ACE - European School of Antennas

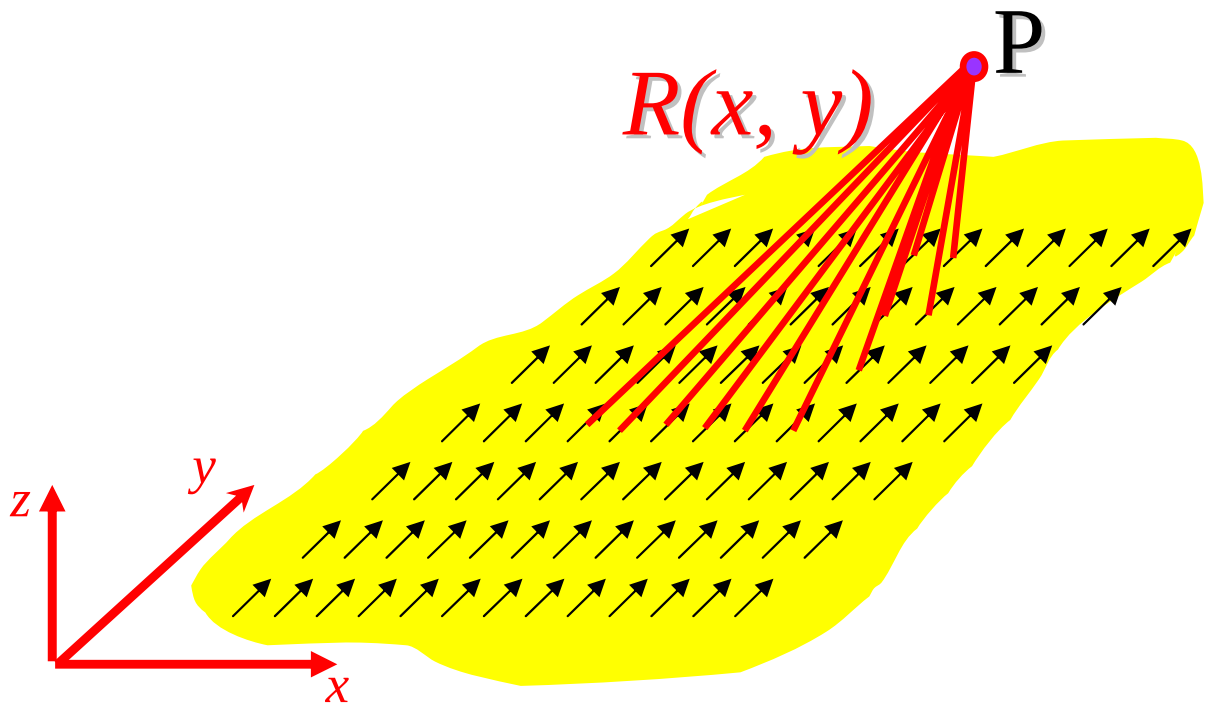
"High-frequency techniques and travelling wave antennas"

Siena – Roma, February 21 – 26, 2005

Truncated FW diffraction and interaction with complex platforms

GREEN'S FUNCTION FOR A PHASED ARRAY OF DIPOLES

- **INFINITE ARRAY GF: ELEMENT BY ELEMENT SUMMATION**



$$A(P) = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} g(P; nd_x, md_y)\, e^{-j(k_{x0} nd_x + k_{y0} md_y)}$$

$g(P;x,y)$ Green's function for the single dipole

Example: in free space

$$g(P;x,y) = \frac{e^{-jkR(x,y)}}{4\pi R(x,y)}$$

- Slowly convergent representation

GREEN'S FUNCTION FOR A PHASED ARRAY OF DIPOLES

- **INFINITE ARRAY GF: FLOQUET WAVE EXPANSION**

$$A(P)=\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} g(P;nd_x,md_y)\,e^{-j(k_{x0}nd_x+k_{y0}md_y)}=\frac{1}{d_x d_y}\sum_{q=-\infty}^{+\infty}\sum_{p=-\infty}^{+\infty} G(\underline{k}_{pq}^{FW})e^{-j\underline{k}_{pq}^{FW}\cdot\underline{r}}$$

$$g(\underline{P};x,y)=\frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G(\underline{k})e^{-j\underline{k}\cdot\underline{r}}dk_x dk_y \qquad G(\underline{k})e^{-j\underline{k}\cdot\underline{r}}=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(\underline{P};x,y)e^{-j\underline{k}\cdot\underline{R}}dxdy$$

$$\sum_{n=-\infty}^{+\infty} f(nd)=\frac{1}{d}\sum_{p=-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(z)e^{-j\frac{2\pi p}{d}z}dz$$

Poisson summation formula

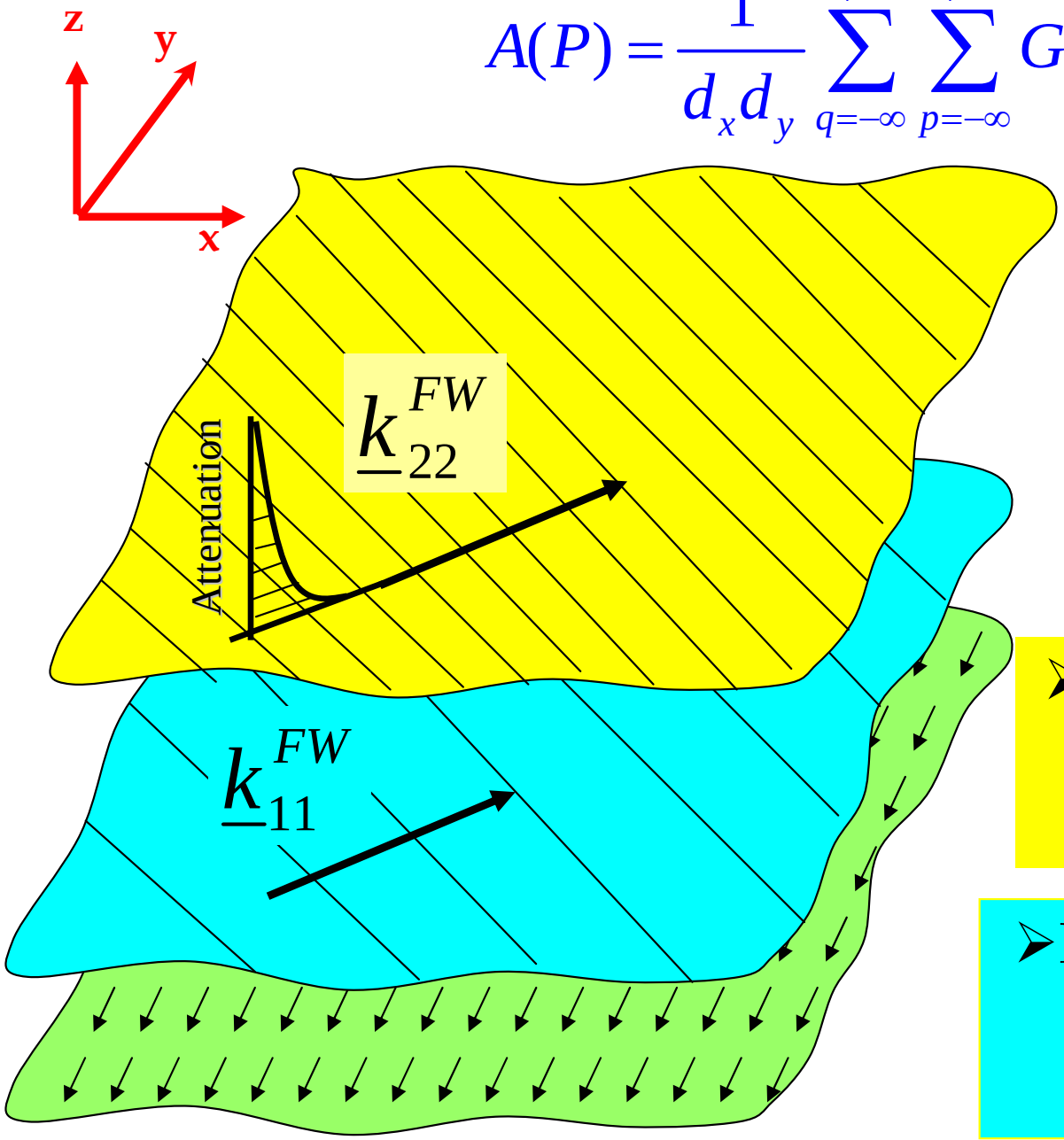
$$\underline{k}_{pq}^{FW}=k_{xp}\hat{x}+k_{yq}\hat{y}+k_{zpq}\hat{z}$$

$$p,q=0,\pm1,\pm2,\ldots$$

$$\begin{cases} k_{xp}=k_{x0}+\dfrac{2\pi p}{d_x} \\ k_{yq}=k_{y0}+\dfrac{2\pi q}{d_y} \\ k_{zpq}=\sqrt{k^2-k_{xp}^2-k_{yq}^2}=\sqrt{k^2-k_{\rho pq}^2} \end{cases}$$

GREEN'S FUNCTION FOR A PHASED ARRAY OF DIPOLES

$$A(P)=\frac{1}{d_x d_y}\sum_{q=-\infty}^{+\infty}\sum_{p=-\infty}^{+\infty}G(\underline{k}_{pq}^{FW})e^{-j\underline{k}_{pq}^{FW}\cdot\underline{r}}$$



- EFW: *Evanescent Floquet waves*

$$k_{zpq}\in C,\quad k_{\rho pq}^2>k^2$$

- PFW: *Propagating Floquet waves*

$$k_{zpq}>0,\quad k_{\rho pq}^2<k^2$$

GREEN'S FUNCTION FOR A PHASED ARRAY OF DIPOLES

- **TRUNCATED ARRAY GF**

The periodicity is broken $\Rightarrow$ the FW basis is not complete

$$\sum_{n=0}^{+\infty} f(nd) = \frac{1}{2} f(0) + \frac{1}{d} \sum_{p=-\infty}^{+\infty} \int_{0}^{+\infty} f(z) e^{-j\frac{2\pi p}{d} z} dz$$

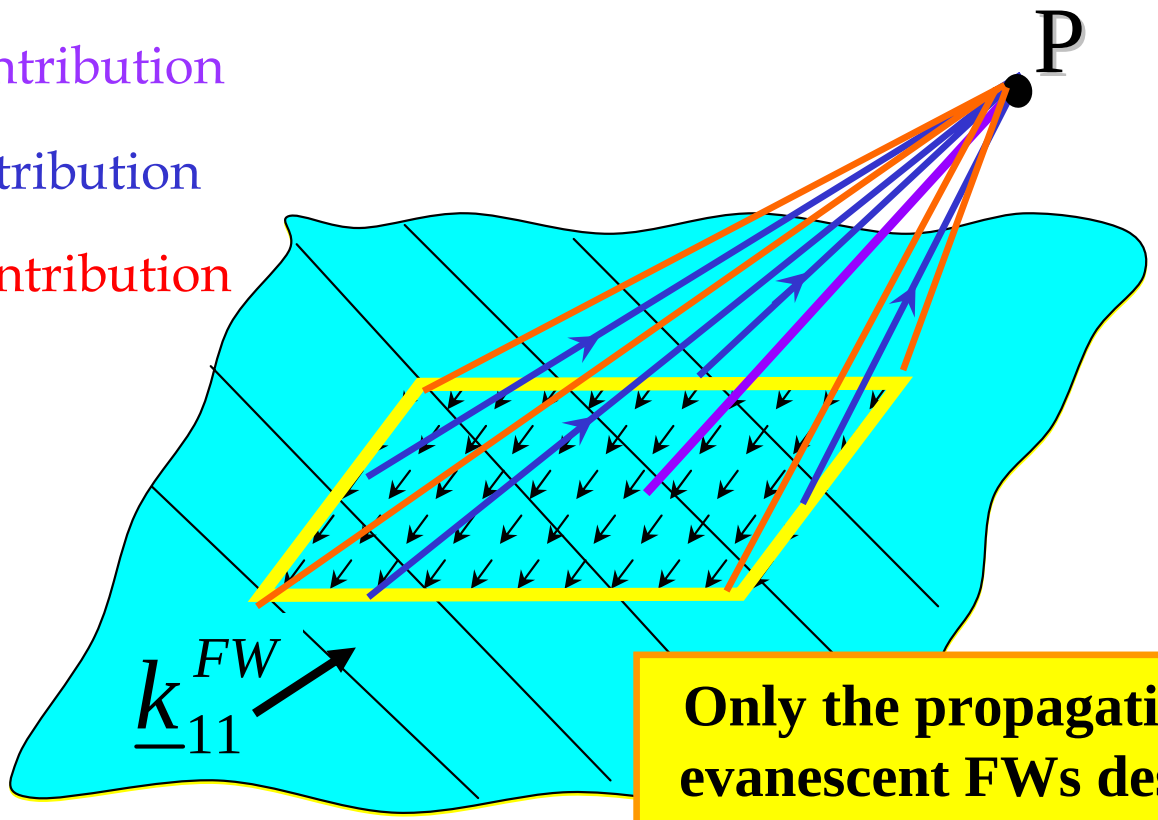
Truncated Poisson summation formula

Radiation integral on the truncated domain

Stationary phase contribution

Edge-end-point contribution

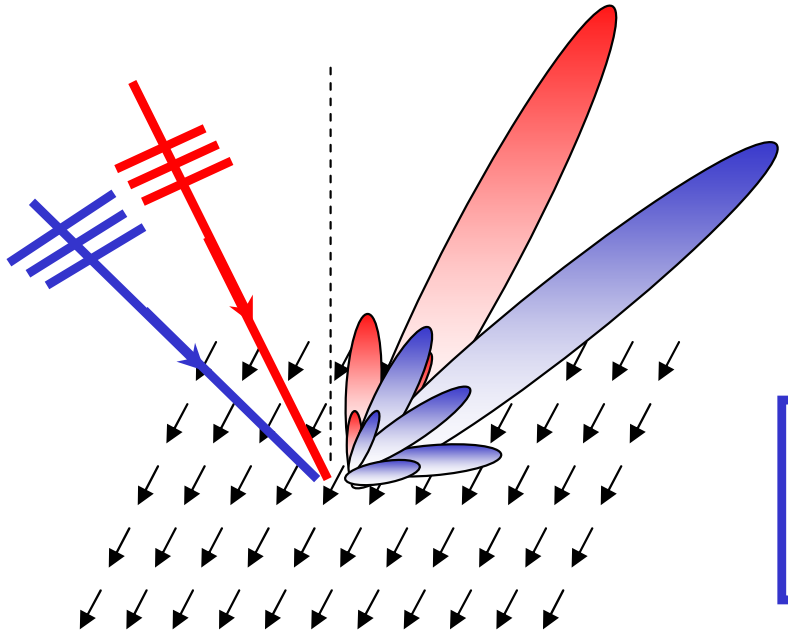
Vertex end-point contribution

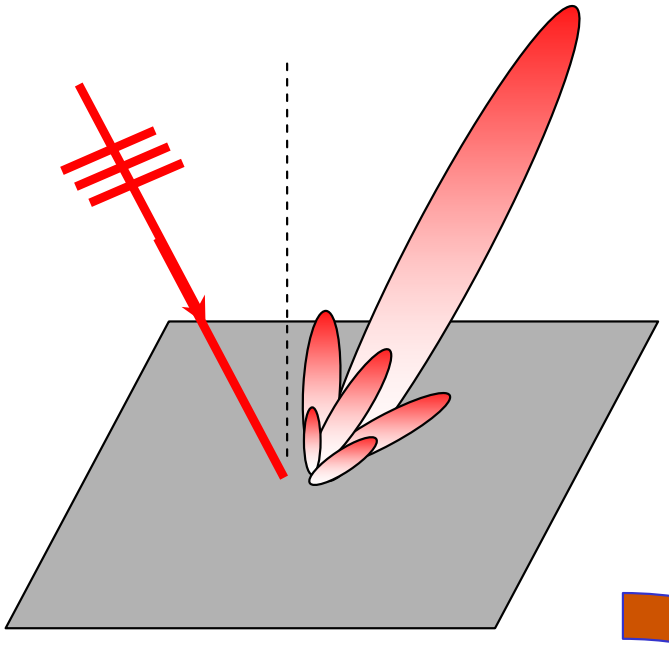


Only the propagating and the first evanescent FWs describes the field accurately in most the cases

