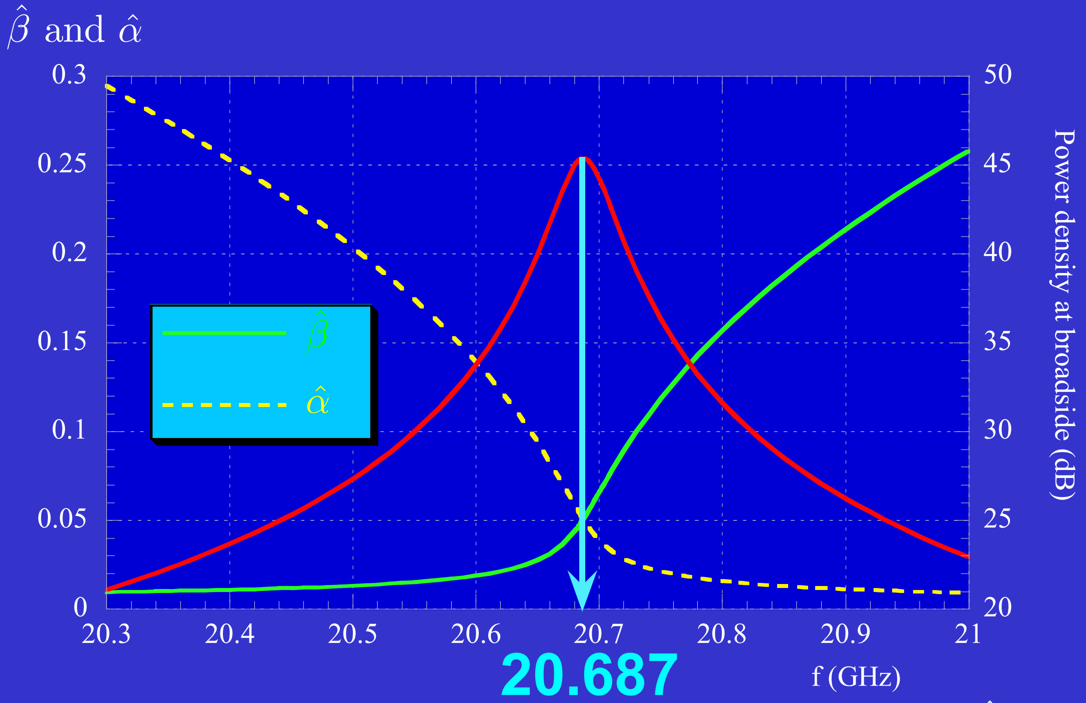
By assuming a closed PPW
with a lossy filling medium
as a model of the structure: $\Rightarrow$

$$\hat{\beta}\hat{\alpha} = \frac{1}{2}\mu_r \varepsilon_r' \tan\delta_{eq} = C$$

(the pole locus is an equilateral hyperbola
in the complex k_x plane)

$\Rightarrow$ $$\beta = \alpha$$

Maximum Power Radiated at Broadside (2)



Parameters of the structure:

