



European School of Antennas

“High-frequency Techniques and Travelling-wave antennas”

LINEAR ARRAYS OF LEAKY-WAVE LINE SOURCES

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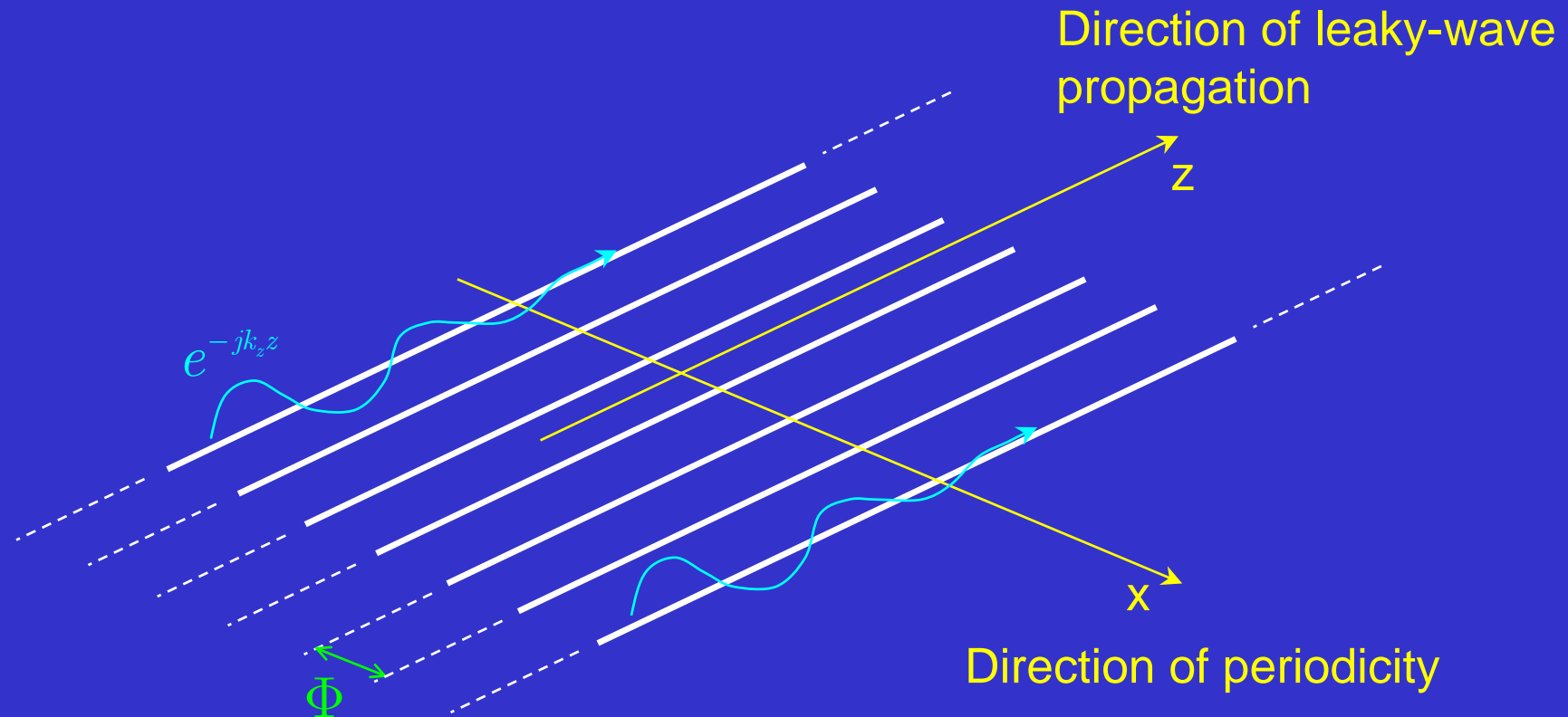
“La Sapienza” University of Rome
Roma, February 25th, 2005

SUMMARY

- Linear arrays of leaky-wave antennas: **description** and examples
- Array **analysis** in the unit cell; main radiative features (effect of a phase-shift between elements, conical beam scanning, grating lobes)
- An example: linear arrays of **microstrip** LWAs