GREEN'S FUNCTION FOR A PHASED ARRAY OF DIPOLES

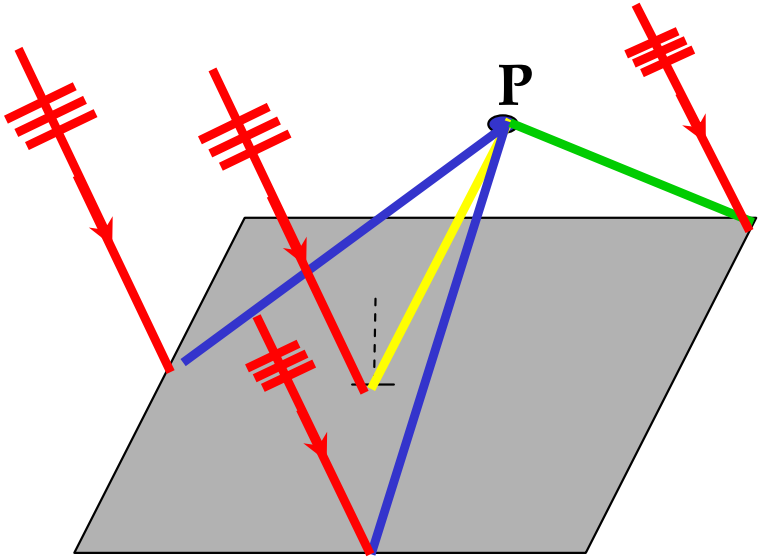
Array in free space: equivalence with far field scattering by a metallic plate





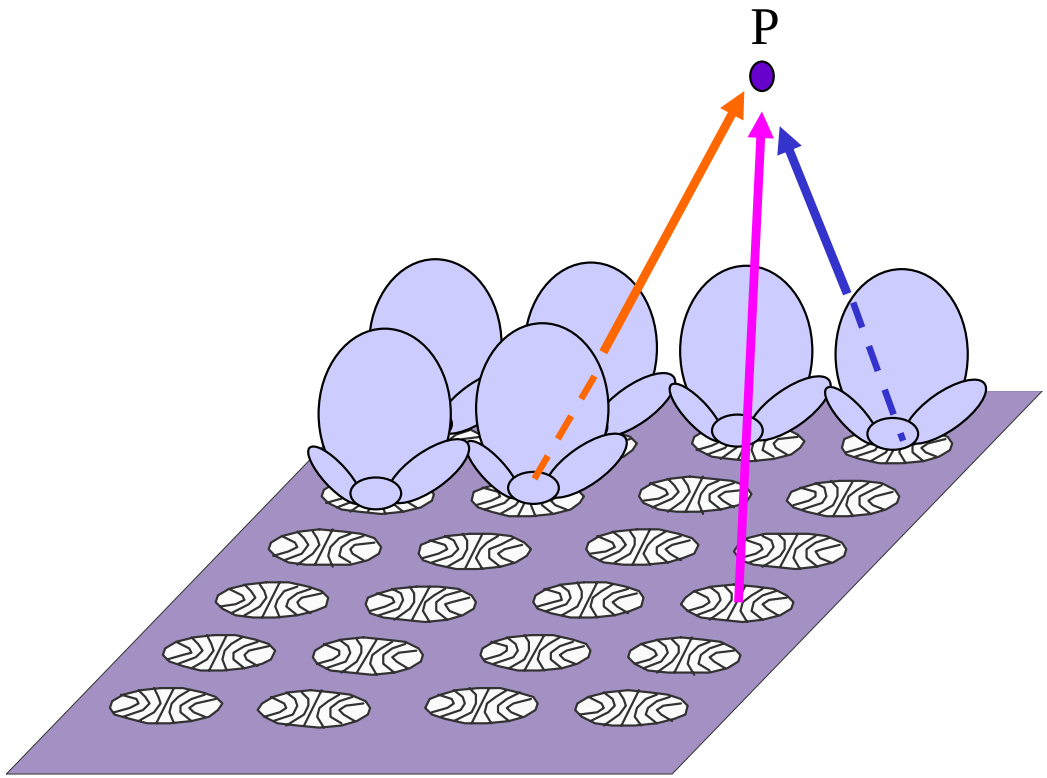
In the far zone, the radiation integral of each propagating FW produces a beam in the specular direction

Generalization to observation point at finite distance: UTD



MOTIVATION: INTERACTION WITH THE ENVIRONMENT

In the far field of the single element
the momentum of each subcell can be replaced by the element factor



ARRAY Green's function
x
Element pattern

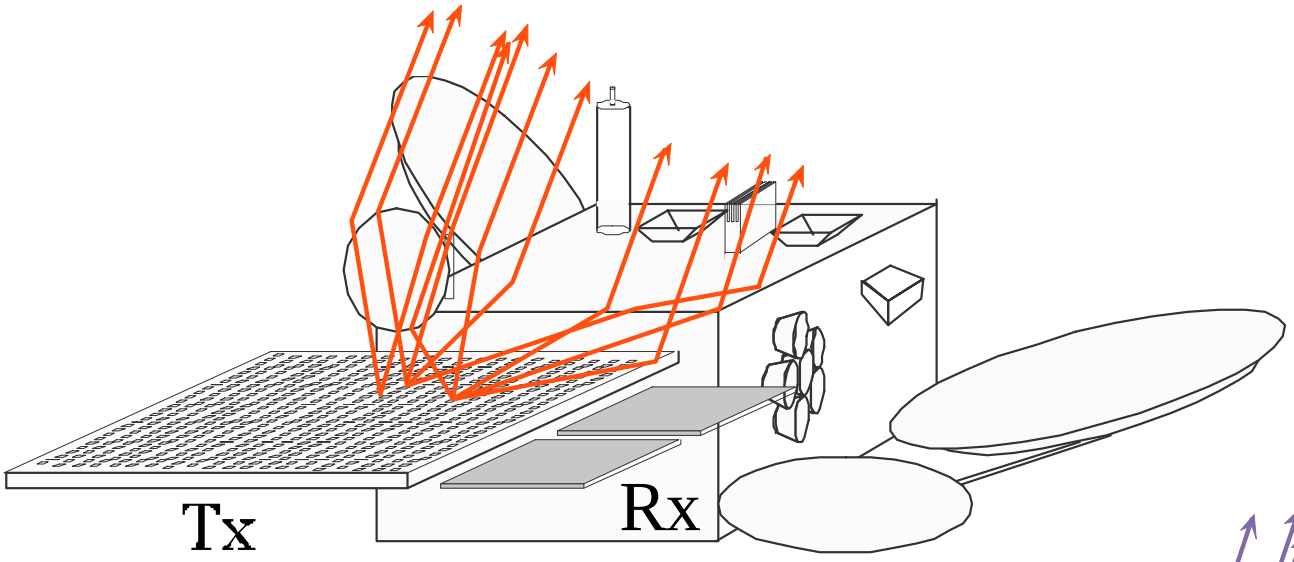
Generalization of the
LAW
ARRAY FACTOR
X
ELEMENT FACTOR
in the FRESNEL zone

The radiation in the Fresnel zone from an array of arbitrary elements can be obtained by weighting each contribution by the proper element factor.

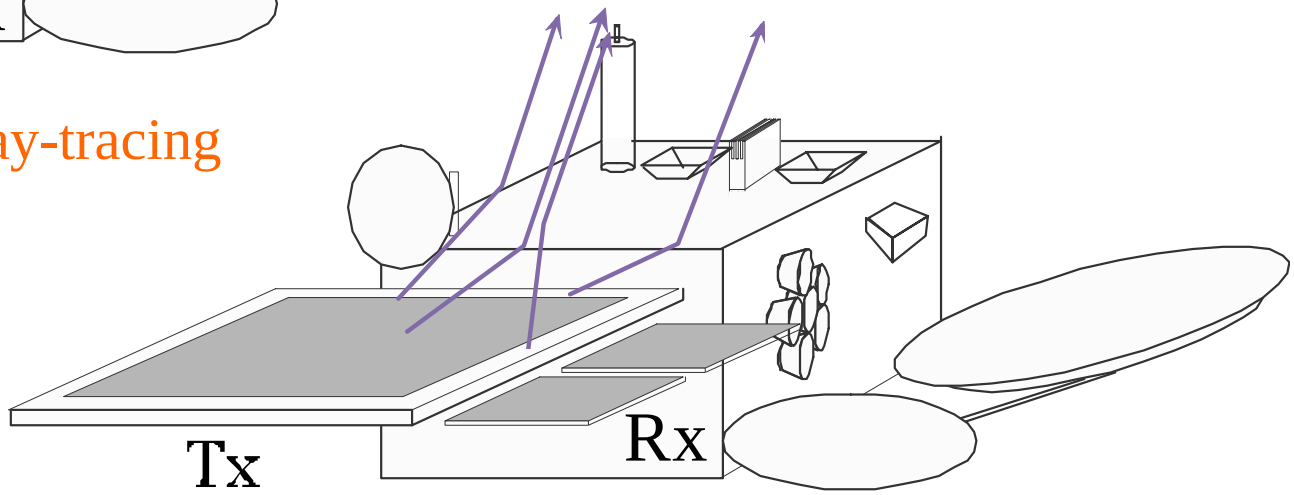
MOTIVATION: INTERACTION WITH THE ENVIRONMENT

The radiation from the array can be collectively described in terms of a few rays associated to the FW of the infinite structure

efficient representation of the interaction with a complex environment

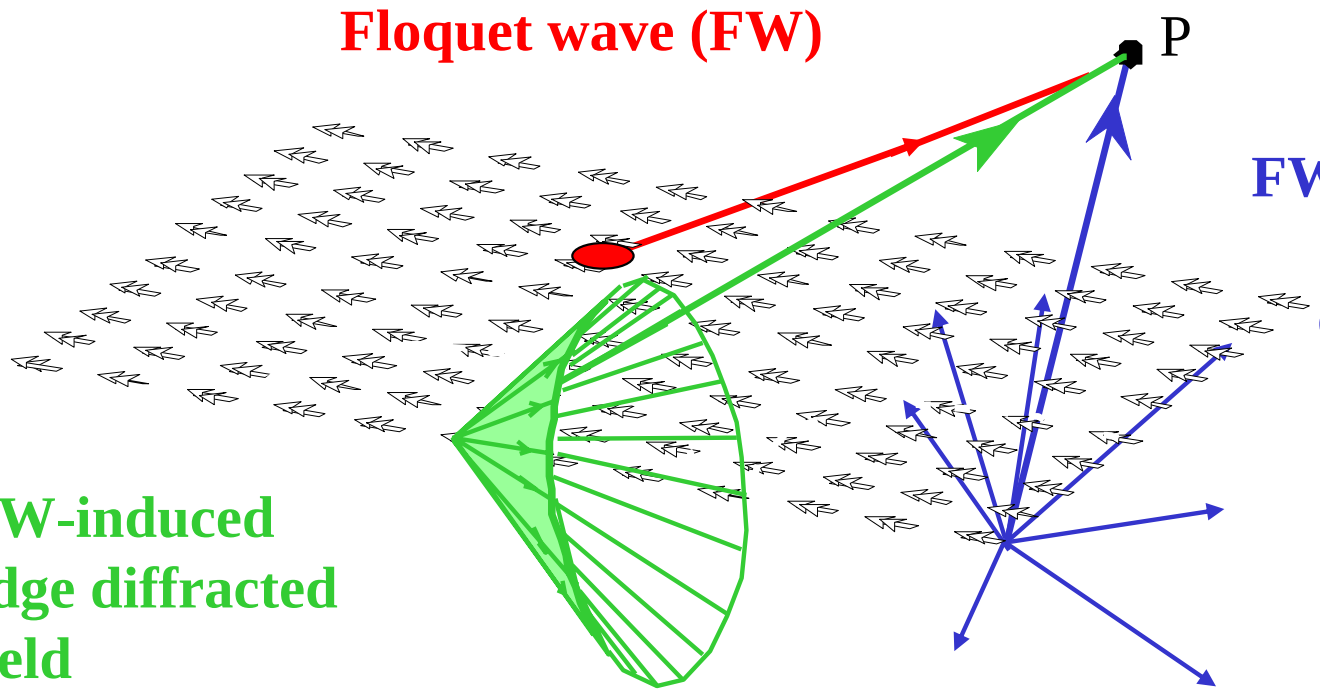


Element by element ray-tracing



Global TFW ray-tracing

RECTANGULAR ARRAY OF DIPOLES IN FREE SPACE



$$\mathbf{E} \approx \sum_{p,q} \mathbf{E}^{FW}_{pq} + \sum_{p,q} \mathbf{E}^{e}_{pq} + \sum_{p,q} \mathbf{E}^{v}_{pq}$$

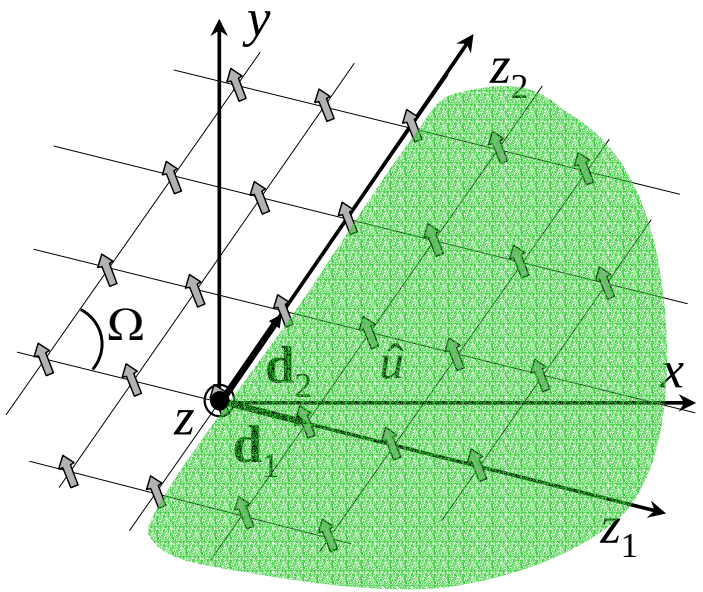
$$\mathbf{E}^{FW}_{pq} \approx U^{FW}_{pq} e^{-j\mathbf{k}^{FW}_{pq}\cdot\mathbf{r}}$$

$$\mathbf{E}^{e}_{pq} \approx U^{e}_{pq} \frac{e^{-j\mathbf{k}^{e}\cdot\mathbf{r}}}{\sqrt{k\rho}} + O\left((k\rho)^{-3/2}\right)$$

$$\mathbf{E}^{v}_{pq} \approx \frac{e^{-jkr}}{kr} + O\left((kr)^{-2}\right)$$

RECTANGULAR ARRAY OF DIPOLES IN FREE SPACE

CANONICAL PROBLEM FOR EDGE DIFFRACTION: SEMI-INFINITE ARRAY



$$A(P)=\sum_{n=0}^{+\infty}\sum_{m=-\infty}^{+\infty} g(P;nd_x,md_y)\,e^{-j(k_{x0}nd_x+k_{y0}md_y)}$$

$$g(P;x,y)=\frac{\exp[-jkR(x,y)]}{R(x,y)}=\frac{1}{(2\pi)^2}\int_{-\infty-\infty}^{+\infty+\infty} G(k)e^{-j\vec{k}\cdot(\vec{r}-\vec{r}')}dk_x dk_y$$

$$\sum_{n=-0}^{+\infty}\sum_{m=-\infty}^{+\infty}\int_{-\infty-\infty}^{+\infty+\infty} \rightarrow \int_{-\infty-\infty}^{+\infty+\infty}\sum_{n=-0}^{+\infty}\sum_{m=-\infty}^{+\infty}$$