$\varepsilon_r = 2.2,\ \tan\delta_e = 0,\ \mu_r = 1$

$h = 5\ \text{mm},\ h_s = h/2$

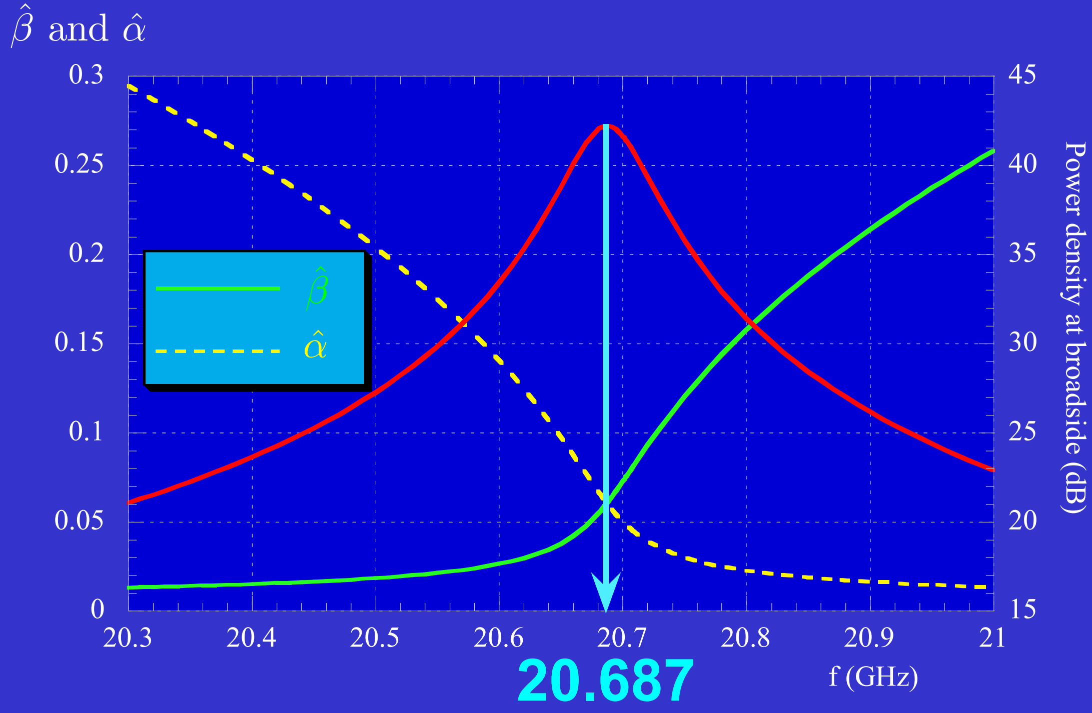
$\bar{B}_S = 20$

Parameters of the structure:

$\varepsilon_r = 2.2,\ \tan\delta_e = 10^{-3},\ \mu_r = 1$

$h = 5\ \text{mm},\ h_s = h/2$

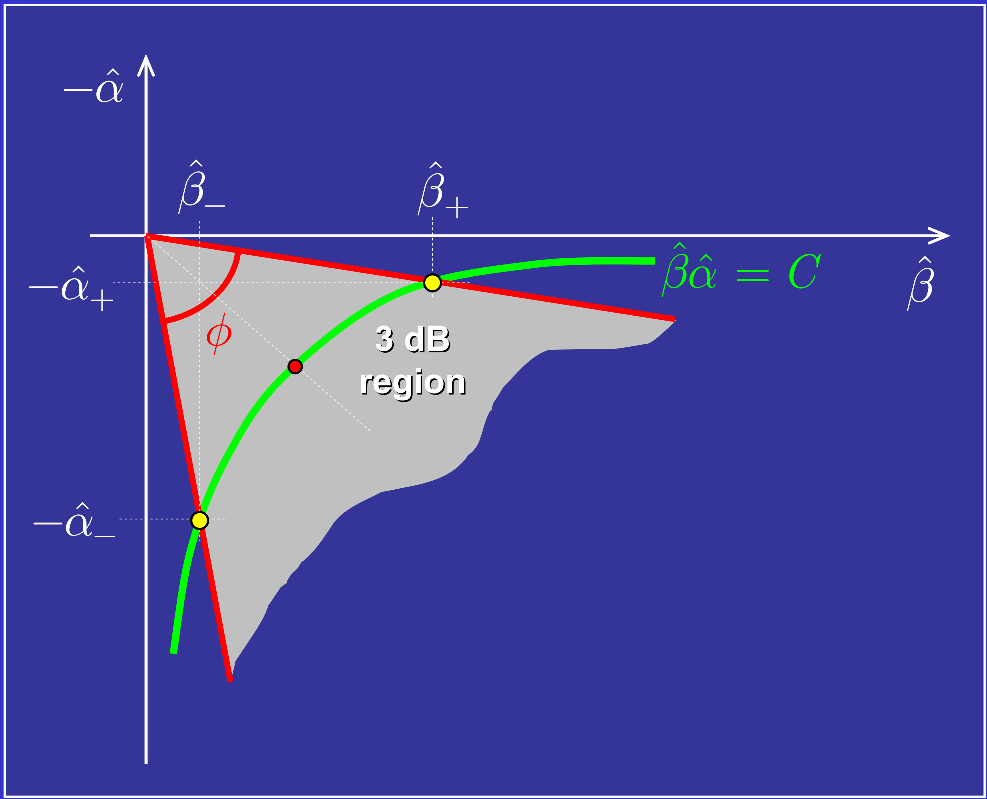
$\bar{B}_S = 20$



Geometrical Characterization of the '3 dB Region'(1)