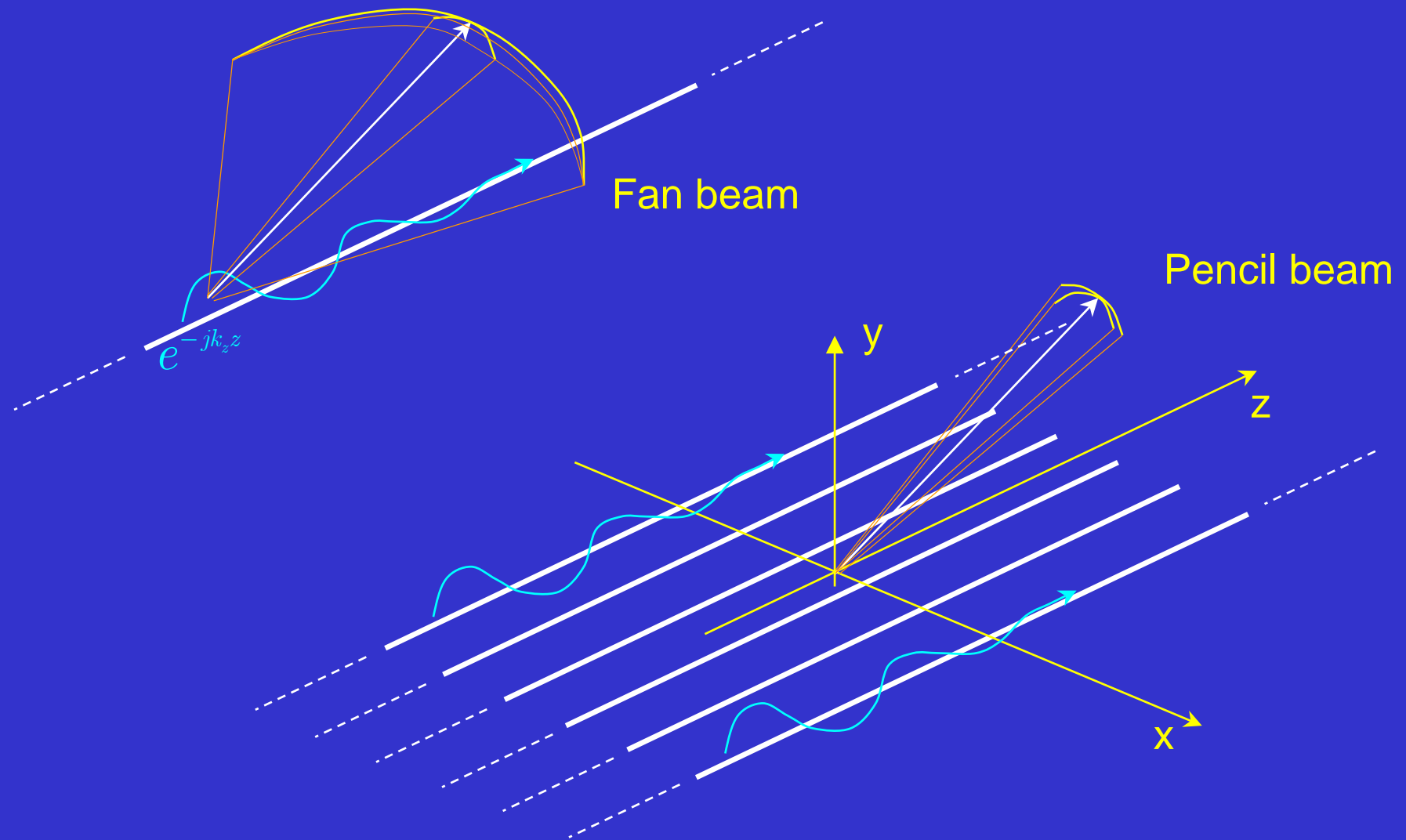
LINEAR ARRAYS OF LEAKY-WAVE LINE SOURCES: DESCRIPTION AND EXAMPLES

GEOMETRY OF THE IDEAL ARRAY

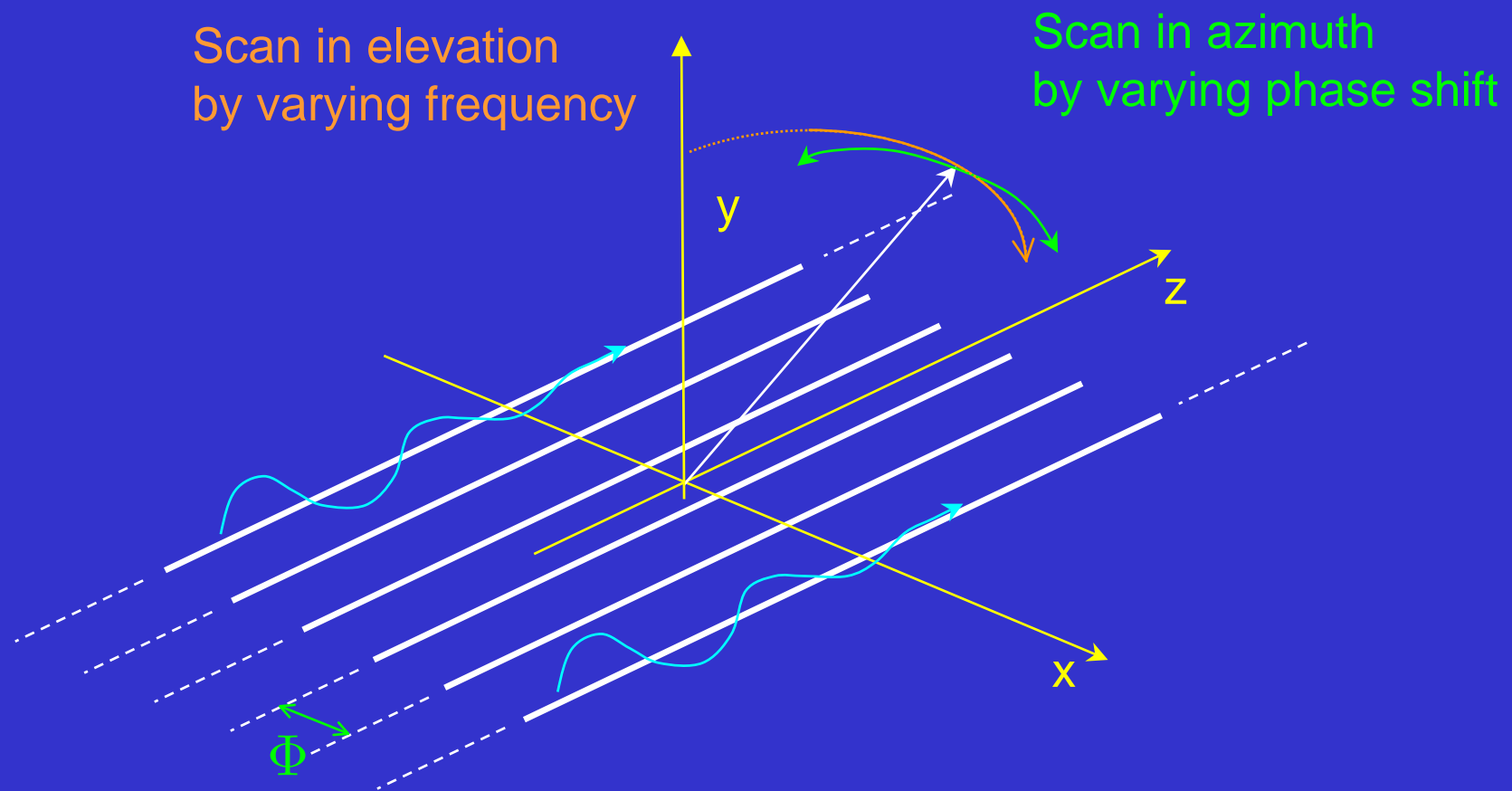


Phase shift
between adjacent elements

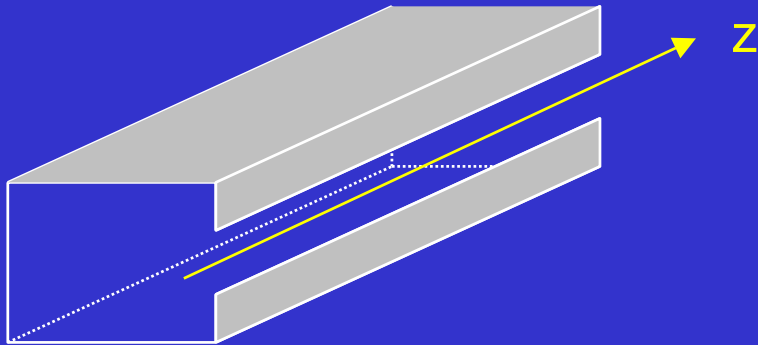
OPERATING PRINCIPLE (1)



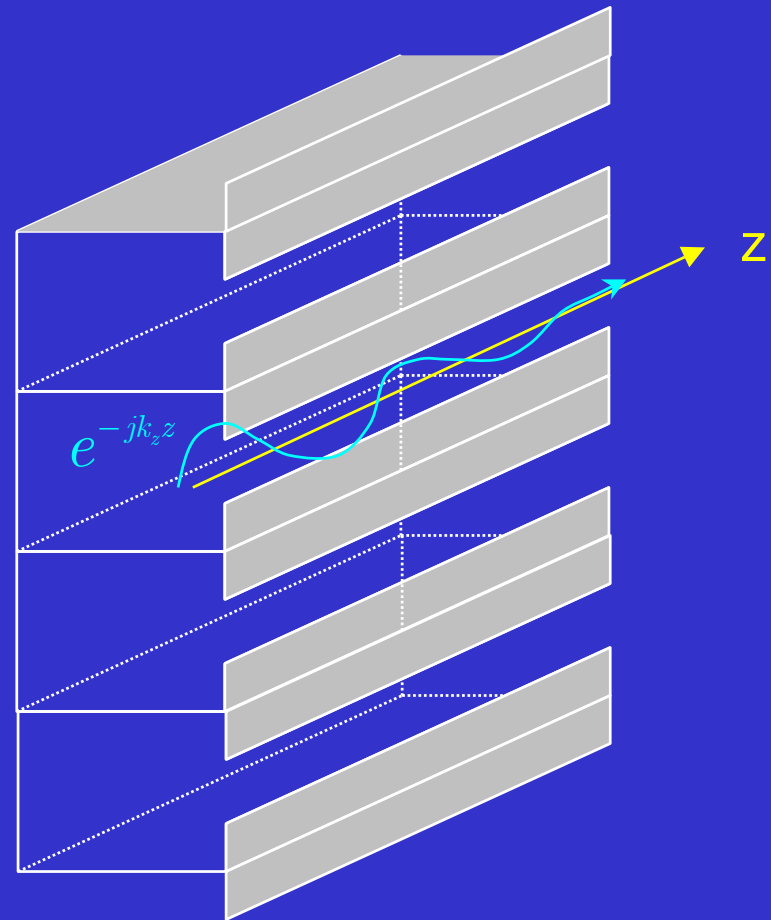
OPERATING PRINCIPLE (2)



EXAMPLES (1)