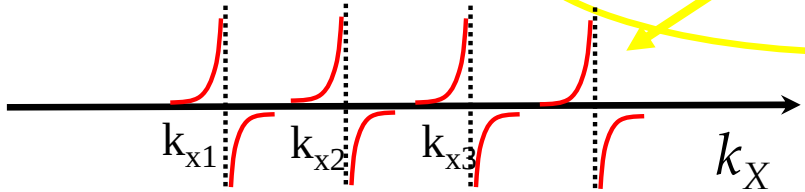
$$k_{xp}=k_{xo}+\frac{2\pi p}{d_x}$$

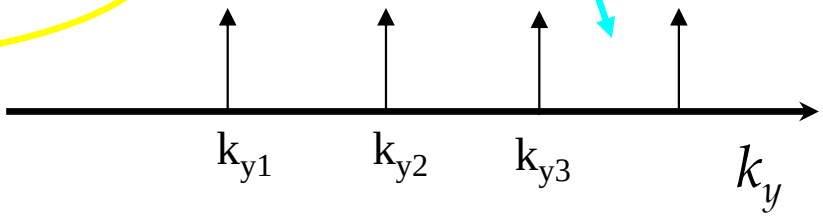
Train of Poles

$$B(k_x)=\frac{1}{1-e^{j(k_x-k_{x0})d_x}}$$

$$k_{yq}=k_{y0}+\frac{2\pi q}{d_y}$$

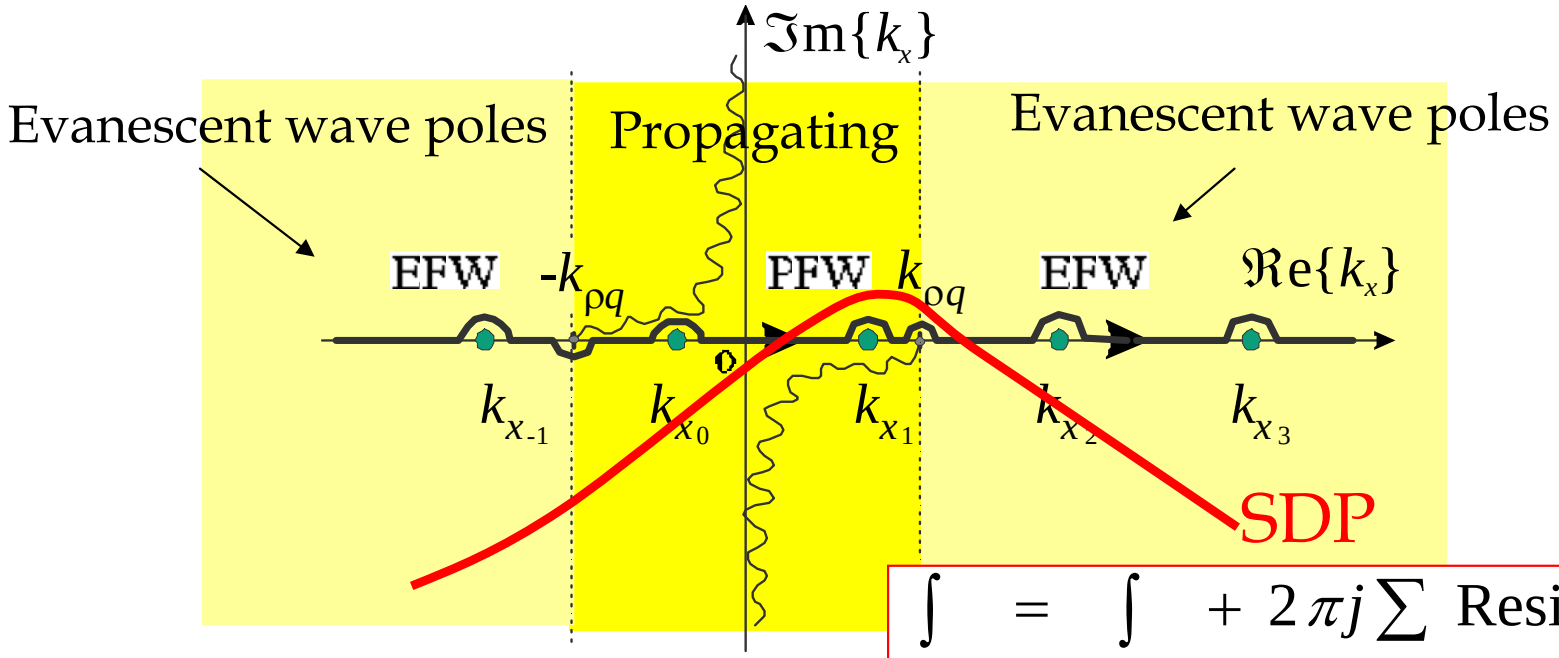
Train of Dirac delta functions



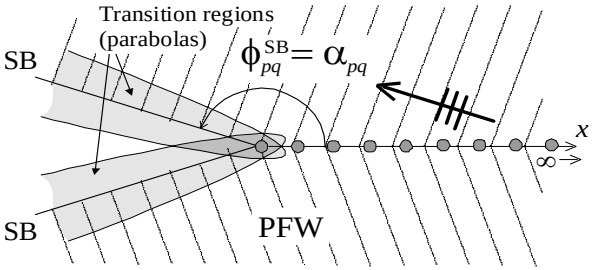


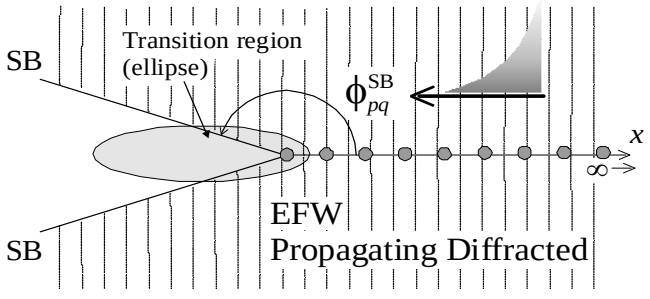
RECTANGULAR ARRAY OF DIPOLES IN FREE SPACE

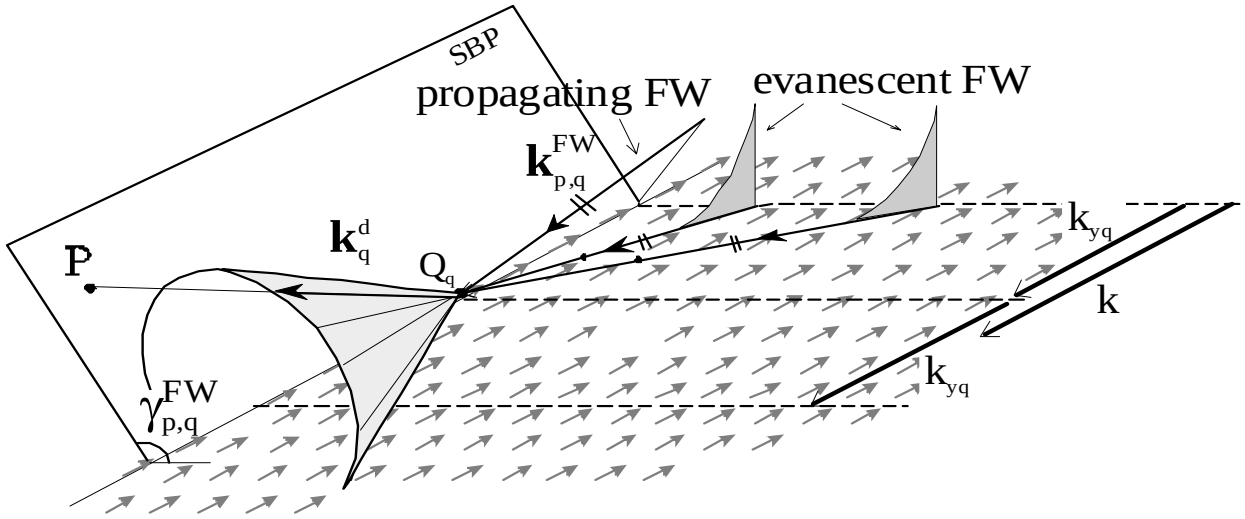
CANONICAL PROBLEM FOR EDGE DIFFRACTION: SEMI-INFINITE ARRAY



$$\int_{C_\alpha} = \int_{SDP} + 2\pi j \sum \text{Residues}$$

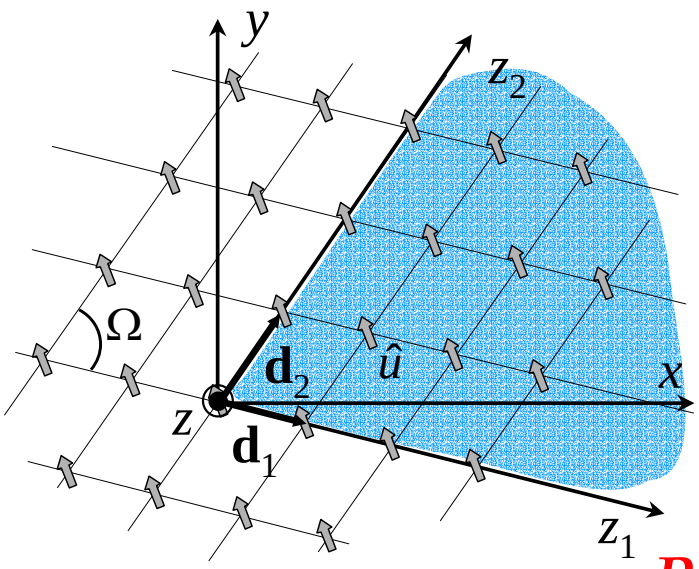


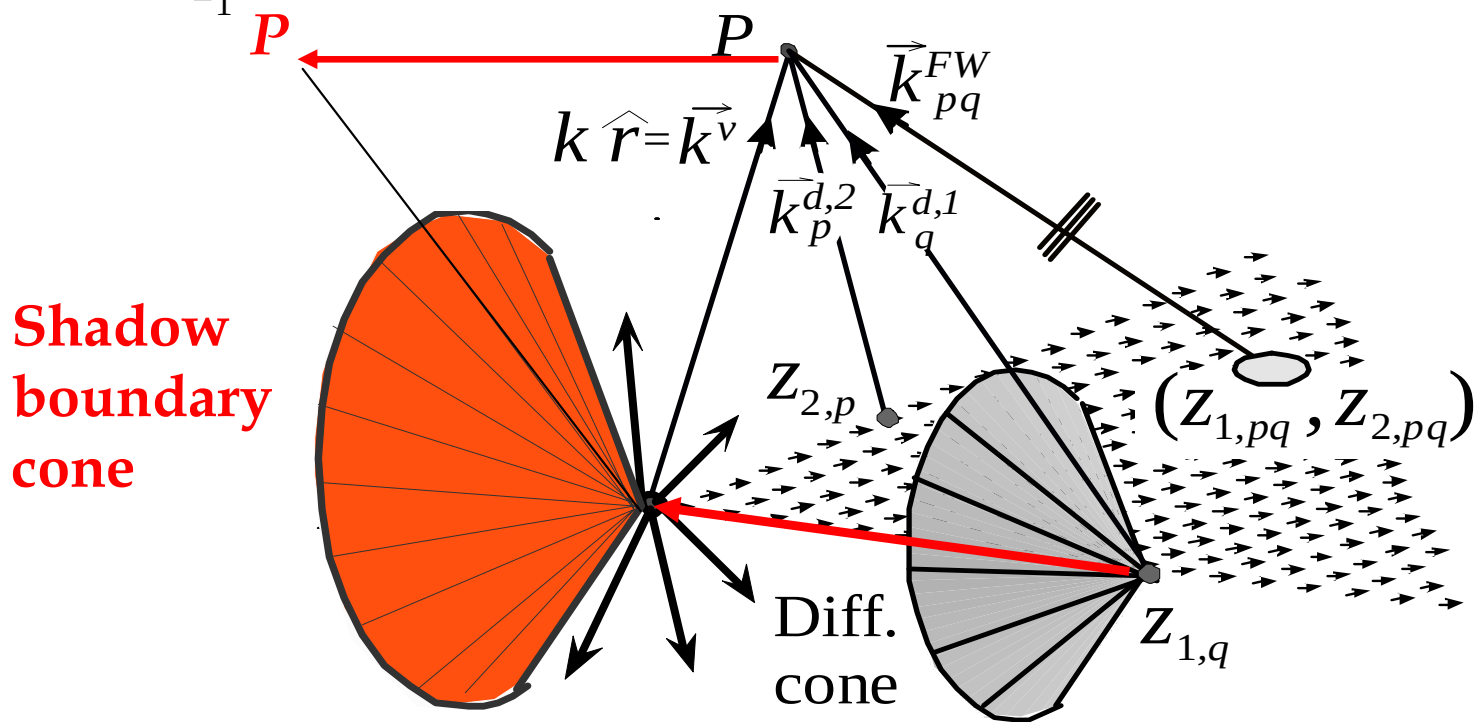




RECTANGULAR ARRAY OF DIPOLES IN FREE SPACE

CANONICAL PROBLEM FOR VERTEX DIFFRACTION: SECTORAL ARRAY





ARRAY OF DIPOLES IN STRATIFIED DIELECTRIC MEDIA

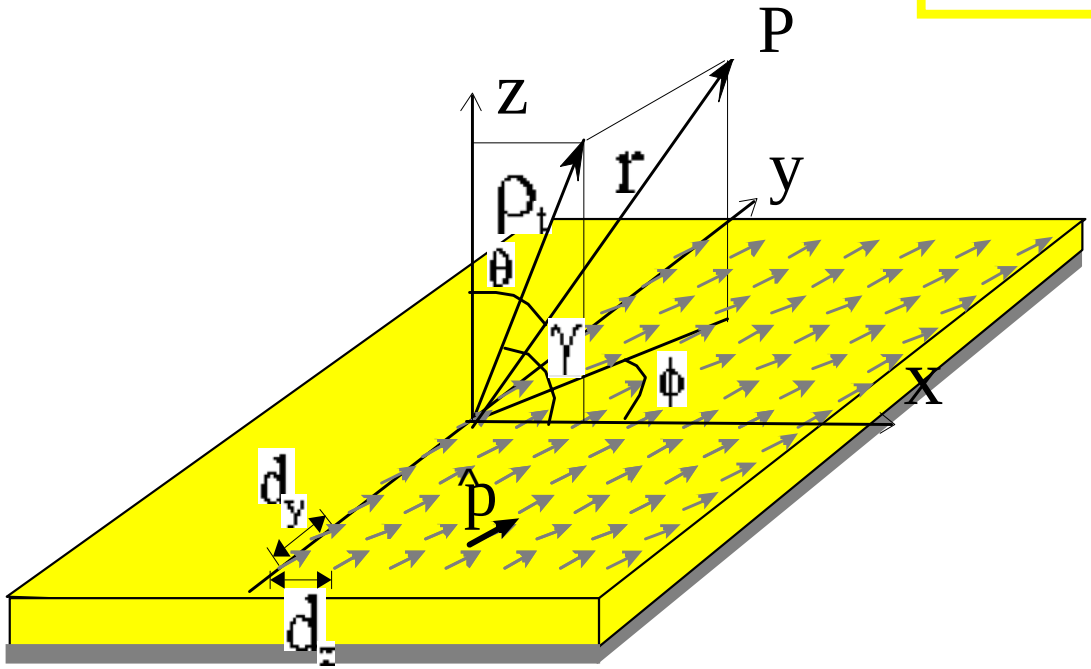
GROUNDED SLAB

$$\vec{A}(P) = \frac{\hat{p}}{2\pi d_y} \sum_{q=-\infty}^{+\infty} \int_{-\infty}^{+\infty} B(k_x) G(\vec{k}_q) e^{-j(\vec{k}_q \cdot \vec{r})} dk_x$$