Determines the location of the pole on the hyperbola $\hat{\beta}\hat{\alpha} = C$ that gives rise to a power density at broadside **3 dB lower** than its maximum value

$$\begin{cases} \dfrac{\left|\hat{R}_0\right|^2}{\left(\hat{\beta}^2 + \hat{\alpha}^2\right)^2} = \dfrac{1}{2}\dfrac{\left|\hat{R}_0\right|^2}{4C^2} \\ \hat{\beta}\hat{\alpha} = C \end{cases} \Rightarrow \begin{array}{l} \hat{\beta}_+ = \sqrt{C}\sqrt{\sqrt{2}+1}, \quad \hat{\alpha}_+ = \dfrac{\sqrt{C}}{\sqrt{\sqrt{2}+1}} \\ \hat{\beta}_- = \sqrt{C}\sqrt{\sqrt{2}-1}, \quad \hat{\alpha}_- = \dfrac{\sqrt{C}}{\sqrt{\sqrt{2}-1}} \end{array}$$



These determine an angular region in the complex $\hat{k}_x$ plane (the '*3 dB region*') where the level of power density at broadside is not more than 3 dB below its maximum

$$\phi = \frac{\pi}{4}$$

$$\mathrm{FBW} \simeq \tan\delta_{eq} = \frac{2}{\pi \overline{B}_S^2}\sqrt{\frac{\varepsilon_r'}{\mu_r}} + \tan\delta_\varepsilon$$

Geometrical Characterization of the ‘3 dB Region’ (2)

Parameters of the structure:

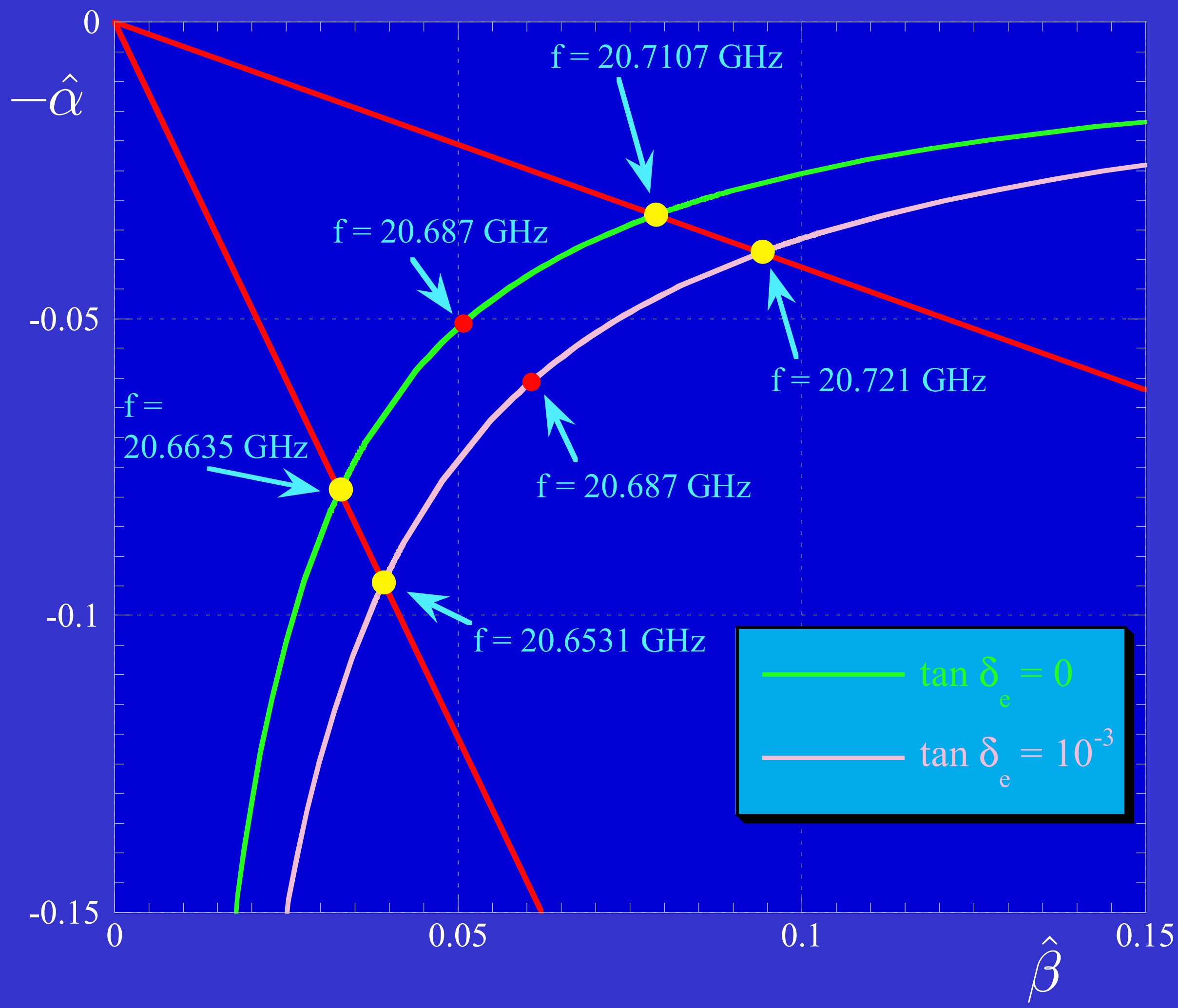
$\varepsilon_r = 2.2, \ \mu_r = 1$

$h = 5 \text{ mm}, \ h_s = h/2$

$\bar{B}_S = 20$

Pole loci in the complex $\hat{k}_x$ plane
are reported for both the lossless and the lossy case

By varying frequency,
the pole trajectories are hyperbolas
in the fourth quadrant of the $\hat{k}_x$ plane



The red circles
indicate the *maximum points*
(exactly on the $\hat{\beta} = \hat{\alpha}$ line)

The yellow circles
indicate the *3 dB points*
(exactly on the edges of
the 3 dB region)

Interpretation of the Optimization Condition (1)

__Hypothesis__: a leaky mode with equal values of its phase and attenuation constants is supported by the considered structure $\Longrightarrow$ $\hat{k}_{xLW} = \hat{\beta} - j\hat{\alpha} = \delta(1-j)$

$\hat{k}_{xLW}$ is solution of the dispersion equation: $\left(\bar{Y}_0 + j\bar{B}_S\right)\sin\left(k_0\hat{k}_{z1}h\right) - j\bar{Y}_1\cos\left(k_0\hat{k}_{z1}h\right) = 0$

$$\begin{cases} \bar{Y}_0 = \hat{k}_{z0} = \sqrt{1-\hat{k}_{xLW}^2} \\ \bar{Y}_1 = \dfrac{\hat{k}_{z1}}{\mu_r} = \dfrac{\sqrt{\varepsilon_r\mu_r - \hat{k}_{xLW}^2}}{\mu_r} \end{cases}$$

In the limit of large $\bar{B}_S$ and small $\tan\delta_e$:

$$\cot\left(k_0\sqrt{\mu_r\varepsilon_r'}h\right) = \sqrt{\frac{\mu_r}{\varepsilon_r'}}\bar{B}_S$$

$$\beta \cong \alpha \cong \sqrt{\underbrace{\frac{\varepsilon_r'}{k_0 h \bar{B}_S^2}}_{\alpha_{rad}^2} + \underbrace{\frac{1}{2}\mu_r\varepsilon_r''}_{\alpha_{loss}^2}}$$

In the lossless case it coincides with the one given in ***Zhao et al., Digest AP-S Int. Symp. 2003, pp. 1134-1137***

Interpretation of the Optimization Condition (2)