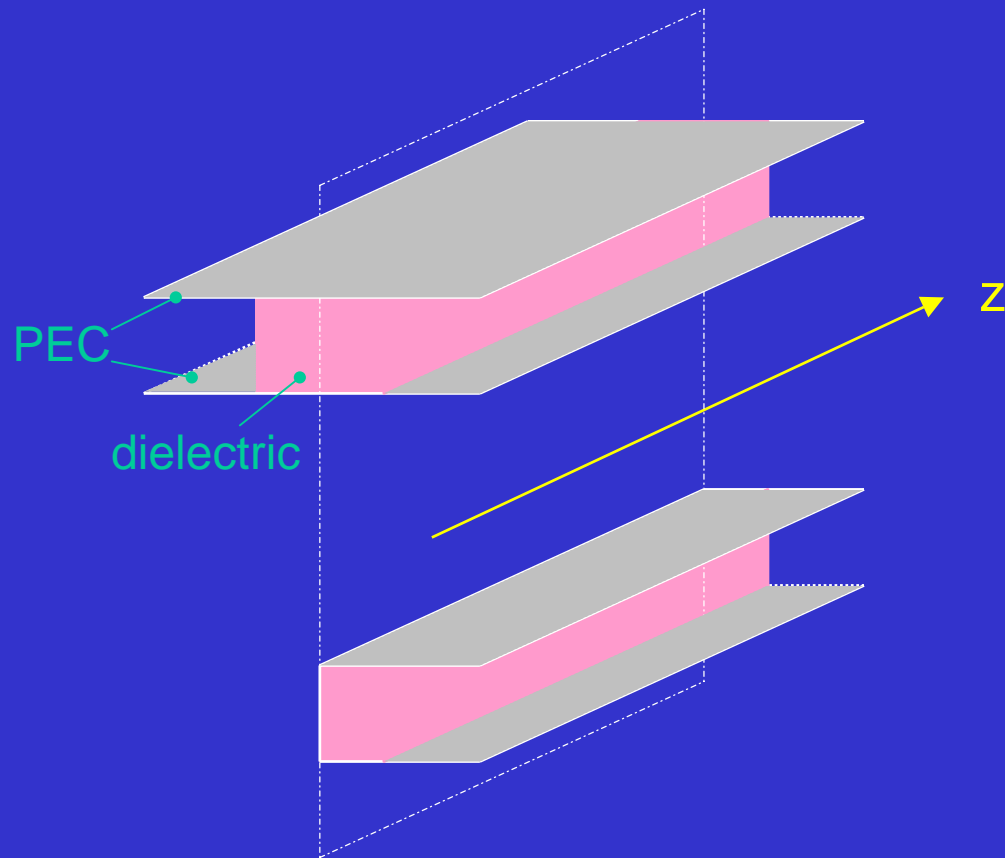


Slotted rectangular waveguide

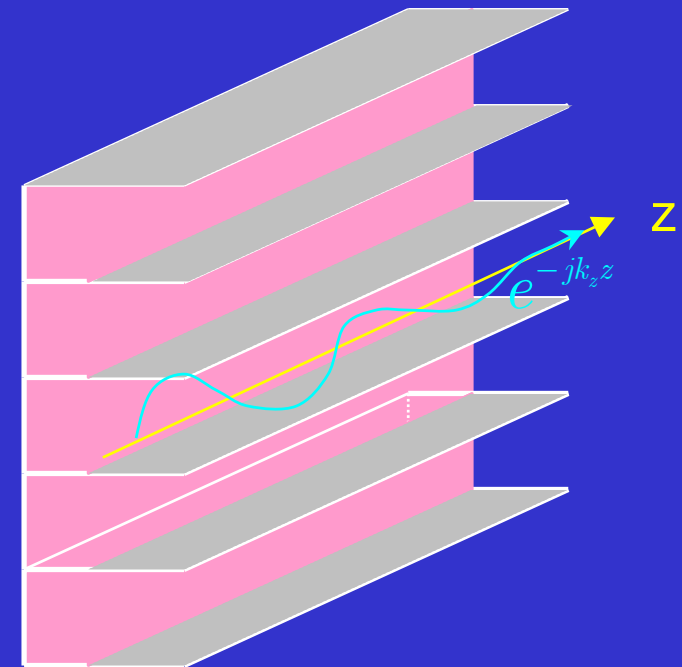


Linear array

EXAMPLES (2)

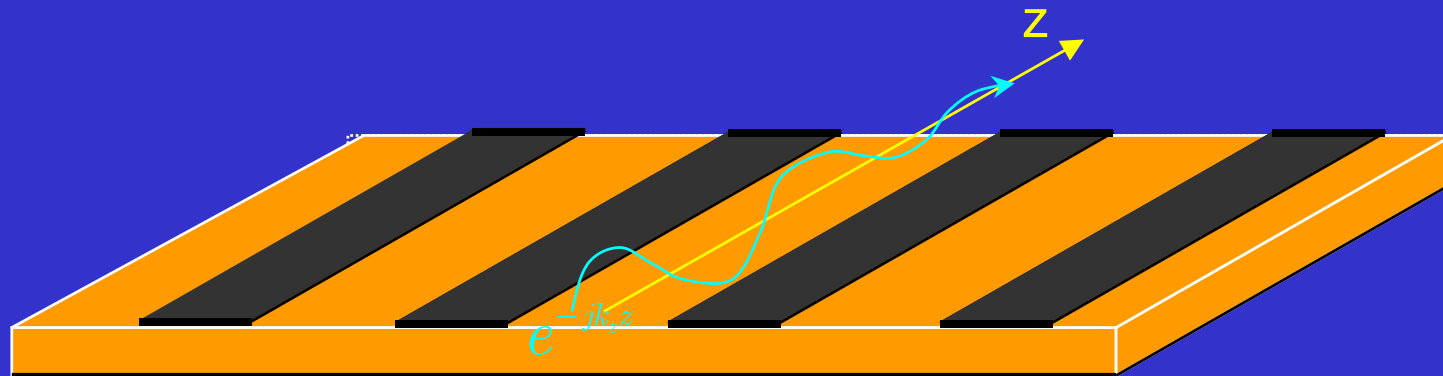
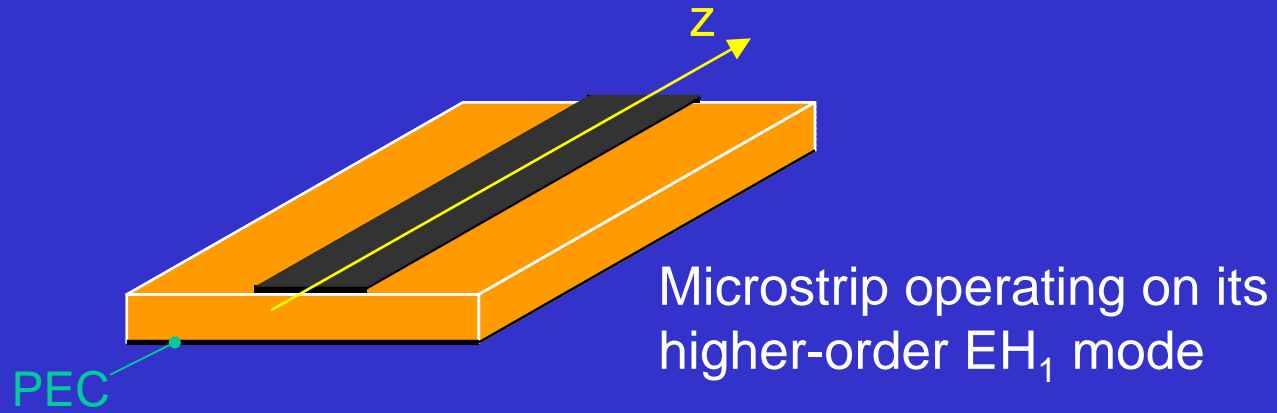