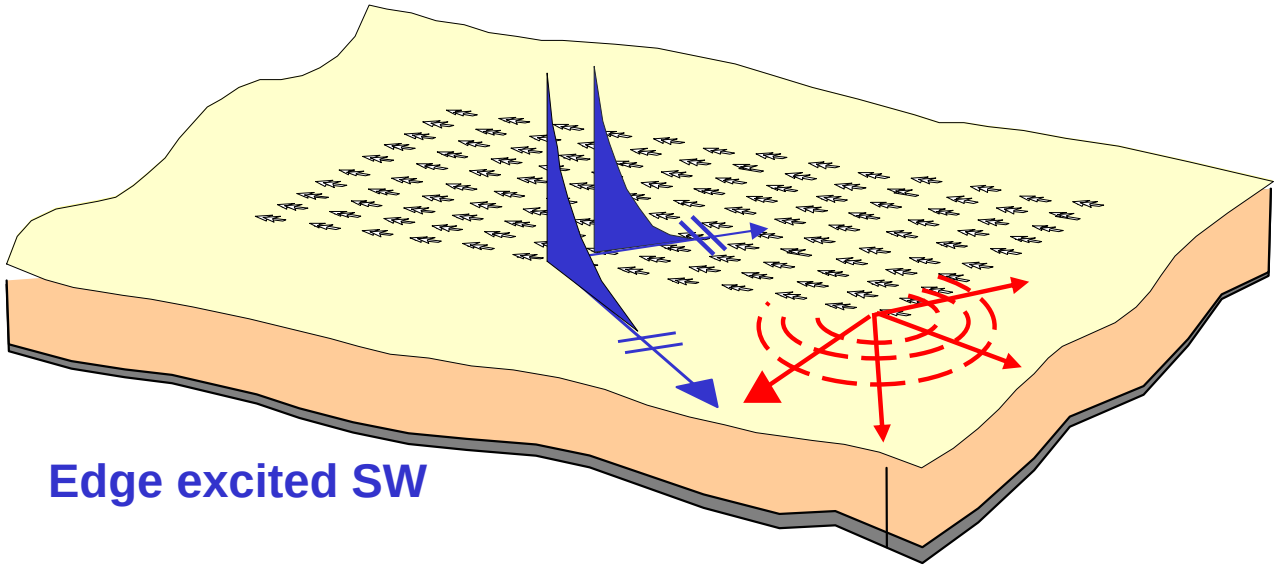
POLES

$$k_{xp} = k_{x0} + \frac{2\pi p}{d_x}$$

The single element Green's function is changed



ARRAY OF DIPOLES IN STRATIFIED DIELECTRIC MEDIA



Edge excited SW

Vertex excited SW

$$\mathbf{E} \approx \sum_{p,q} \mathbf{E}_{pq}^{FW} + \sum_{p,q} \mathbf{E}_{pq}^{e} + \sum_{p,q} \mathbf{E}_{pq}^{v} + \sum_{p,q} \mathbf{E}_{pq}^{e,SW} + \sum_{p,q} \mathbf{E}_{pq}^{v,SW}$$

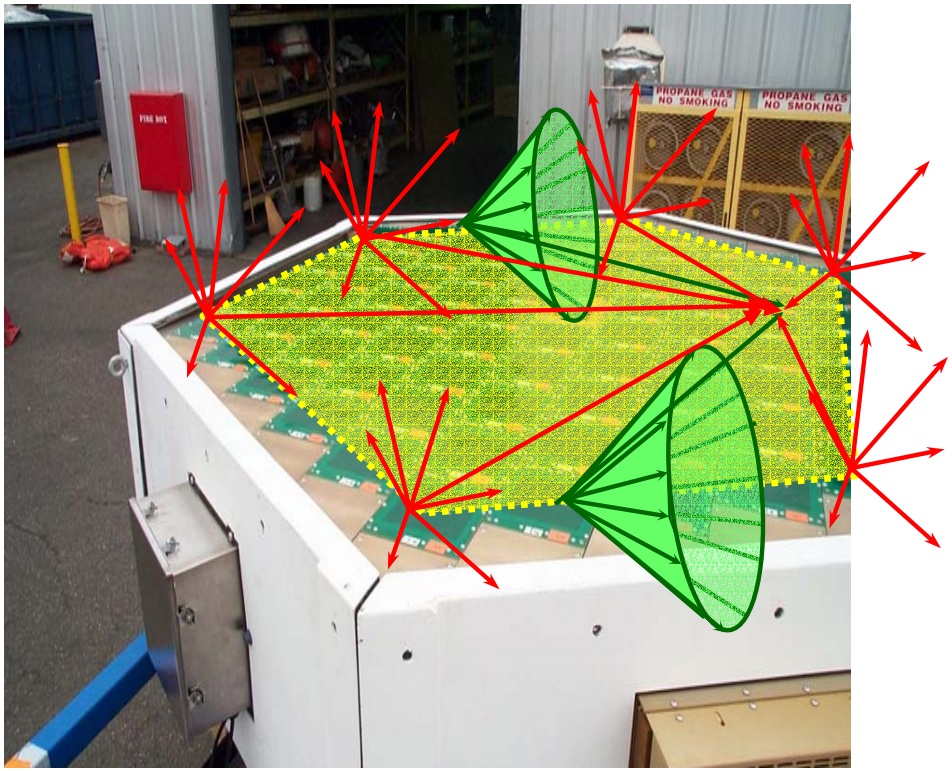
$$\mathbf{E}_{pq}^{e,SW} \approx U_{pq}^{e,SW} e^{-j\mathbf{k}_{pq}^{SW}\cdot\mathbf{r}}$$

$$\mathbf{E}_{pq}^{v,SW} \approx \frac{1}{\sqrt{r}} e^{-jk^{SW}r}$$

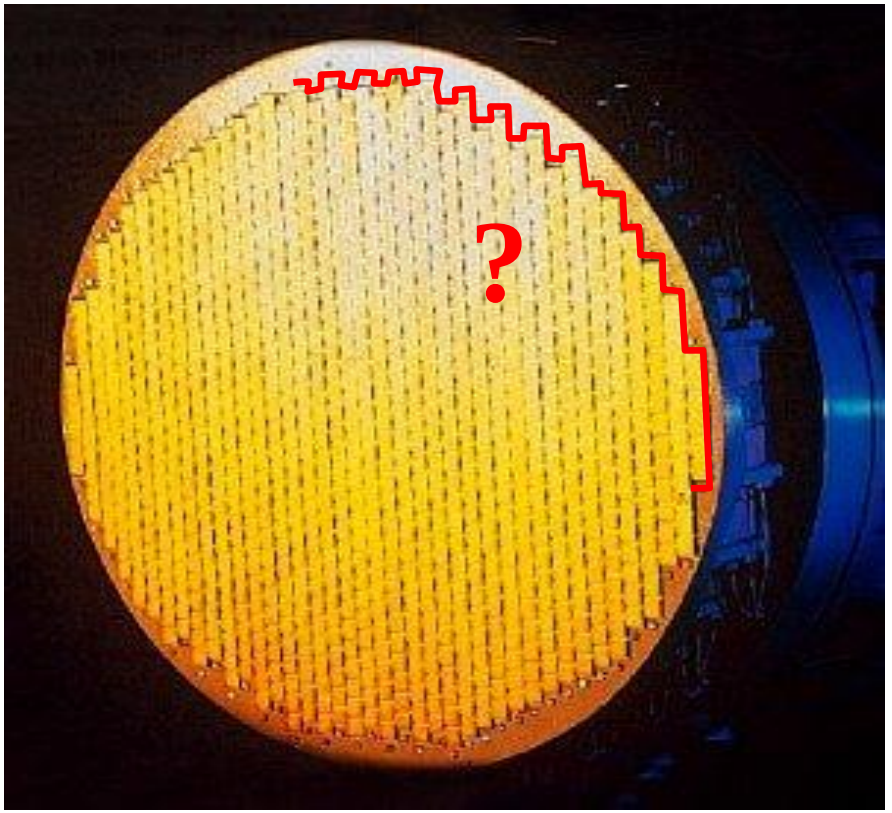
ARBITRARILY CONTOURED ARRAYS

The description in terms of truncated FW has proven to be very efficient for rectangular arrays.

However, as the contour of the array becomes irregular, the number of flash points increases and the contour may no longer be well defined