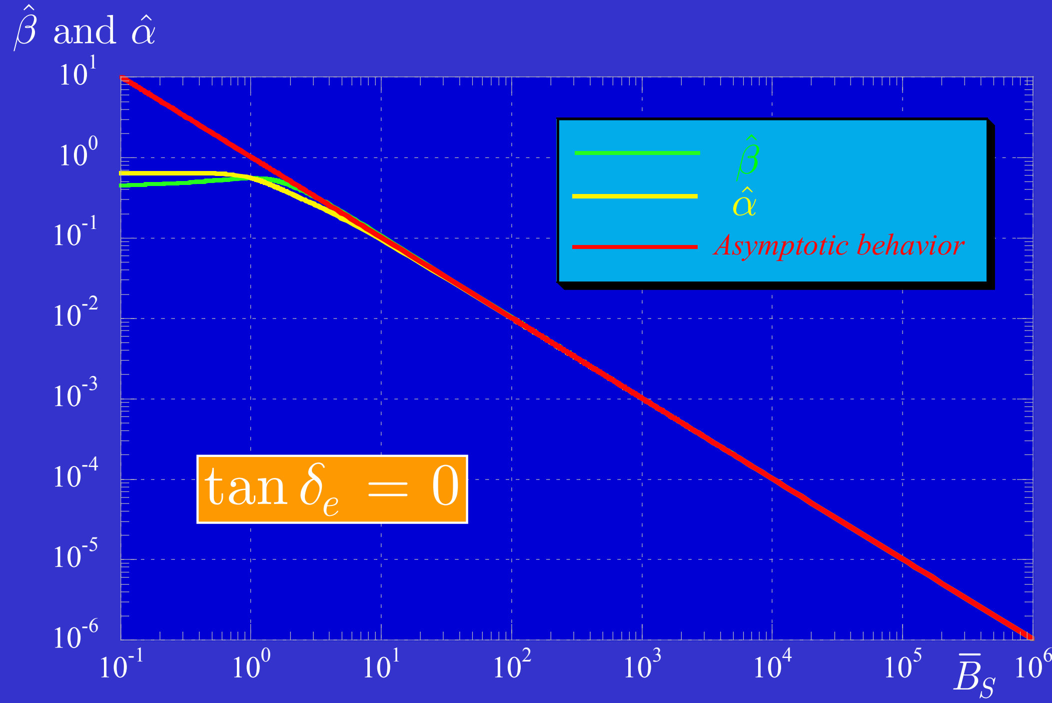
Parameters of the structure:

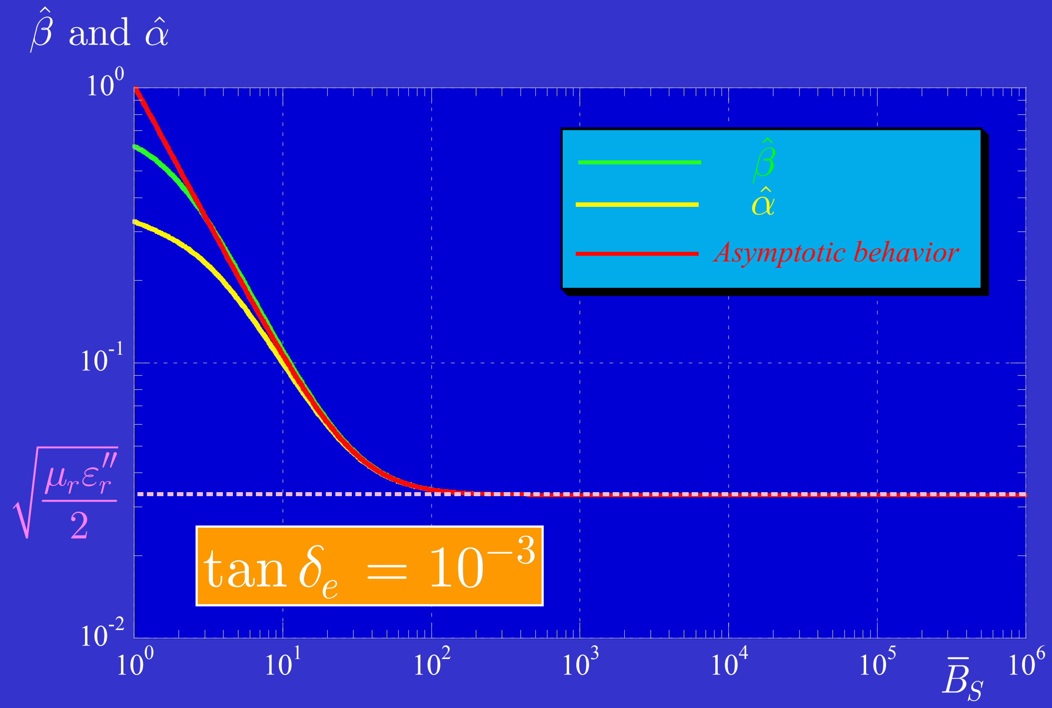
$\varepsilon_r' = 2.2,\ \mu_r = 1$

$f = 10\ \text{GHz}$

For each value of $\overline{B}_S$
the slab height h is designed
according to the optimization condition:

$$\cot\left(k_0\sqrt{\mu_r\varepsilon_r'}h\right) = \sqrt{\frac{\mu_r}{\varepsilon_r'}}\overline{B}_S$$





Maximization of Power Density in the Lossy Case (1)

In the lossless case, the power density radiated at broadside by an optimized structure, considered as a function of $\bar{B}_S$, increases indefinitely as $\bar{B}_S \to \infty$

In fact: $\hat{\beta} = \hat{\alpha} \to 0 \qquad P(0) \propto \dfrac{\left|\hat{R}_0\right|^2}{\left(\hat{\beta}^2 + \hat{\alpha}^2\right)^2}$

When losses are present in the substrate, the relevant leaky-mode attenuation constant does not tend to zero when $\bar{B}_S \to \infty$. Since the structure tends to become a closed PPW with PEC walls when $\bar{B}_S \to \infty$, the radiation efficiency of the antenna tends to zero.

An optimum value for $\bar{B}_S$ is to be expected

By considering the exact far field radiated by an electric-line source, under the optimization condition, and in the limit of small $\tan\delta_e$ and large $\bar{B}_S$:

$$\bar{B}_S^{opt} = \sqrt{\frac{2}{\pi \tan\delta_e}\sqrt{\frac{\varepsilon_r'}{\mu_r}}}$$

Maximization of Power Density in the Lossy Case (2)