Bisected NRD guide



Linear array

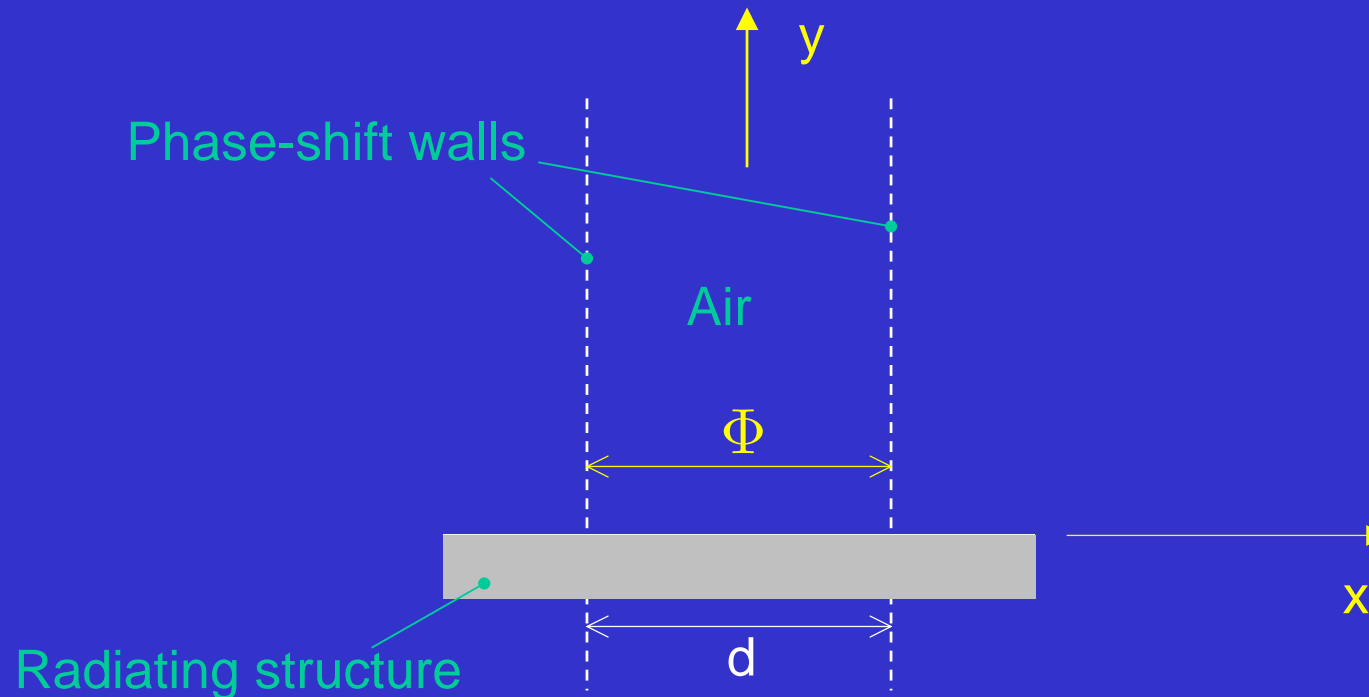
EXAMPLES (3)



Linear array

ARRAY ANALYSIS AND MAIN RADIATIVE FEATURES

UNIT-CELL APPROACH



The phase-shift Φ between adjacent unit cells is established by the excitation. It is **not** an unknown. The unknown is the leaky-wave propagation constant:

$$k_z = k_z(f, \Phi)$$

PROPERTIES OF THE UNIT-CELL WAVEGUIDE

The Floquet expansion in space harmonics corresponds to the modal expansion in a PPW with phase shift walls

$$\mathbf{E} = e^{-jk_z z} \sum_{n=-\infty}^{+\infty} \mathbf{e}_n(y) e^{-j\left(k_{x0} + \frac{2\pi n}{d}\right)} \quad \Phi = k_{x0} d$$

In air: $\mathbf{e}_n(y) = \mathbf{e}_n e^{-jk_{yn}y}$

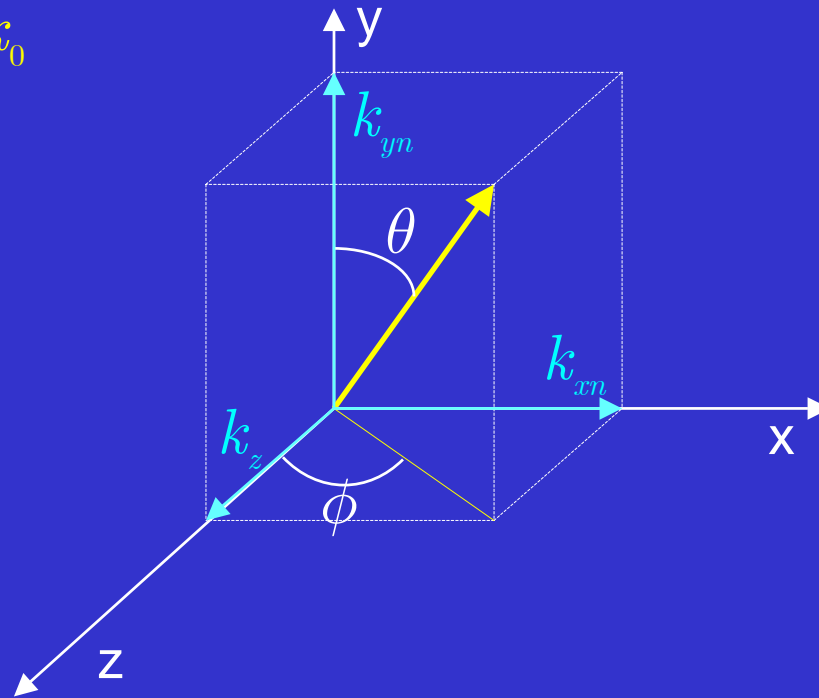
$$k_{yn}^2 = k_0^2 - k_{xn}^2 - k_z^2$$


$$k_z = \beta_z - j\alpha_z$$

Usually $\alpha_z \ll \beta_z$ so that $k_z \simeq \beta_z$

SPACE HARMONICS AS PLANE WAVES

Hyp: $k_z \simeq \beta_z < k_0$



Each mode corresponds to a plane wave propagating at an angle to both the x and z directions

$$k_{yn}^2 = k_0^2 - k_{xn}^2 - k_z^2$$

If $k_{yn}^2 > 0$, then the plane wave propagates in air along y

→ Radiated beam

GRATING LOBES (1)

$$\text{Hyp: } k_z \simeq \beta_z < k_0 \quad \longrightarrow \quad k_{y0}^2 > 0$$

$$\Phi = k_{x0}d = 0$$

We generally wish to operate the system so that only **one beam** is radiated, and we need to know the value of period d to avoid the second mode being above cutoff in the unit-cell waveguide:

$$n = -1 : \quad k_{y,-1}^2 = k_0^2 - k_z^2 - \left(k_{x0} - \frac{2\pi}{d}\right)^2 \leq 0$$

Assuming $k_{x0} > 0$, this is
the next mode
to go above cutoff

$$d \leq \frac{\lambda_0}{\left[1 - \left(\frac{k_z}{k_0}\right)^2\right]^{1/2} + \frac{k_{x0}}{k_0}}$$

GRATING LOBES (2)

Special cases:

When $k_z = 0$ (no leaky-wave propagation along z)