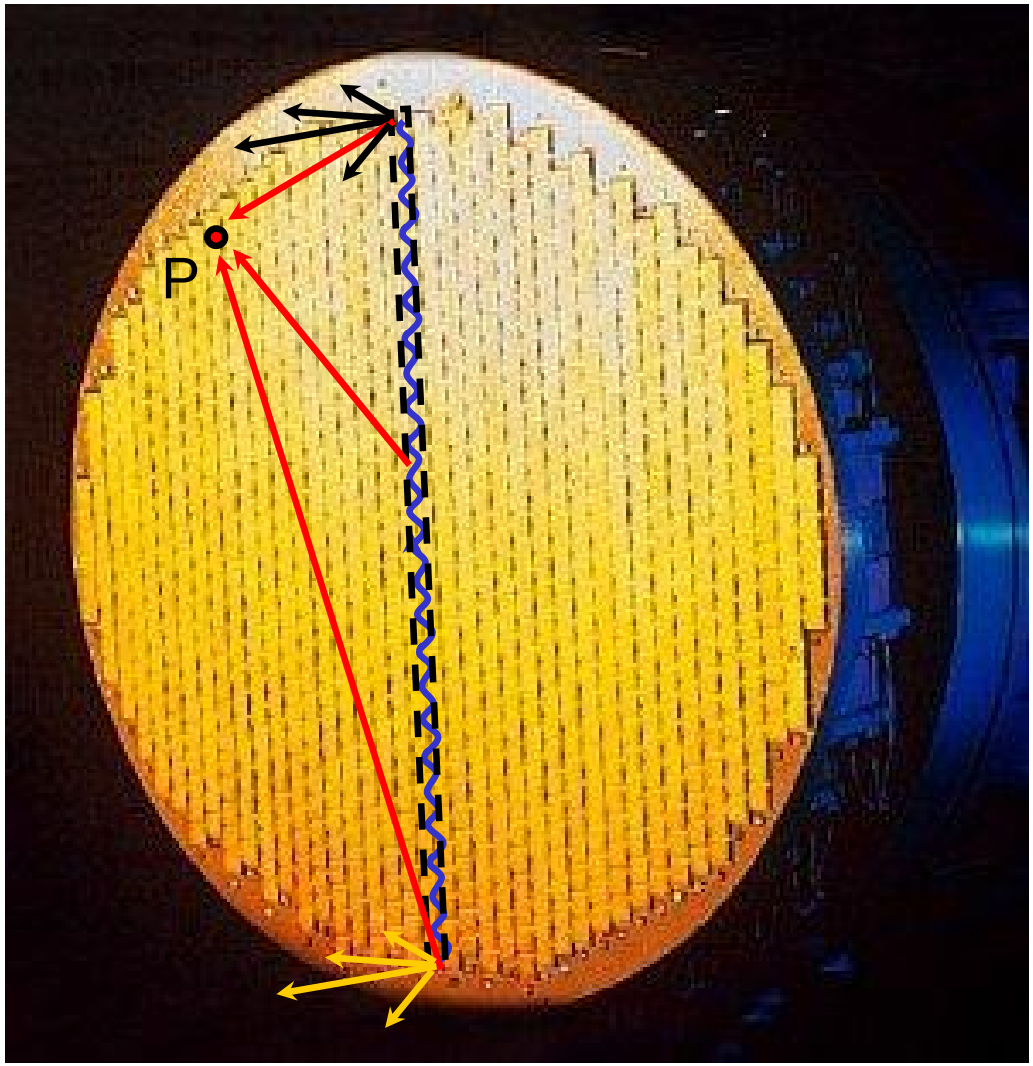


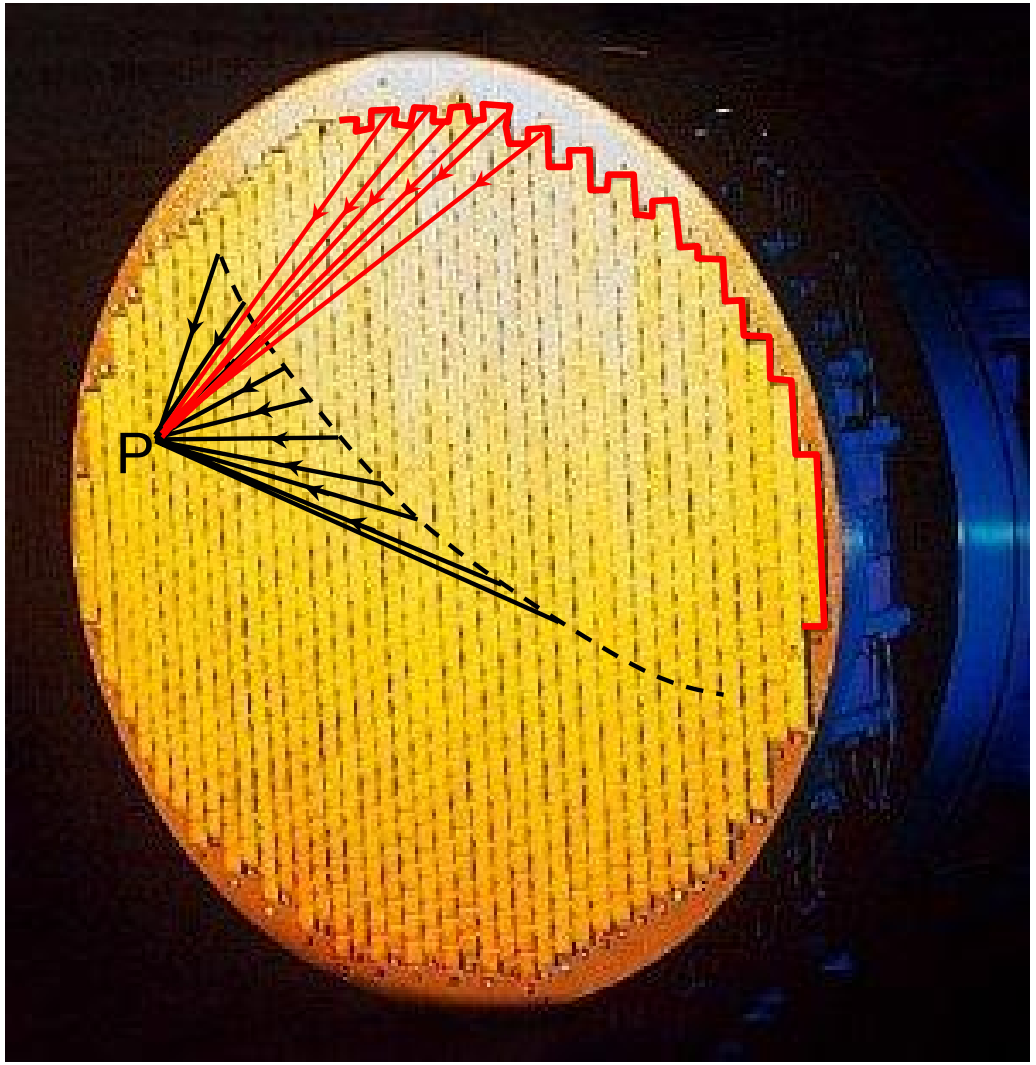
Polygonal contours



Irregular contours

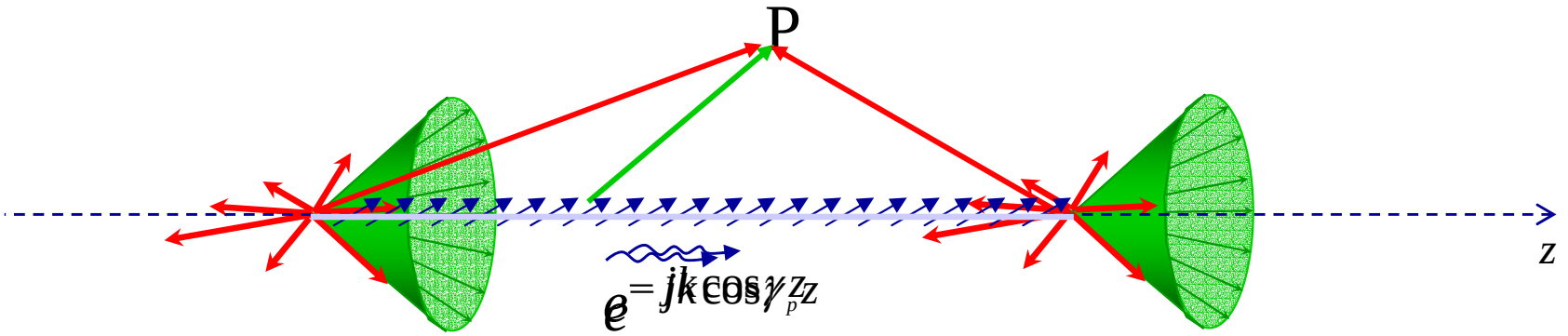
LINE BY LINE STRATEGY FOR ARBITRARILY CONTOURED ARRAYS



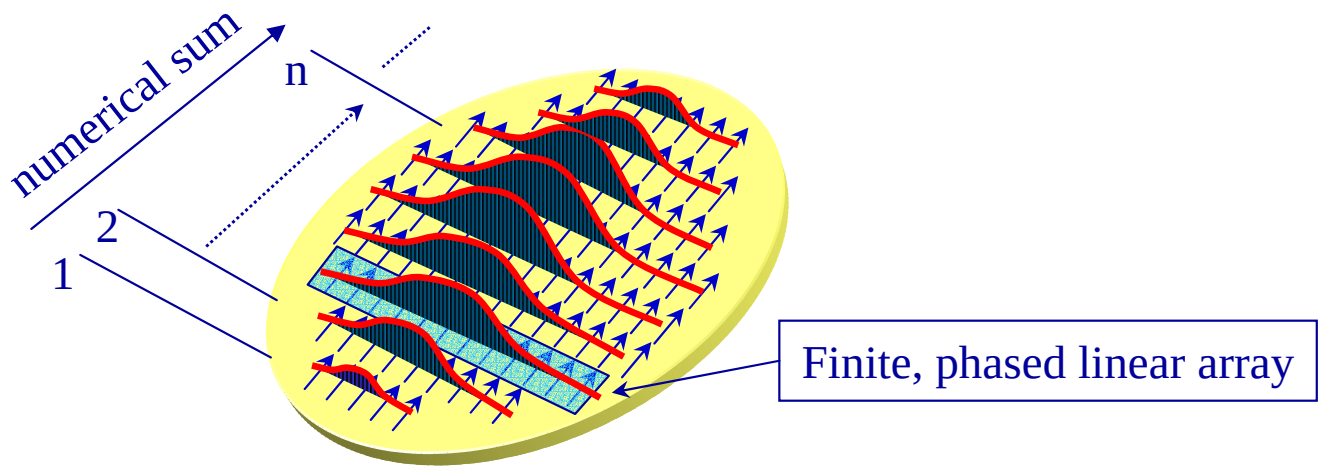


LINE BY LINE STRATEGY FOR ARBITRARILY CONTOURED ARRAYS

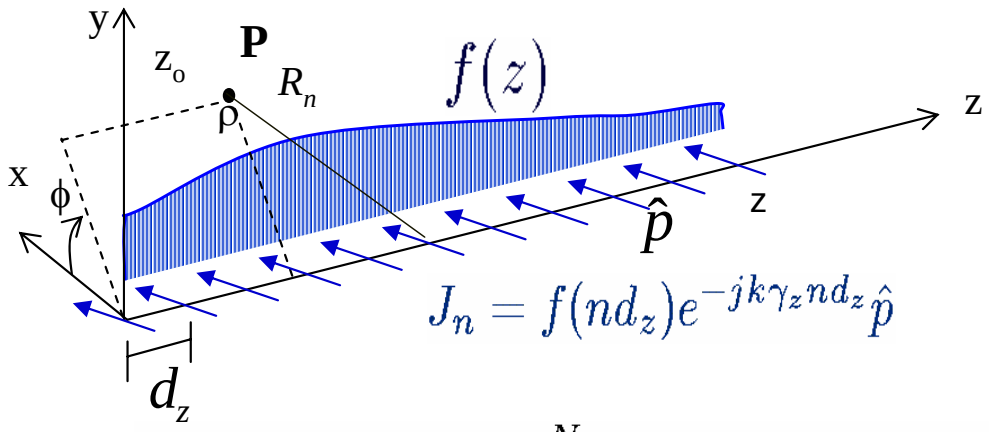
The radiation from a uniform finite linear array can be represented in terms of truncated cylindrical FWs, augmented by the relevant end-point diffracted fields



A uniform planar array radiation can be obtained as a one-dimensional superposition of the dominant high-frequency contributions from the constituting finite phased linear arrays



LINE BY LINE STRATEGY FOR TAPERED ARRAYS

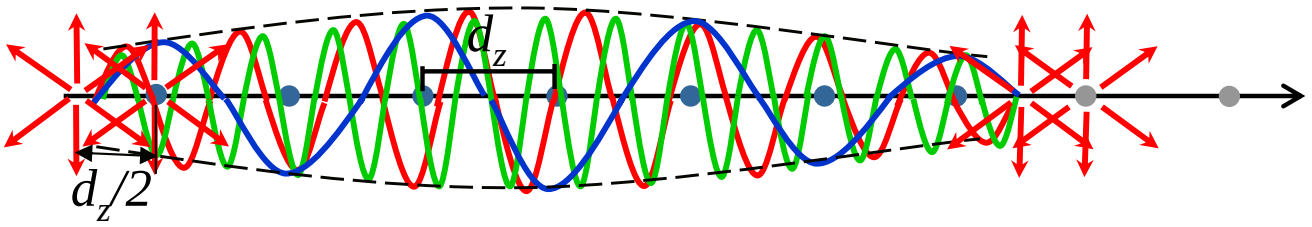


$$\mathbf{A}(P) = A(P)\hat{p} = \sum_{n=1}^{N} \frac{e^{-jkR_n}}{4\pi R_n} f(nd_z) e^{-j\gamma_z n d_z} \hat{p}$$

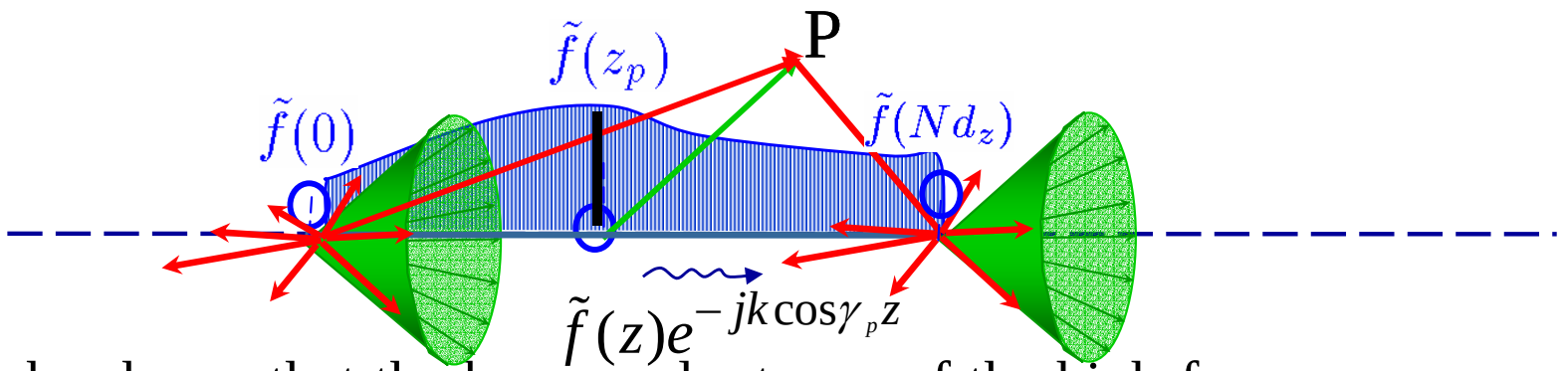
By applying the Poisson summation formula the array can be seen as the superimposition of suitably phased and tapered continuous lines:

$$A(P) = \sum_{p=-\infty}^{\infty} \int_{-\frac{d_z}{2}}^{Nd_z - \frac{d_z}{2}} \frac{e^{-jkR}}{4\pi d_z R} f(\nu) e^{-jk_{zp}\nu} d\nu = \sum_{p=-\infty}^{\infty} A_p(P)$$

with $k_{zp} = \gamma_z + \dfrac{2\pi p}{d_z}$ infinite linear array FW wavenumbers



THE CANONICAL PROBLEM OF THE TAPERED LINE



It can be shown that the lower-order terms of the high-frequency representation of the potential are:

$$A_p(P) = \tilde{f}(z_p)\, A_p^{\infty}(P) U(\gamma_p - \beta) + \tilde{f}(z_p)\, A_p^{U}(P) + \tilde{f}(0)\, A_p^{R}(P) + \tilde{f}(Nd_z)\, A_p^{R}(P)$$

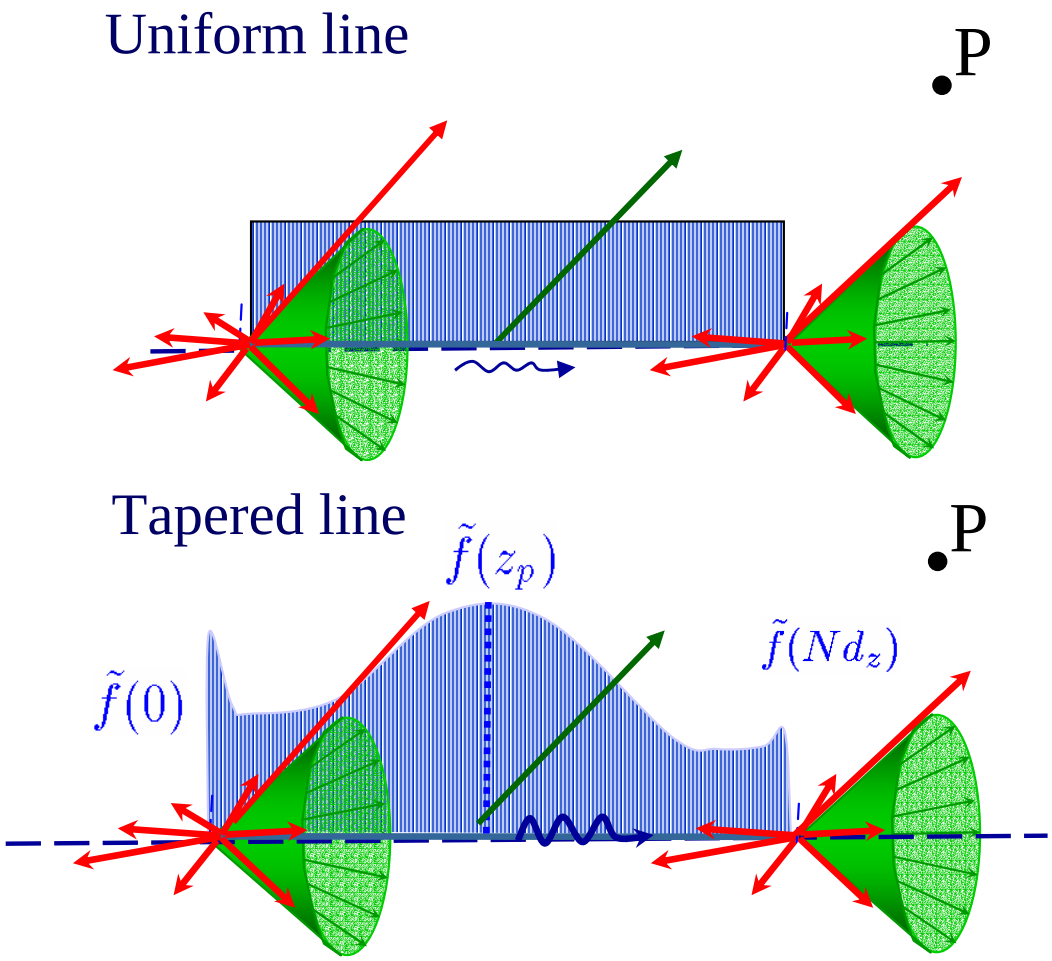
$$\propto \frac{e^{-jkr\cos(\beta-\gamma_p)}}{\sqrt{k\rho}}$$ truncated FW

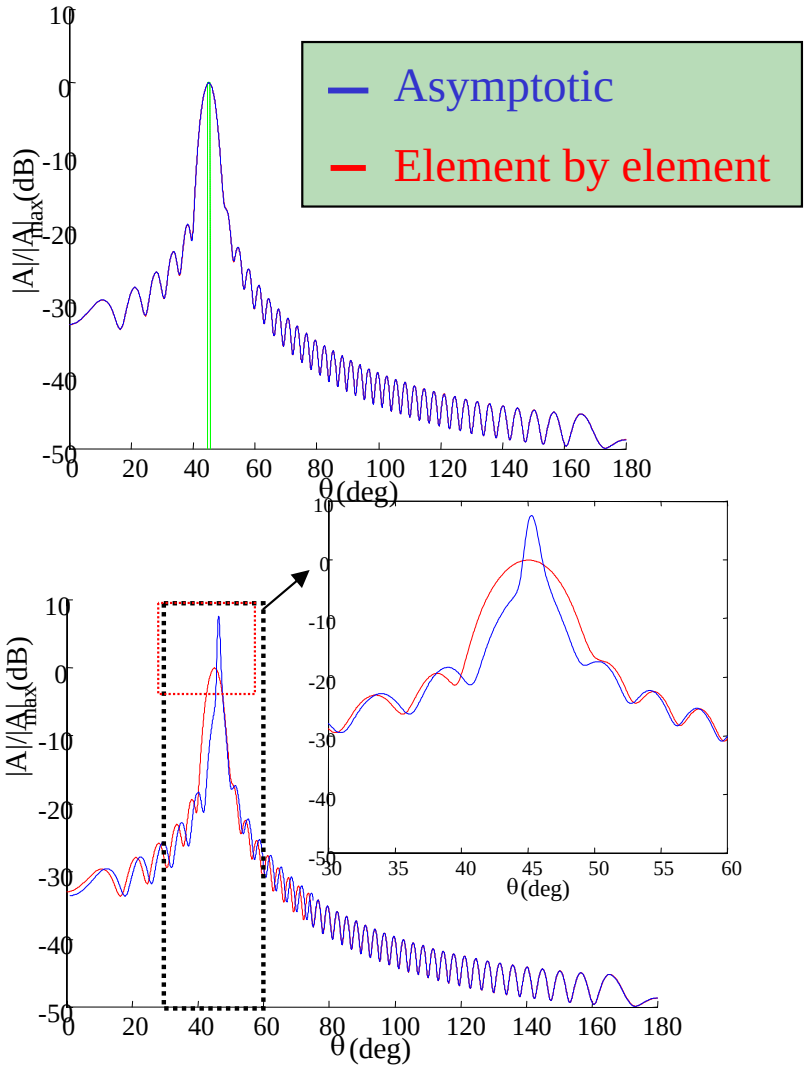
$$\propto \frac{e^{-jkr}}{kr}\frac{(-1)^p}{(\cos\beta - \cos\gamma_p)}$$ spherical diffracted wave

$$\propto \frac{e^{-jkr}}{kr}\frac{(-1)^p}{\sin\left(\frac{\beta-\gamma_p}{2}\right)}\left\{F\left[2kr\sin^2\left(\frac{\beta-\gamma_p}{2}\right)\right]-1\right\}$$ uniform contribution

UNIFORM VS. TAPERED LINE

This approach provides excellent results for uniform arrays, both in the near and in the far field regions. Accurate results are obtained also for the field radiated in the near zone by a weakly tapered array. Accuracy, however, decreases when the observation point moves to the far zone.