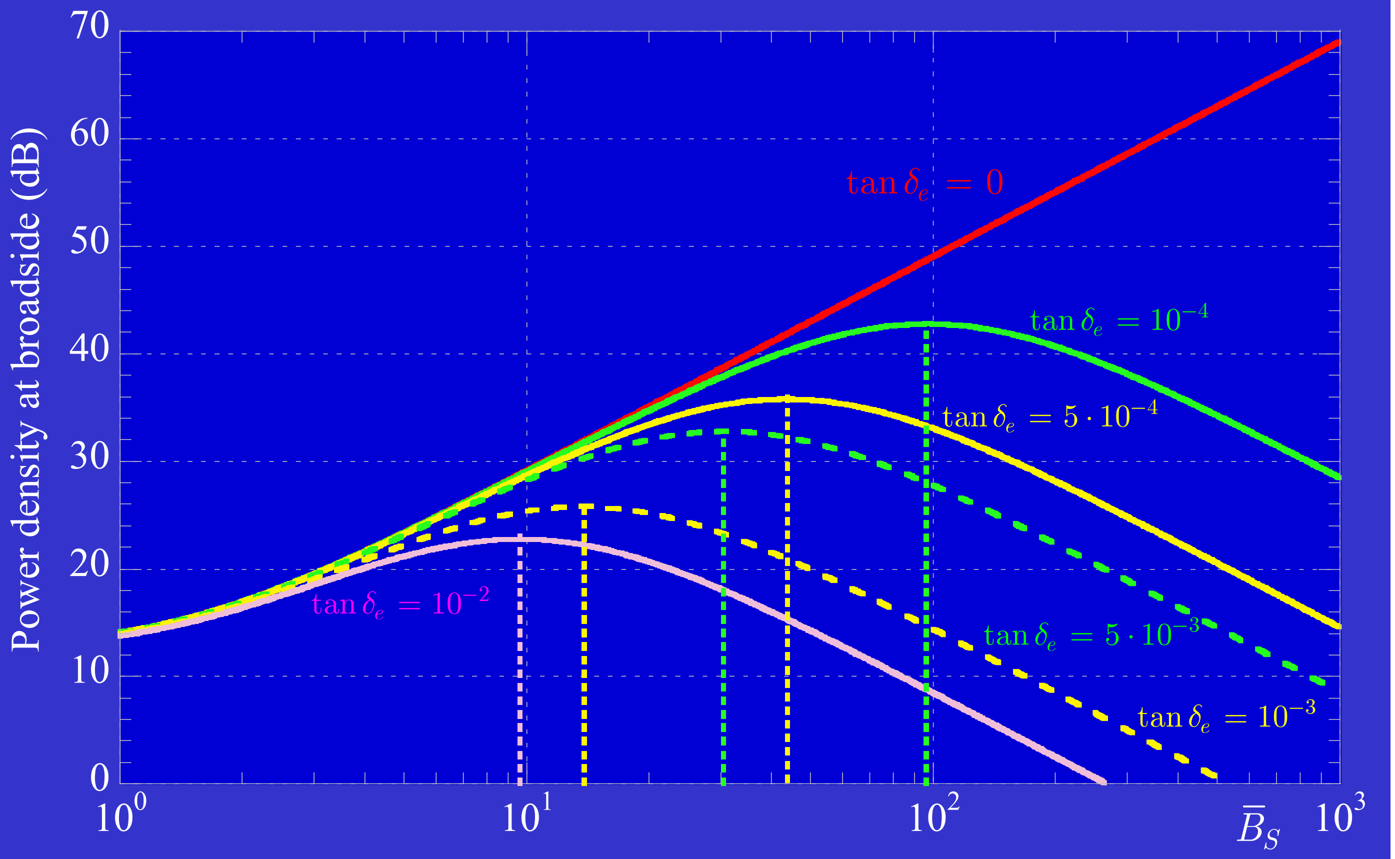
Parameters of the structure:

$\varepsilon_r' = 2.2,\ \mu_r = 1$

$f = 10$ GHz

$h_s = 1$ mm

For each value of $\bar{B}_S$ the slab height h is designed according to the optimization condition



- The optimum value of $\bar{B}_S$ increases by decreasing the value of $\tan\delta_e$
- The maximum value of the power is proportional to $1/\tan\delta_e$

Extension to the 2D Case (1)

Observation:

The far-field pattern of a 1 $A \cdot m$ electric dipole on its broadside plane (*H* plane) is the same as the far-field pattern of a 1A electric line source

We aim at showing that the far fields radiated by the electric dipole in both principal planes (*E* and *H* planes) are *the same* in a neighborhood of broadside at frequencies close to the optimum frequency derived for the 1D case

- For large values of $\bar{B}_S$
- At frequencies close to the optimum frequency

$\text{At } f_{opt} \text{ and } f_{3dB}^{-}$ same beamwidth

$\text{At } f_{3dB}^{+}$ same pointing angle, peak-field value, and beamwidth

The shapes of the radiation patterns in the two principal planes are *the same* in the neighborhood of broadside

Extension to the 2D Case (2)