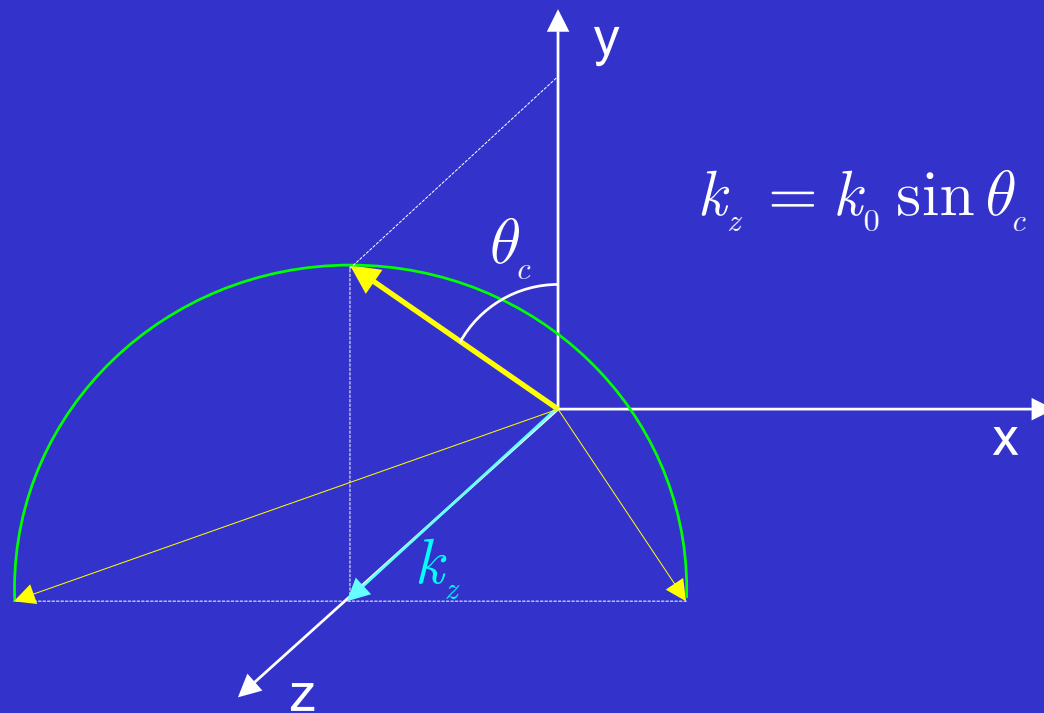
$$d \leq \frac{\lambda_0}{1 + \frac{k_{x0}}{k_0}} \begin{cases} \nearrow d \leq \lambda_0 & (\text{broadside: } k_{x0} = 0) \\ \searrow d \leq \frac{\lambda_0}{2} & (\text{endfire: } k_{x0} = k_0) \end{cases}$$

When $k_z > 0$ (leaky-wave propagation along z) the period may be further increased with respect to the case $k_z = 0$ before a grating lobe appears

$$d \leq \frac{\lambda_0}{\left[1 - \left(\frac{k_z}{k_0}\right)^2\right]^{1/2} + \frac{k_{x0}}{k_0}} > \frac{\lambda_0}{1 + \frac{k_{x0}}{k_0}}$$

CONICAL SCAN (1)

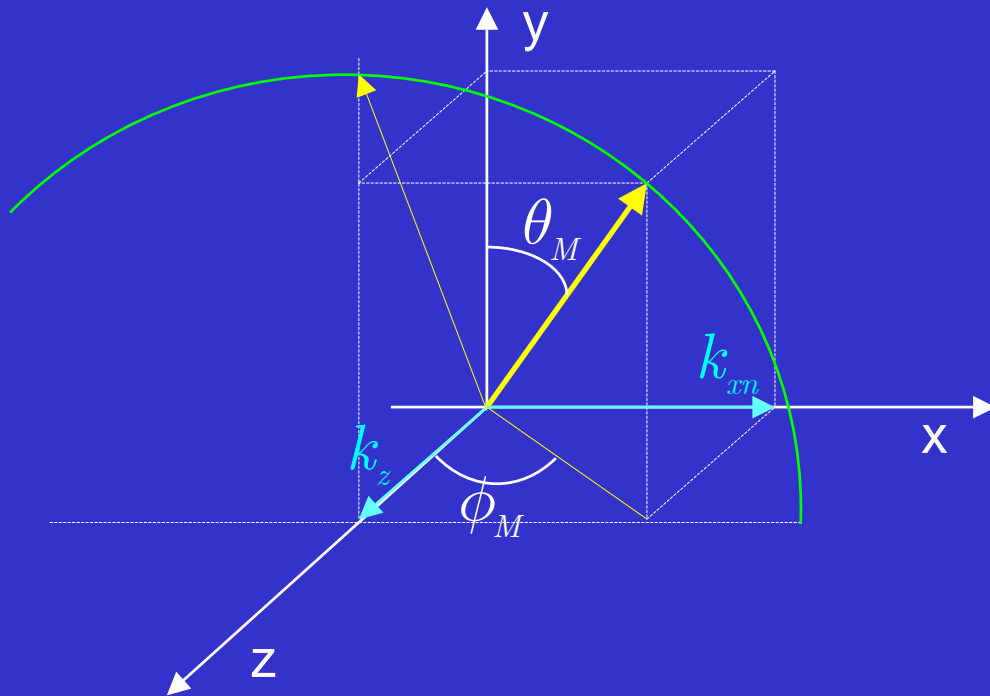
Assuming that the leaky-mode propagation constant k_z does **not** depend on the phase shift Φ :



By varying the phase shift, the beam describes a conical surface

CONICAL SCAN (2)

Pointing angles as a function of the transverse and longitudinal wavenumbers



$$\phi_M = \tan^{-1} \left(\frac{\beta_{xn}}{\beta_z} \right)$$

$$\theta_M = \sin^{-1} \left\{ \left[\left(\frac{\beta_z}{k_0} \right)^2 + \left(\frac{\beta_{xn}}{k_0} \right)^2 \right]^{1/2} \right\}$$

SPACE HARMONICS: SPECTRAL PROPERTIES

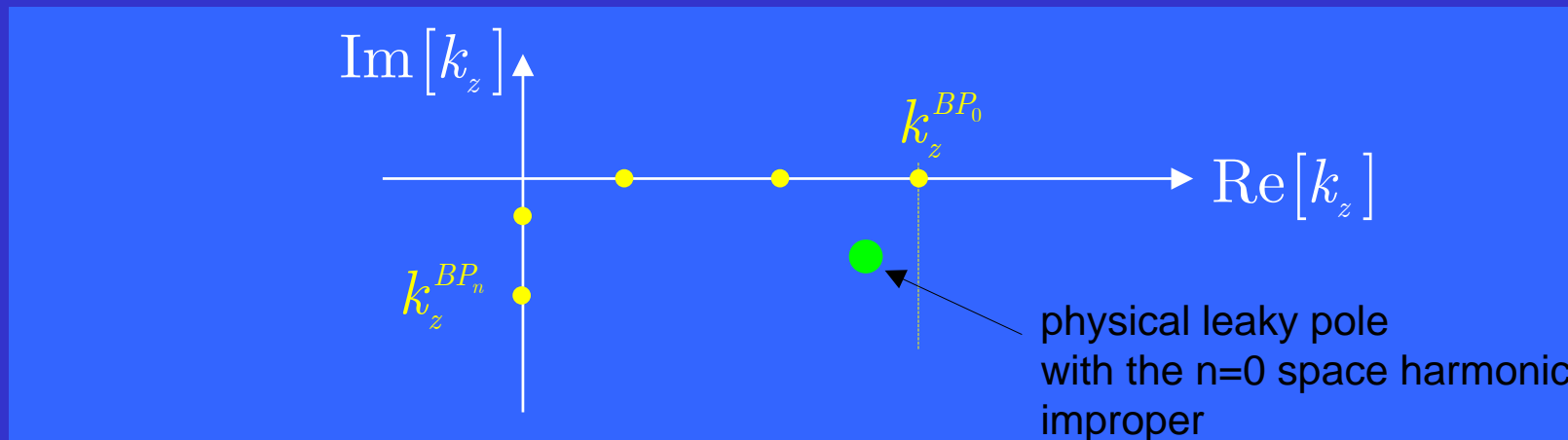
$$k_{yn} = \pm \sqrt{k_0^2 - k_{xn}^2 - k_z^2}$$

\swarrow
 \searrow

$\text{Im}[k_{yn}] < 0$ (proper)
 $\text{Im}[k_{yn}] > 0$ (improper)