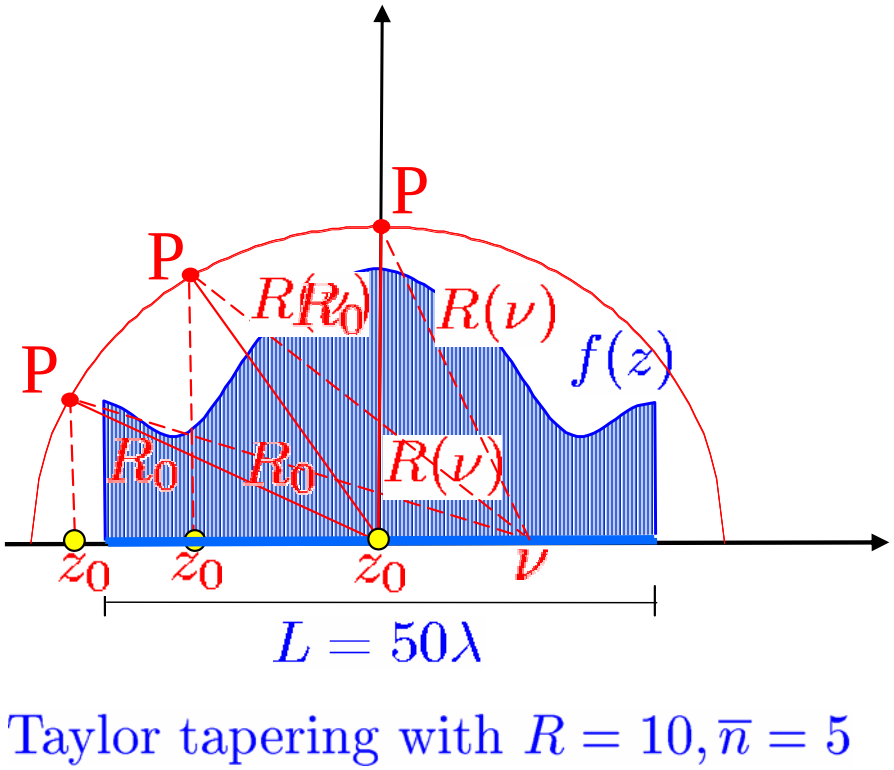




UNIFORM VS. TAPERED LINE

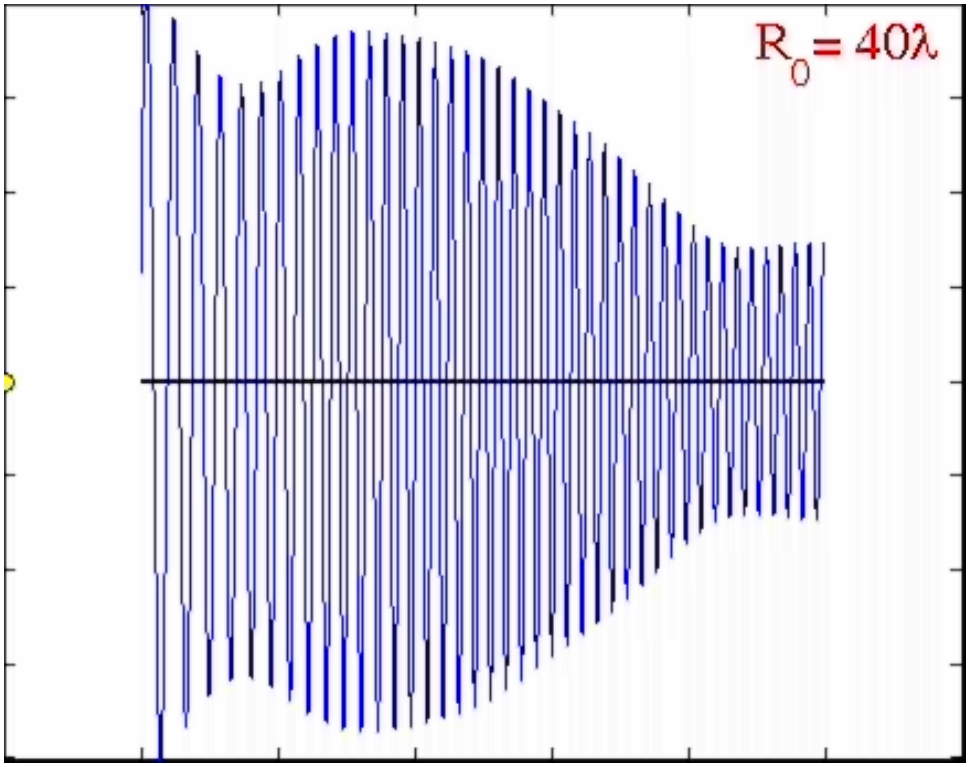
This phenomenon can be clearly explained by observing the behavior of the argument of the radiation integral for different positions of the observer .

- The observer moves in the near field zone.



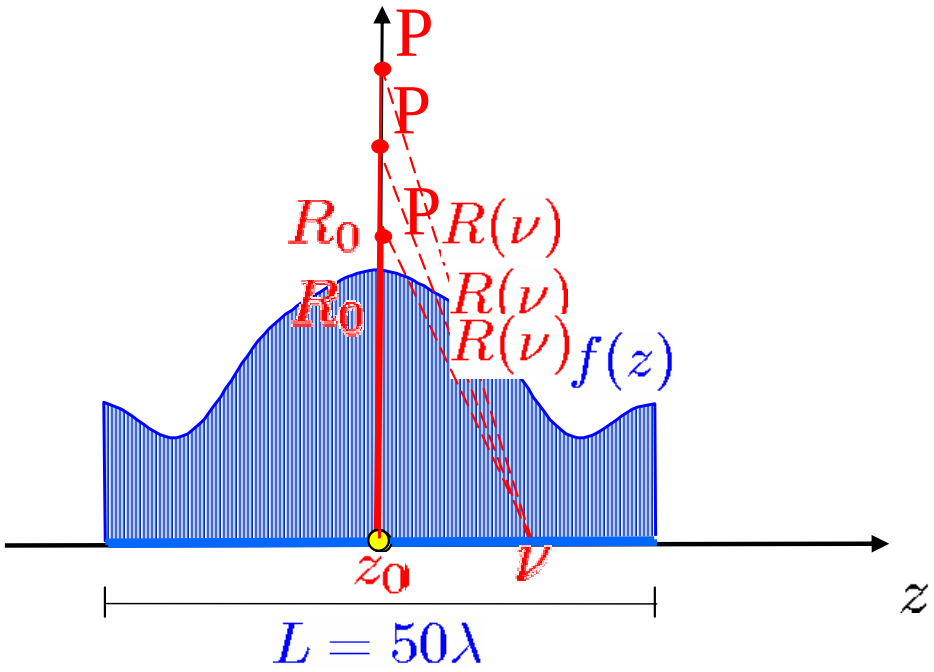
Taylor tapering with $R = 10, \overline{n} = 5$

Integrand function $\frac{e^{-jkR(\nu)}}{4\pi R(\nu)} f(\nu)$



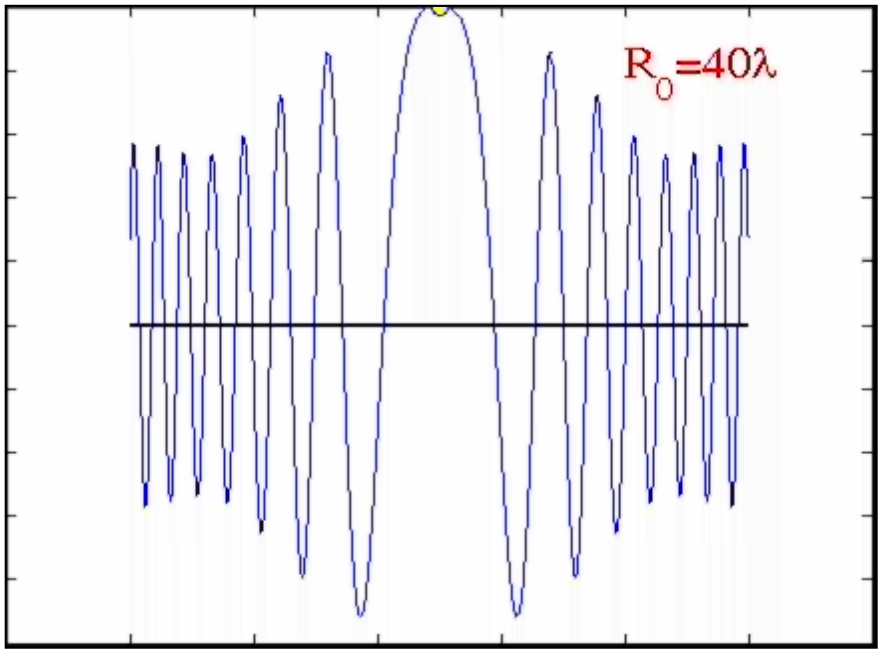
UNIFORM VS. TAPERED LINE

- The observer moves from the near to the far field zone.



Taylor tapering with $R = 10, \overline{n} = 5$

Integrand function $\dfrac{e^{-jkR(\nu)}}{4\pi R(\nu)} f(\nu)$



The strong localization of radiation contributions is no longer valid for far field observation!

HOW TO OVERCOME THE IMPAIRMENT?

Approximating the actual current distribution through a suitable superimposition of uniform excitations allows to go from the near to the far field region.

This has been done in the past by resorting to DFT or to spatial Fourier Transforms.

- *Nepa, Pathak, Civi, and Chou, 1999*
- *Cicchetti,Faraone,and Balzano, 2003*

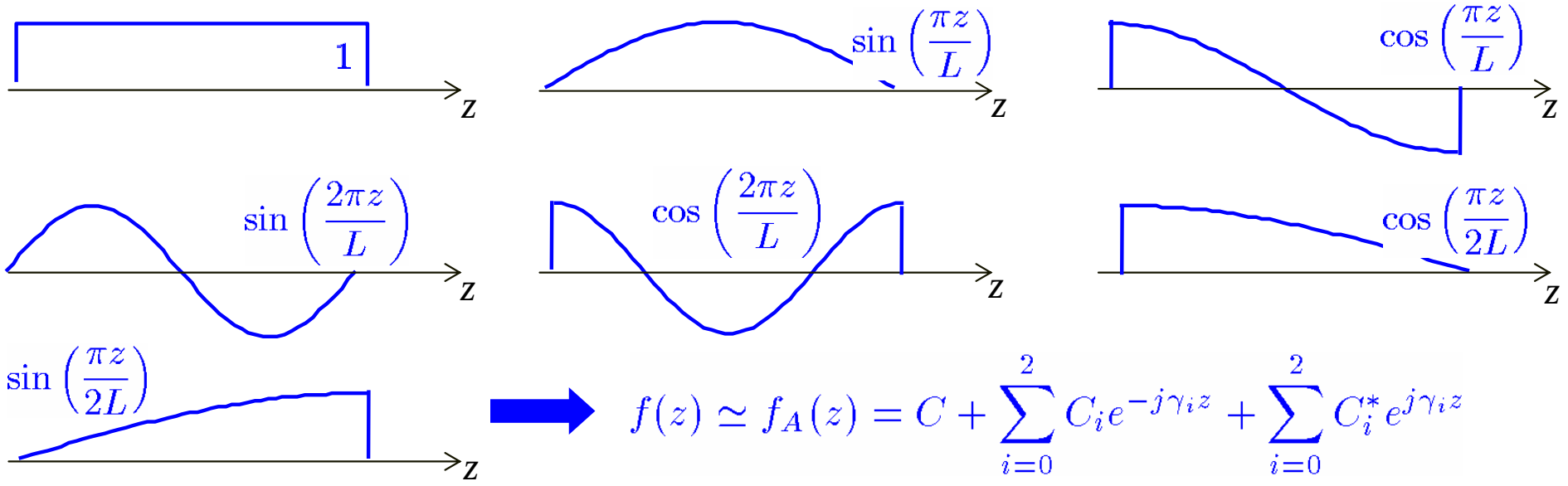
A more efficient representation of the radiated field is obtained by linking the coefficients of the expansion to the characteristics of the actual tapering associated with the dominant asymptotic contributions.

Specifically, the approximate tapering function is defined to match:

- the value and the derivative of *f(z)* at the two end-points
- the value and the derivative of *f(z)* at the stationary phase point (if it lies on the line)
- the area subtended by *f(z)*

APPROXIMATION OF THE LINE TAPERING

To approximate the current distribution the following set of basis functions is employed, which can be immediately rewritten in terms of equiamplitude linearly phased excitations.



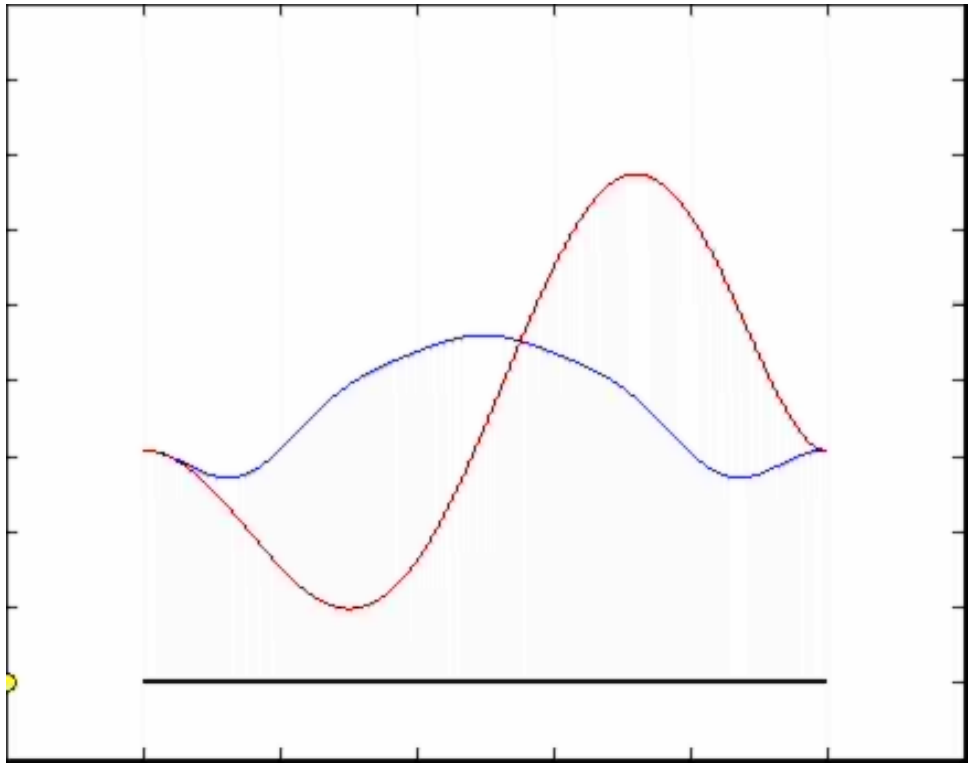
$$f(z) \simeq f_A(z) = C + \sum_{i=0}^{2} C_i e^{-j\gamma_i z} + \sum_{i=0}^{2} C_i^* e^{j\gamma_i z}$$

$$\gamma_i = \frac{\pi}{L} 2^{i-1}$$

The expression of the expansion coefficients is obtained in closed form by solving a linear system of equations.