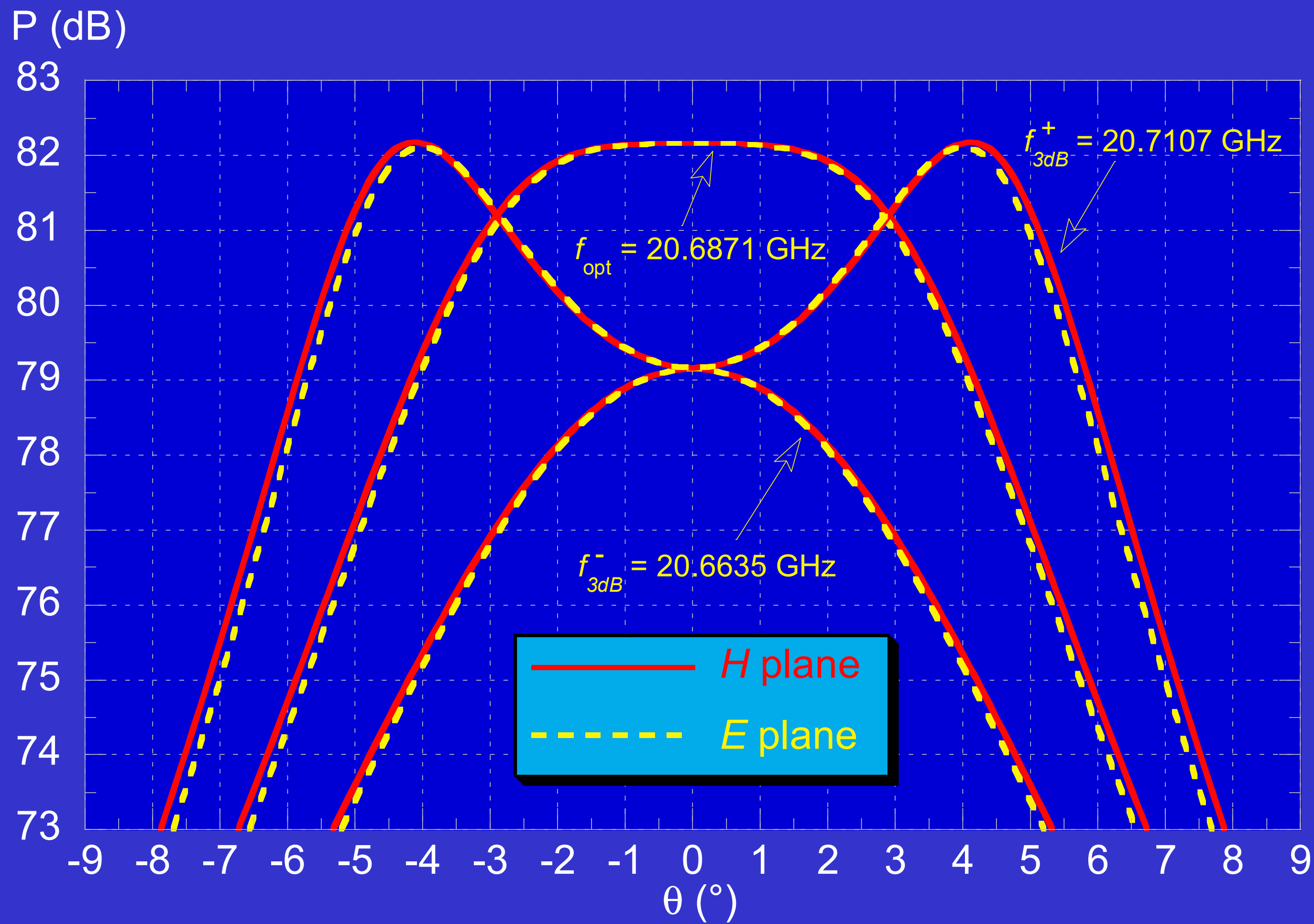
Parameters of the structure:

$$\varepsilon_r = 2.2,\ \mu_r = 1$$

$$h = 5\ \text{mm},\ h_s = h/2$$

$$\bar{B}_S = 20$$

Even though the pattern shapes and the level of power density at broadside change considerably in the shown frequency range, the patterns in the principal planes are almost superimposed in the shown angular range



Conclusions

- Various features of radiation at broadside from 1D and 2D uniform LWAs have been illustrated with reference to a class of antennas based on a grounded slab with a cover modeled as a PRS

- A fundamental condition is $\beta = \alpha$
  - Determines the splitting of a single beam pointing exactly at broadside into a beam with two distinct peaks pointing off broadside
  - Gives rise to a maximum value of radiated power density at broadside
  - Has been used to derive a design formula for the considered structures and approximate asymptotic expression for the leaky-mode propagation constant

- In the lossy case, an optimum value of the shunt susceptance equivalent to the PRS has been derived to achieve maximum power density at broadside

- The frequency range inside which the power density at broadside is not more than 3 dB lower than its maximum has been geometrically characterized in terms of the location of the leaky poles in the complex $\hat{k}_x$ plane and a closed form expression for the fractional bandwidth has been obtained