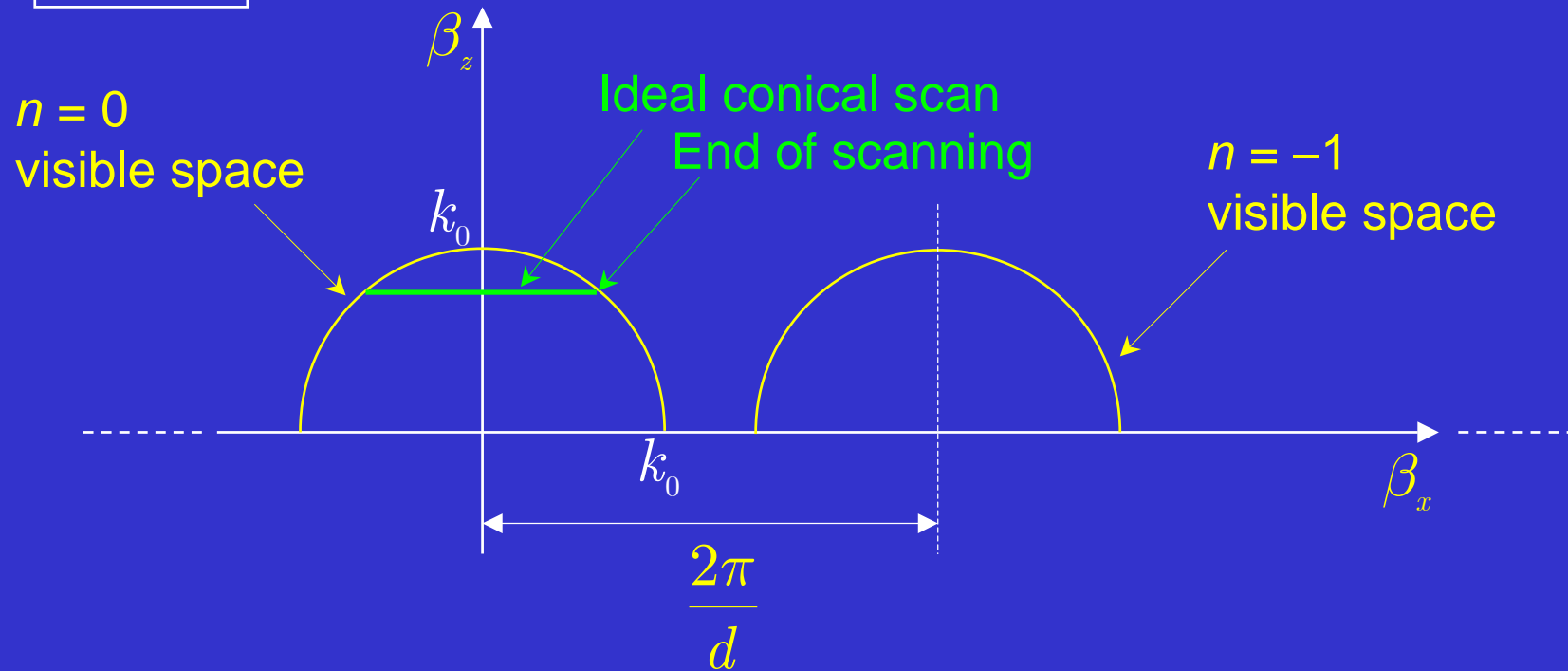
An infinite number of branch points $k_z^{BP_n} = \pm \sqrt{k_0^2 - k_{xn}^2}$



A modal solution is **physical** (i.e., the relevant pole is captured in a SDP representation of the field along the array plane) if its phase constant β_z is less than the real part of all the branch points on the positive real axis associated to the improper space harmonics

THE β_x - β_z PLANE (1)

$$d < \frac{\lambda_0}{2}$$



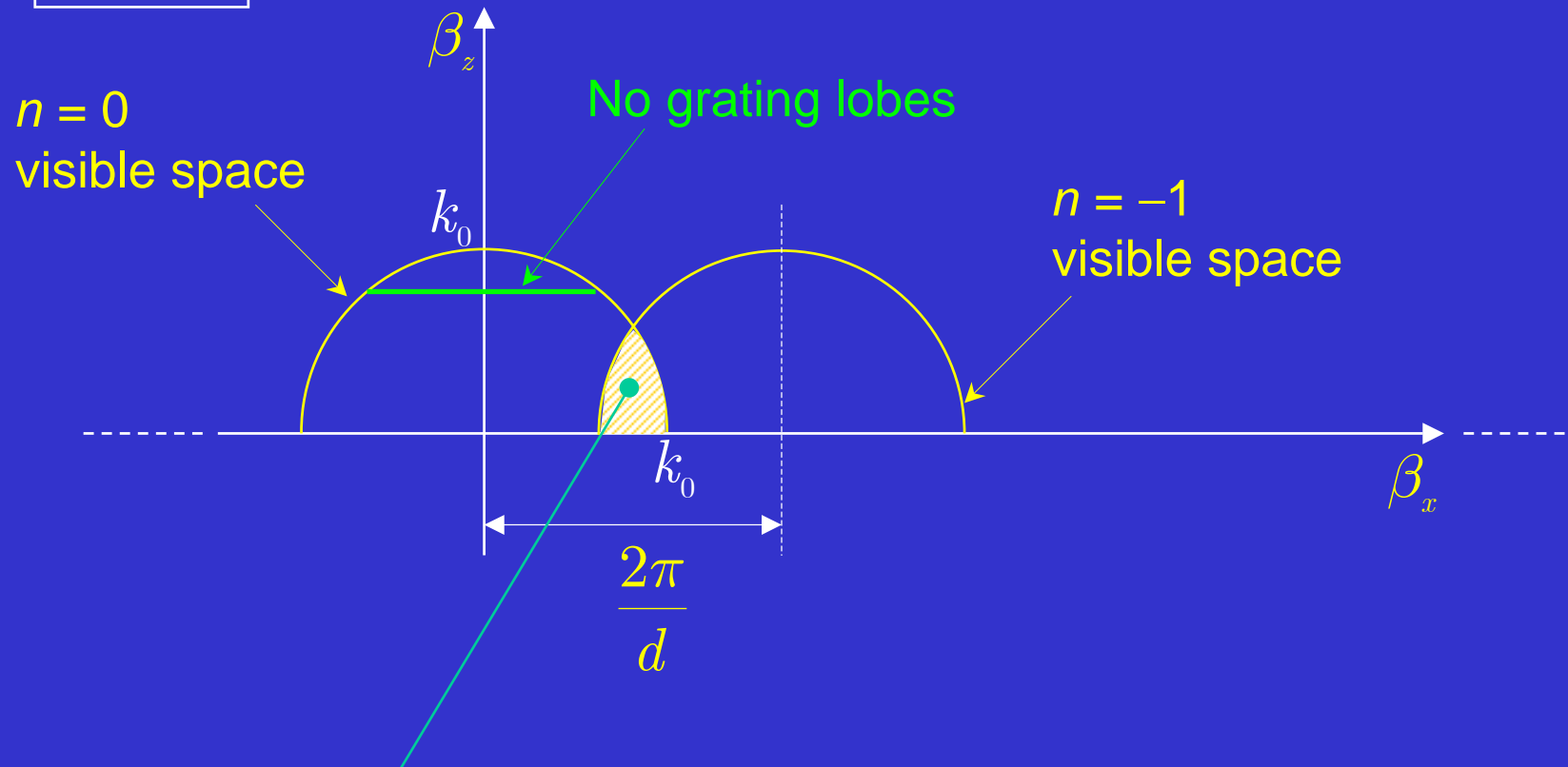
n -th visible space:

$$\beta_{xn}^2 + \beta_z^2 \leq k_0^2 \quad (\text{circles})$$

They are also the loci of the branch points

THE β_x - β_z PLANE (2)