APPROXIMATION OF THE LINE TAPERING

The conditions relevant to the stationary phase point imply a dependence of the equivalent current on the position of the observation point, however they are only required when the stationary phase point lies on the finite line.



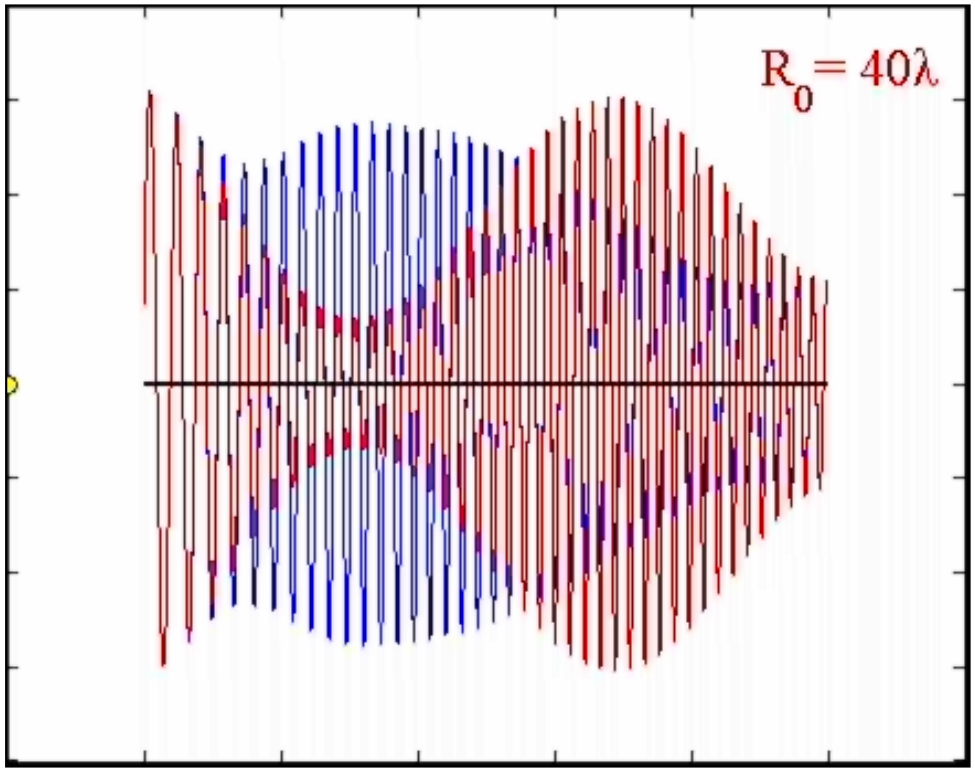
— Actual current distribution $f(\nu)$

— Approximated current distribution $f_A(\nu)$)

○ Stationary phase point

APPROXIMATION OF THE INTEGRAND FUNCTION

Although the approximated tapering function does not equate the actual current distribution on the whole line, there is a good agreement between the relevant radiation integrals:



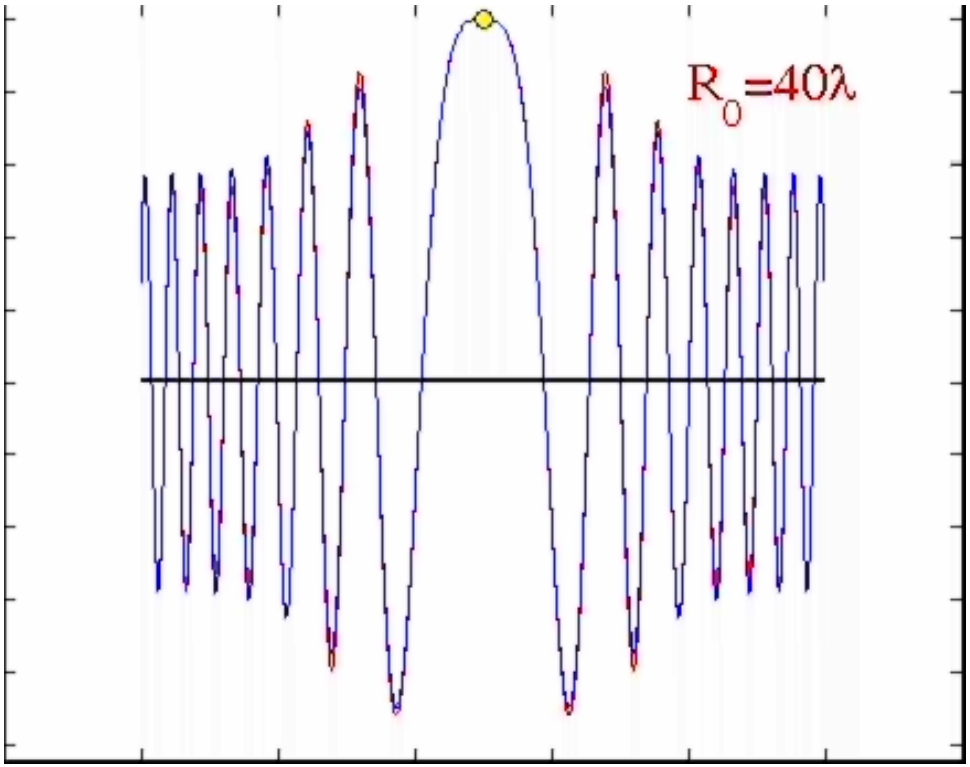
— Actual integrand $\frac{e^{-jkR}}{4\pi R} f(\nu)$

— Approximated integrand $\frac{e^{-jkR}}{4\pi R} f_A(\nu)$

○ Stationary phase point

APPROXIMATION OF THE INTEGRAND FUNCTION

Thanks to the condition on the subtended area, the agreement between the relevant radiation integrals is preserved when the observation point moves to the far field region



— Actual integrand $\frac{e^{-jkR}}{4\pi R} f(\nu)$

— Approximated integrand $\frac{e^{-jkR}}{4\pi R} f_A(\nu)$

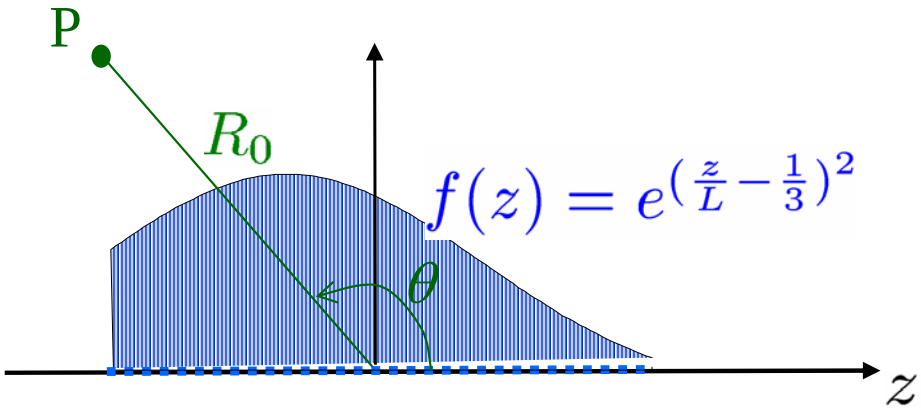
○ Stationary phase point

A COMPARISON OF THE TWO APPROACHES

Numerical results confirm that the proposed approach yields more accurate results when the observation point approaches the far field region:

$L = 20\lambda$

$\gamma_p = \pi/4$

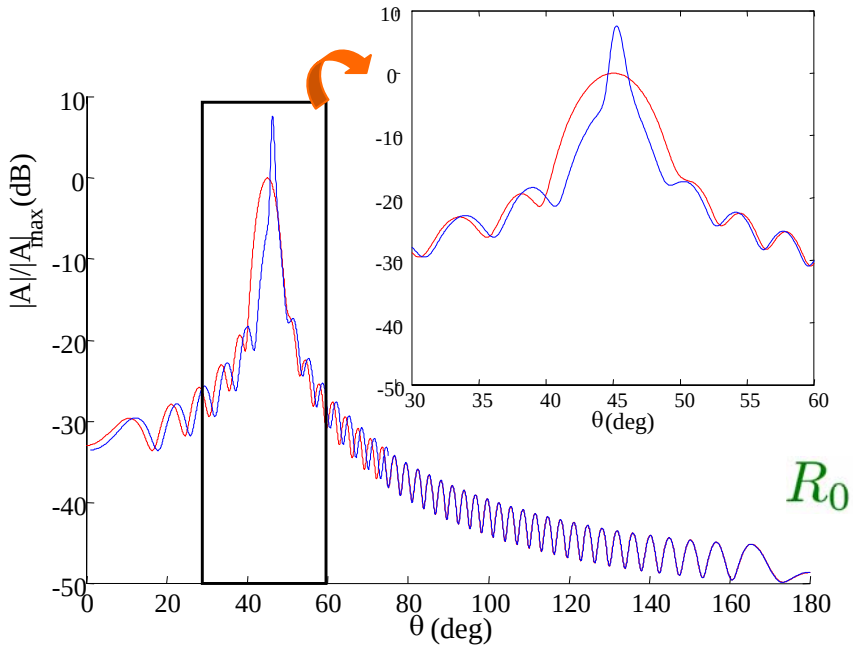


— Asymptotic

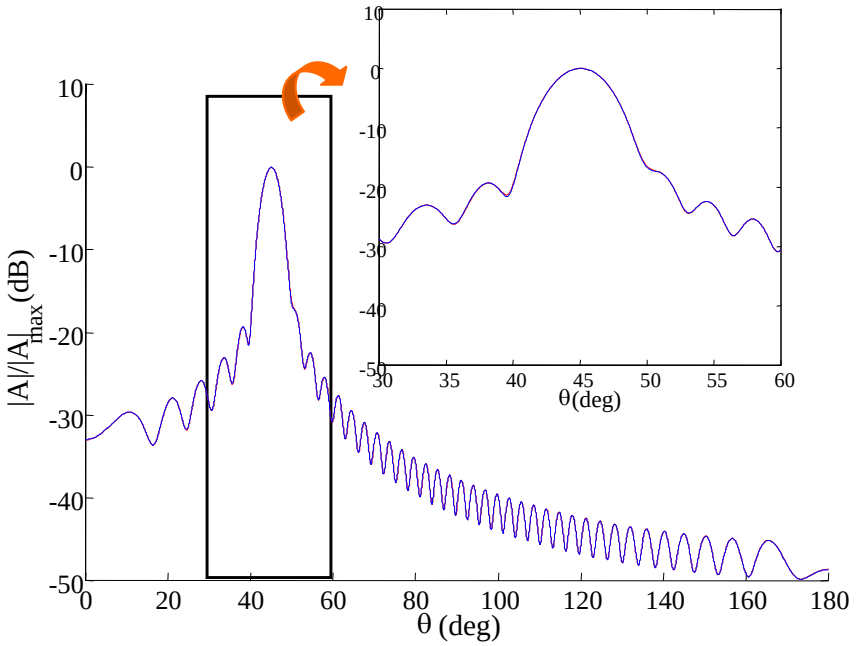
— Element by element

Actual current

Approximated current



$R_0 = 800\lambda$



LINEAR ARRAY SYNTHESIS

Tapered linear array

Poisson

$\sum_p$ tapered continuous lines

$$\sum_{n=0}^{\infty} f(nd_z)e^{-jk_z nd}\frac{e^{-jkR_n}}{4\pi R_n}$$

$$\sum_{p=-\infty}^{+\infty}\int f(z)e^{-jk_{zp}z}\frac{e^{-jkR(z)}}{4\pi dR(z)}dz$$

Propagating modes

$$f(z) \simeq C_{0p} + \sum_{m=1}^{3} C_{mp}e^{j\frac{2\pi}{L2^{m-1}}} + \sum_{m=1}^{3} C_{mp}^{*}e^{-j\frac{2\pi}{L2^{m-1}}}$$

Coefficients are chosen to match:

- the value and the first derivative of *f* at the end-points and at the saddle point
- the area subtended by *f* to the line

↙ $_m$ uniform continuous lines

Evanescent modes

$$f(z) \simeq C_{0} + \sum_{m=1}^{2} C_{m}e^{j\frac{2\pi}{L2^{m-1}}} + \sum_{m=1}^{2} C_{m}^{*}e^{-j\frac{2\pi}{L2^{m-1}}}$$

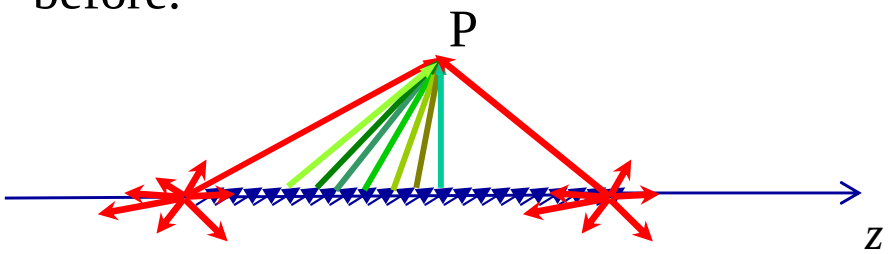
Coefficients are chosen to match:

- the value and the first derivative of *f* at the end-points

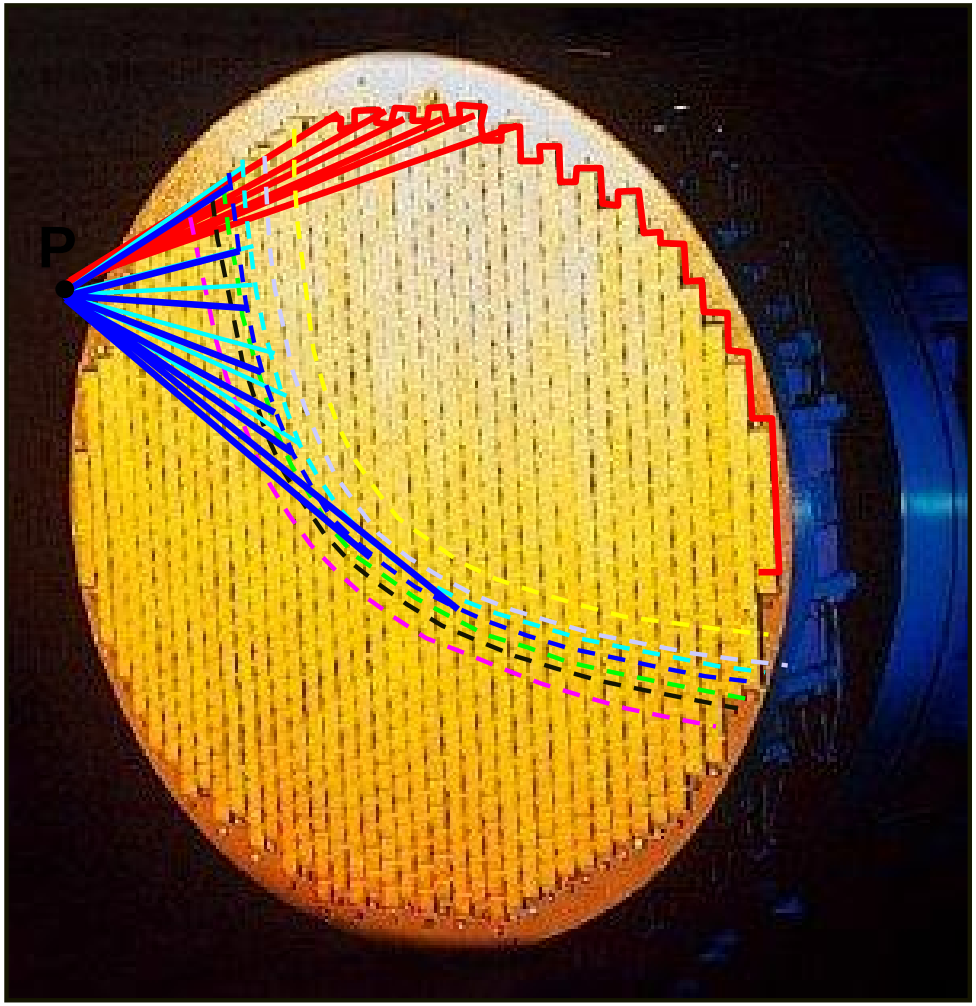
PLANAR ARRAY SYNTHESIS

- The radiation from the planar array is reconstructed by numerical summation of the linear arrays contributions.

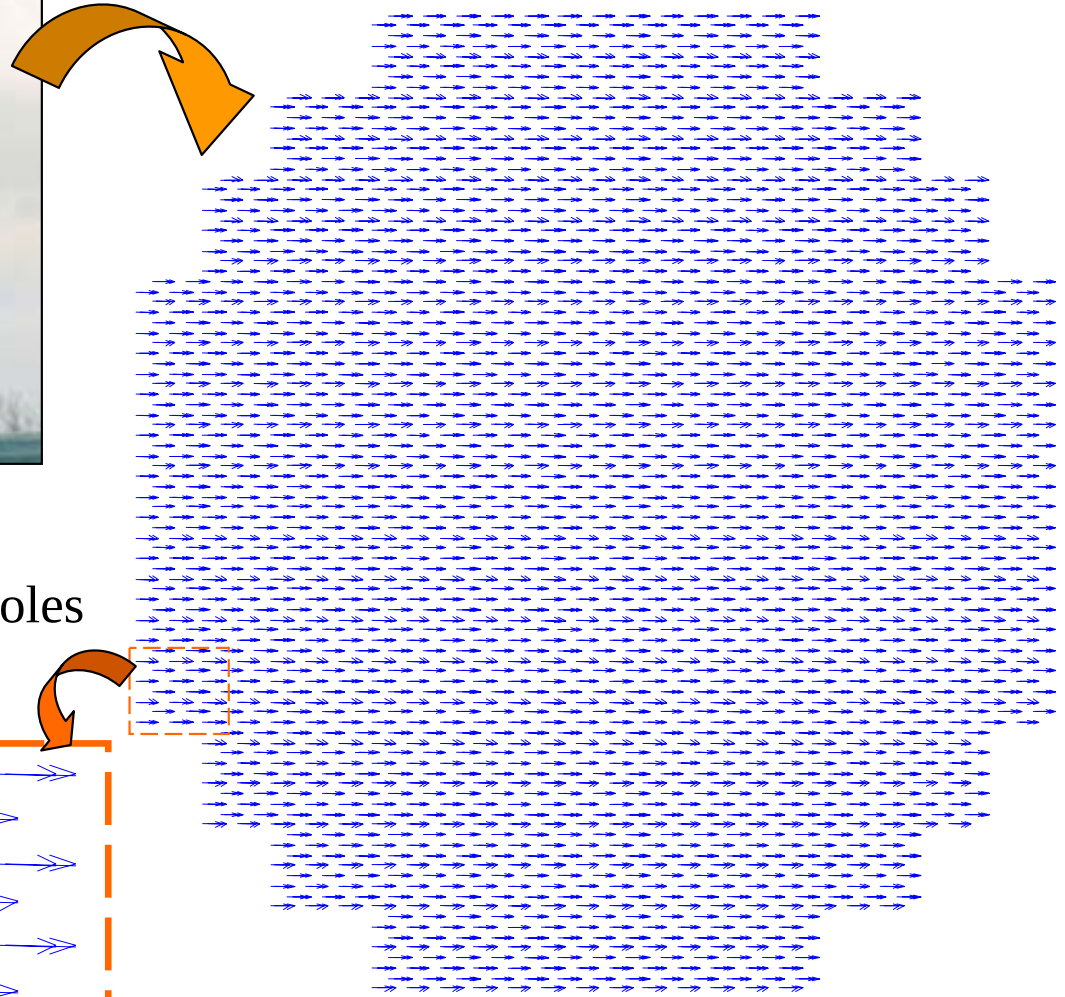
- These contributions are uniformly asymptotically evaluated as shown before.



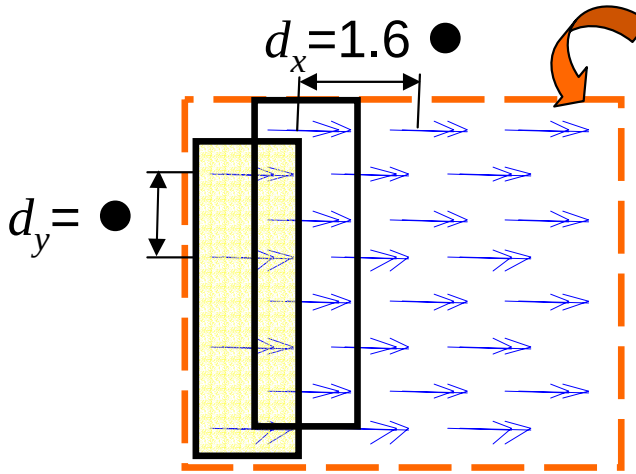
- Finally, the field radiated by the planar array is represented as the sum of diffracted fields from the actual rim plus (when needed) a limited number of conical waves arising from points located on the array surface.



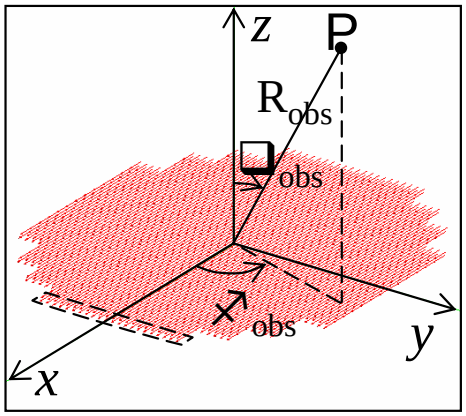
NUMERICAL RESULTS: ARRAY CONFIGURATION

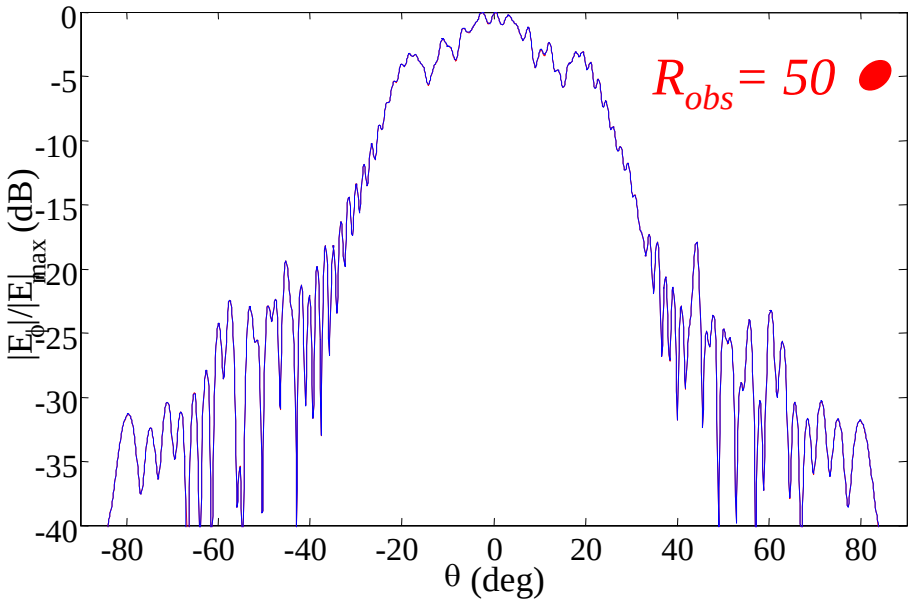


Array of 2160 elementary magnetic dipoles



NUMERICAL RESULTS: FREE SPACE RADIATION

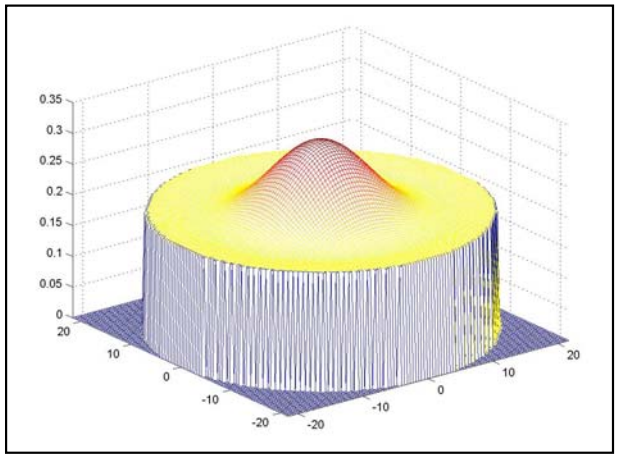




Bearing angles: $\varphi_0=0°$, $\theta_0=0°$

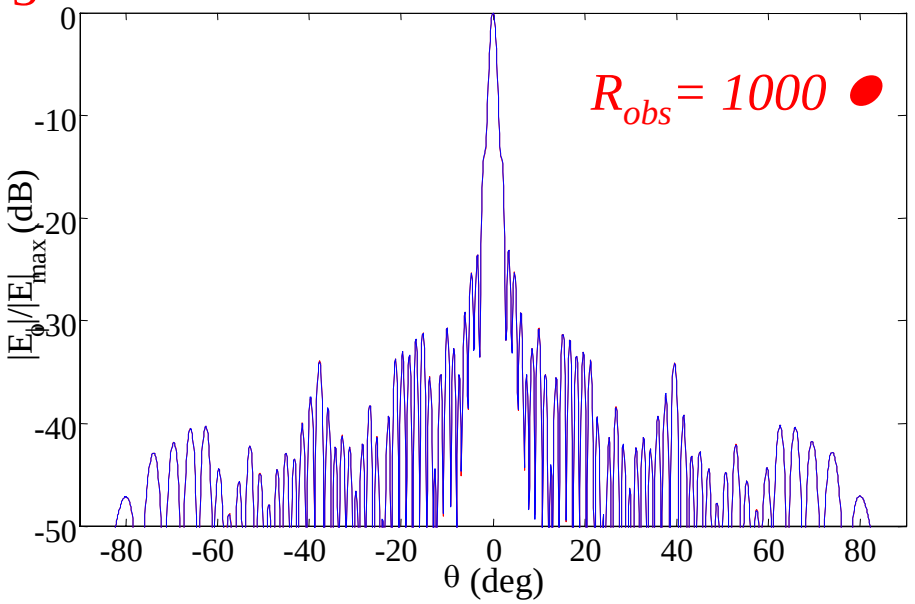
Observation plane: $\theta_{obs}=0°$

Taylor distribution with SLL = 20dB and n = 3

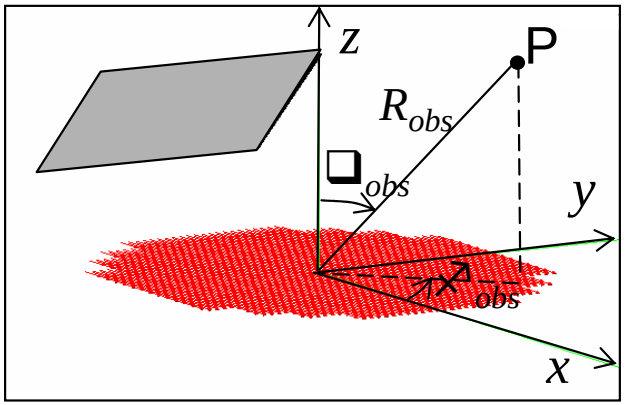


Line by line

Element by element



NUMERICAL RESULTS: INTERACTION WITH THE ENVIRONMENT

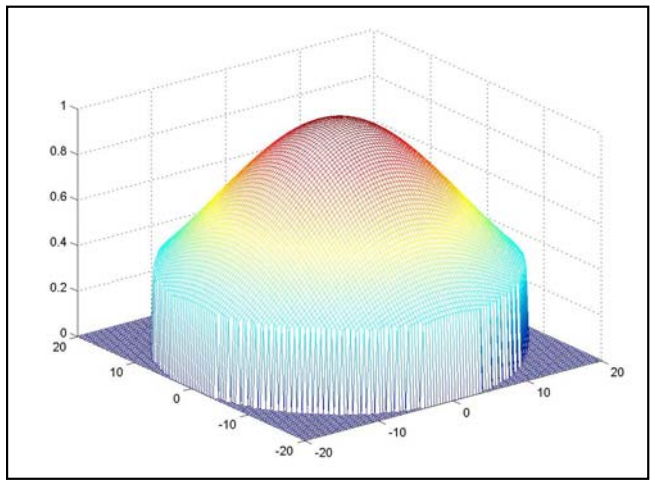


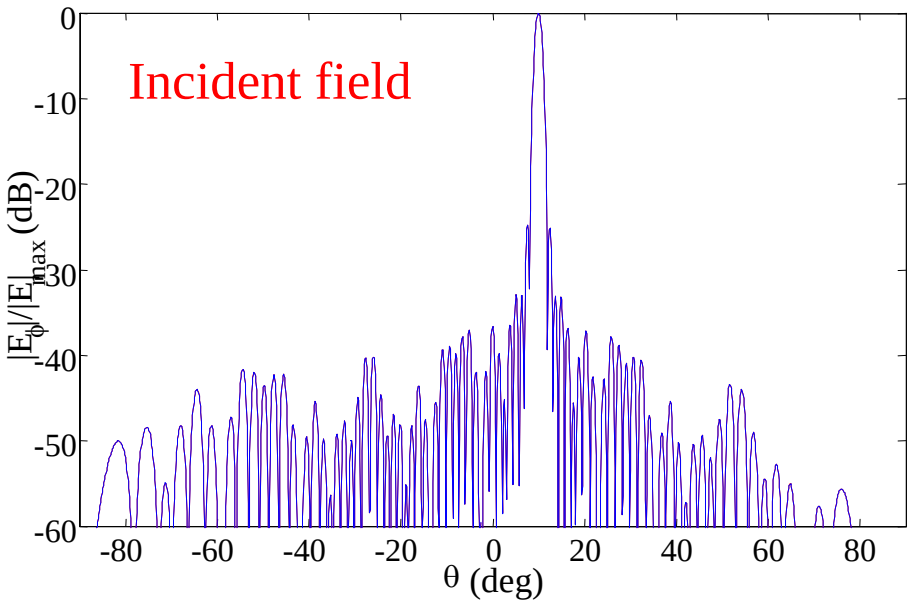
Bearing angles: φ_0= 10°, θ_0=0°

Observation: θ_{obs}=0°, R_{obs}=10000 λ

Gaussian distribution with σ = 14 λ

24 λ x 17 λ PEC plate





— Line by line + DFT

— FEKO (Element by element + PO)