$$d > \frac{\lambda_0}{2}$$

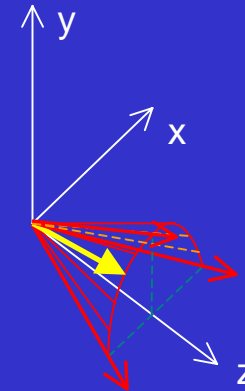
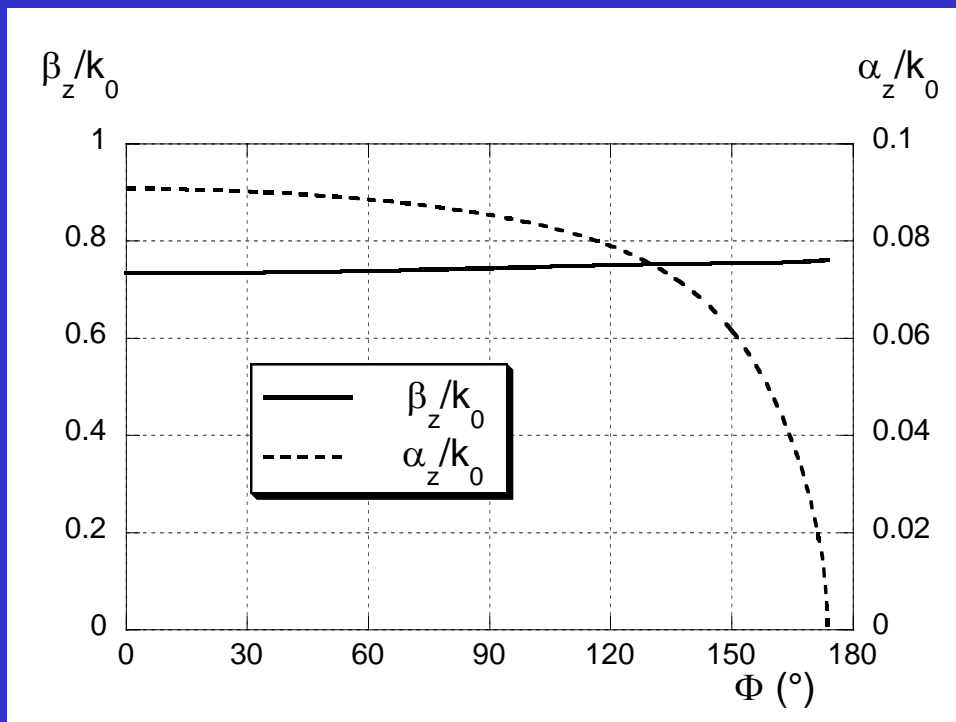


Here a physical leaky mode
(with the $n = 0$ and $n = -1$ space harmonics improper)
radiates **two** beams

LINEAR ARRAYS OF MICROSTRIP LWAs

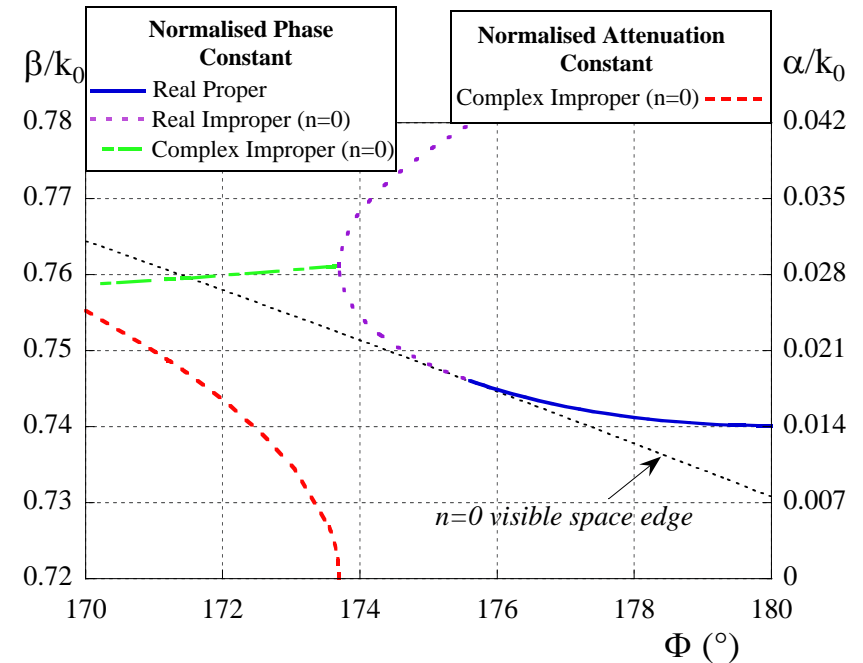
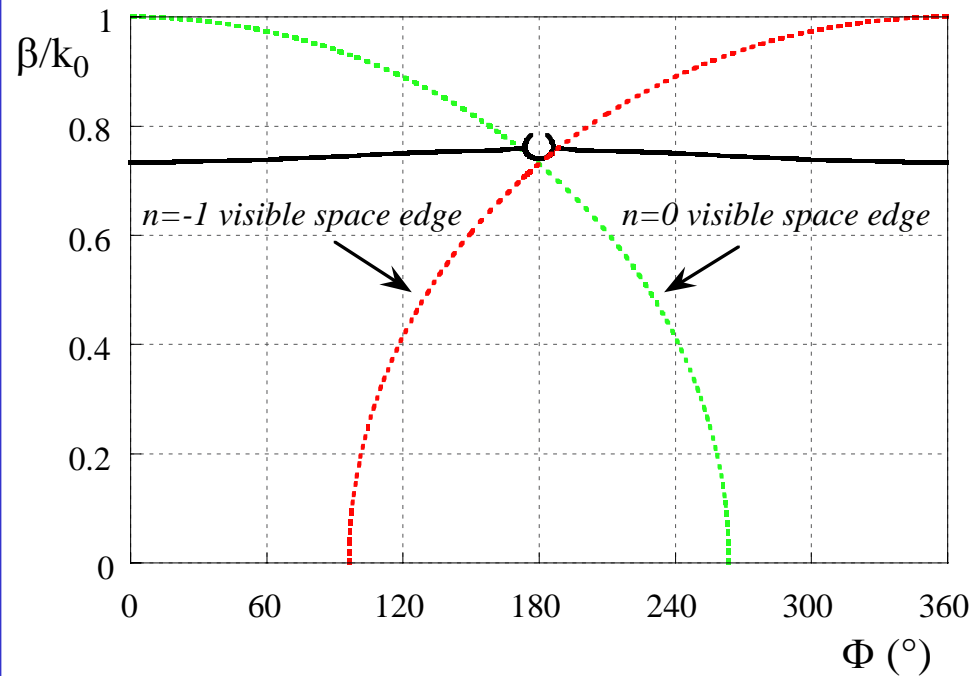
CONICAL BEAM SCANNING

$f = 12.2$ GHz; $d/\lambda_0 = 0.73$; $\epsilon_r = 10.2$; $h = 0.635$ mm



- Nearly conical 2D scanning process
- Progressive lowering of the attenuation constant

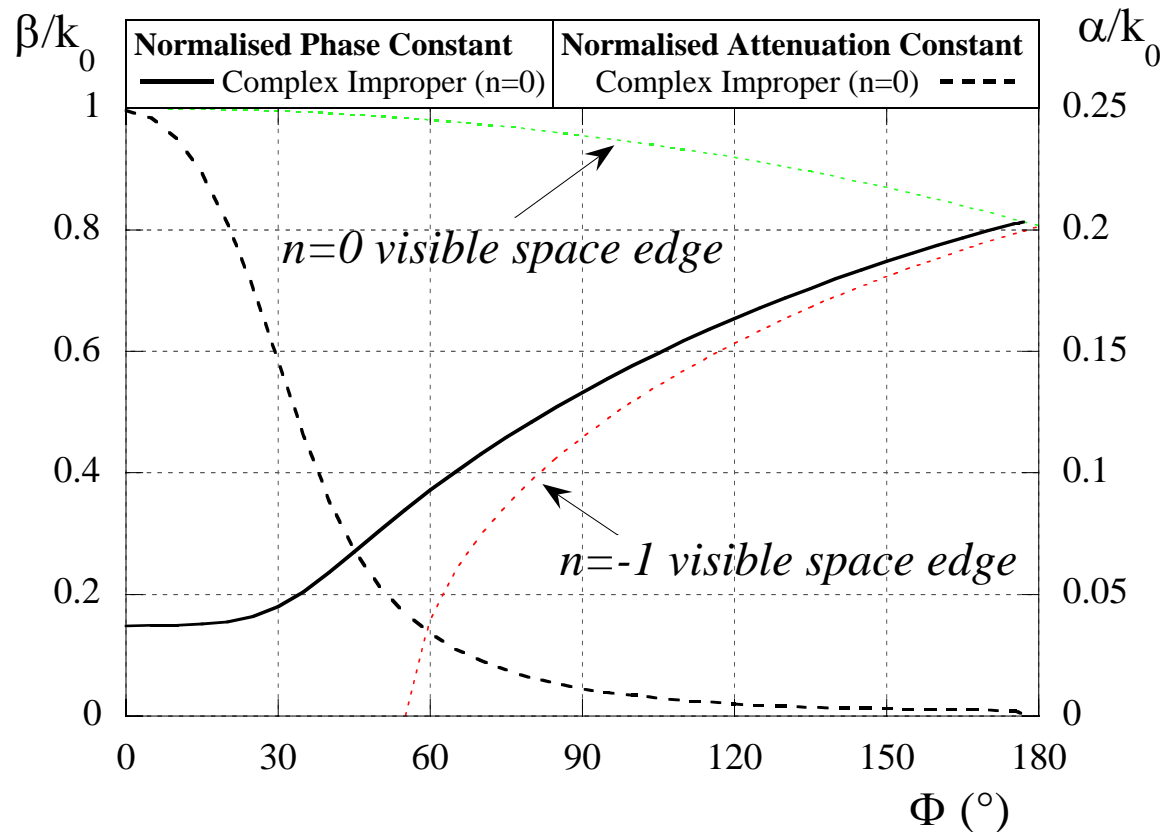
LEAKY-TO-SURFACE WAVE TRANSITION



At the end of scanning, the radiated beam *hits the plane of the array*, and then, after a transition region, it evolves into a *surface wave*, which propagates at an angle with respect to the microstrip lines.

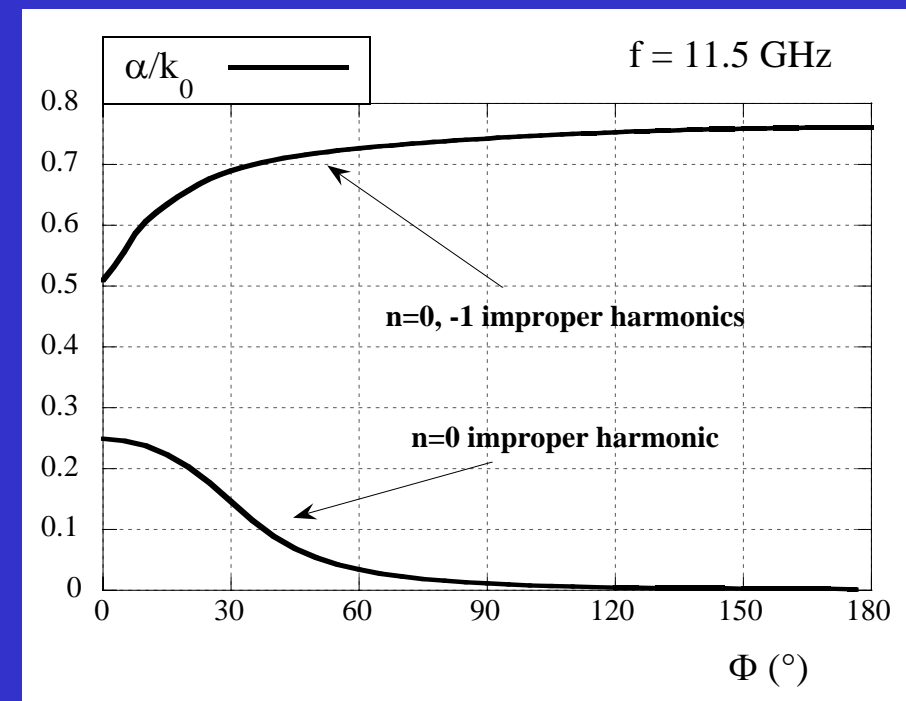
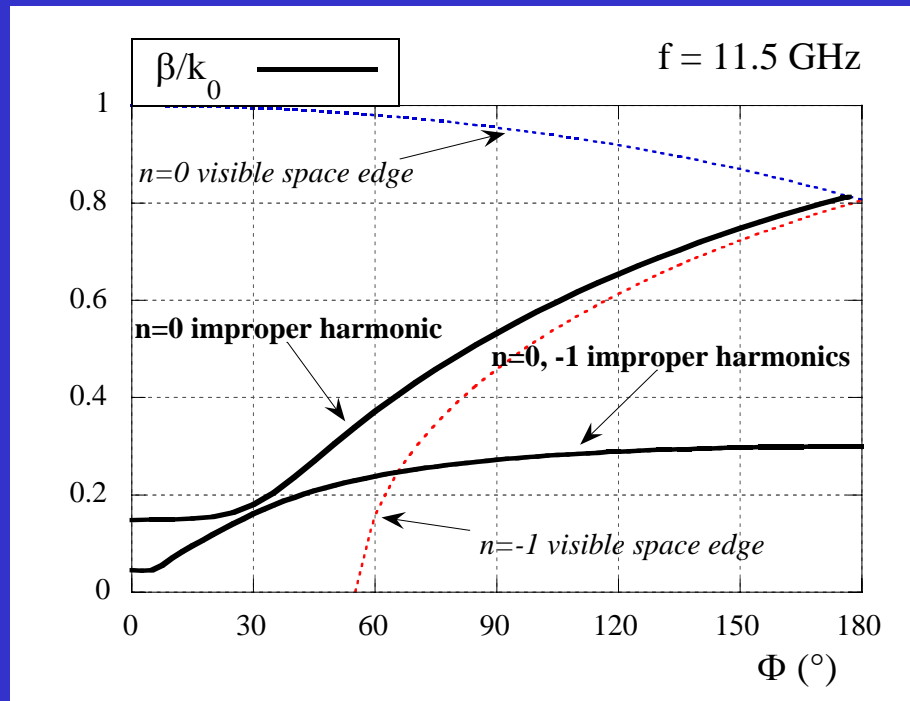
EFFECT OF INCREASING THE ARRAY SPACING

$$f = 11.5 \text{ GHz}; d = 22 \text{ mm}; d/\lambda_0 = 0.84$$



- The phase constant varies as the phase shift is increased
- The phase-constant curve does not enter the visible space of the $n = -1$ spatial harmonic: *there does not seem to be any grating lobe...*

GRATING LOBES: A NEW LEAKY MODE



- A **new leaky mode** appears, with both the $n = 0$ and $n = -1$ spatial harmonics improper
- The new mode is physical only when its phase-constant curve enters the common region between the visible spaces for the $n = 0$ and $n = -1$ spatial harmonics, giving rise to a second radiated beam (grating lobe).

REFERENCES

- A. A. Oliner, Leaky-Wave Antennas, in R. C. Johnson (Ed.), *Antenna Engineering Handbook*, New York: McGraw-Hill, 1993.
- A. A. Oliner, Principal Investigator, *Scannable millimeter-wave arrays*, Polytechnic University, New York, Final Report on RADC Contract no. F19628-84-K-0025, 1988.
- P. Baccarelli, P. Burghignoli, F. Frezza, A. Galli, and P. Lampariello, "Novel modal properties and relevant scanning behaviors of linear arrays of microstrip leaky-wave antennas", *IEEE Trans. Antennas Propagat.*, vol. 51, n. 12, pp. 3228-3238, Dec. 2003.