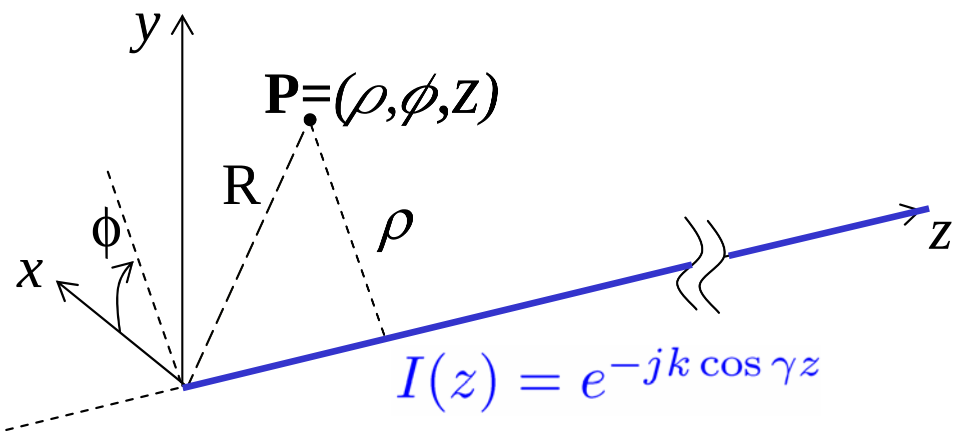
EXERCISE 1: SEMI-INFINITE CURRENT LINE

Semi-infinite current line with constant amplitude and linear phase



Magnetic vector potential $\mathbf{A}(P) = A_z(P)\hat{z}$

- Spatial integral representation

$$A_z(P) = \int_0^\infty \frac{e^{-jkR'}}{4\pi R'} e^{-jk\cos\gamma z'} dz'$$

$$R' = \sqrt{\rho^2 + (z-z')^2}$$

$$k_\rho = \sqrt{k^2 - k_z^2}$$

- Spectral integral representation

$$\frac{e^{-jkR'}}{4\pi R'} = \frac{1}{8\pi j}\int_{-\infty}^{\infty} H_0^{(2)}(k_\rho \rho) e^{-jk_z(z-z')} dk_z$$

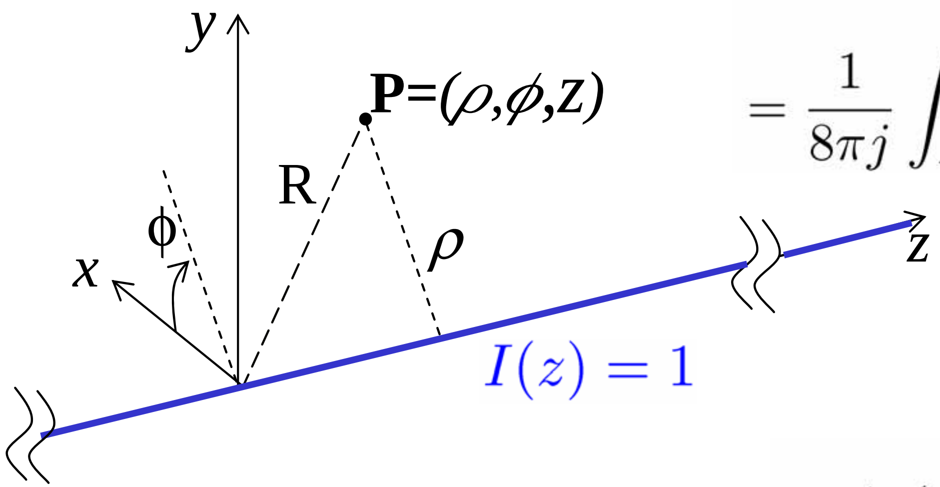
$$A_z(P) = \frac{1}{8\pi j}\int_{-\infty}^{\infty} H_0^{(2)}(k_\rho\rho)\, e^{-jk_z z}\int_0^\infty e^{j(k_z - k\cos\gamma)z'} dz'\, dk_z = \frac{1}{8\pi}\int_{-\infty}^{\infty} \frac{H_0^{(2)}(k_\rho\rho)\, e^{-jk_z z}}{k_z - k\cos\gamma} dk_z$$

BACKGROUND: INFINITE CURRENT LINE

Constant current

$$A_z(P) = \frac{1}{8\pi j}\int_{-\infty}^{\infty} H_0^{(2)}\left(k_\rho \rho\right) e^{-jk_z z}\int_{-\infty}^{+\infty} e^{jk_z z'}\, dz'\, dk_z =$$

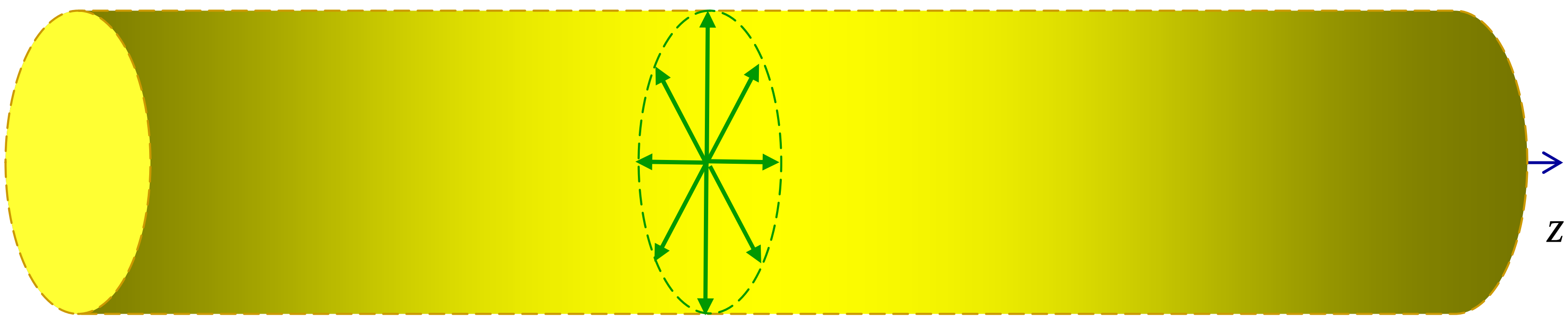
$$= \frac{1}{8\pi j}\int_{-\infty}^{+\infty} H_0^{2}(k_\rho \rho) e^{-jk_z z} 2\pi \delta(k_z) dk_z = \frac{1}{4j} H_0^{(2)}(k\rho)$$



Green's function for 2D problems

At large distances ($k\rho$>>1) $\Rightarrow A_z(P) \simeq \frac{1}{2\sqrt{2\pi j}} \frac{e^{-jk\rho}}{\sqrt{k\rho}}$ $\quad \left[H_0^{(2)}(x) \simeq \sqrt{\frac{2j}{\pi x}} e^{-jx} \quad x \gg 1 \right]$

Cylindrical wave



Cylindrical equiphase surfaces

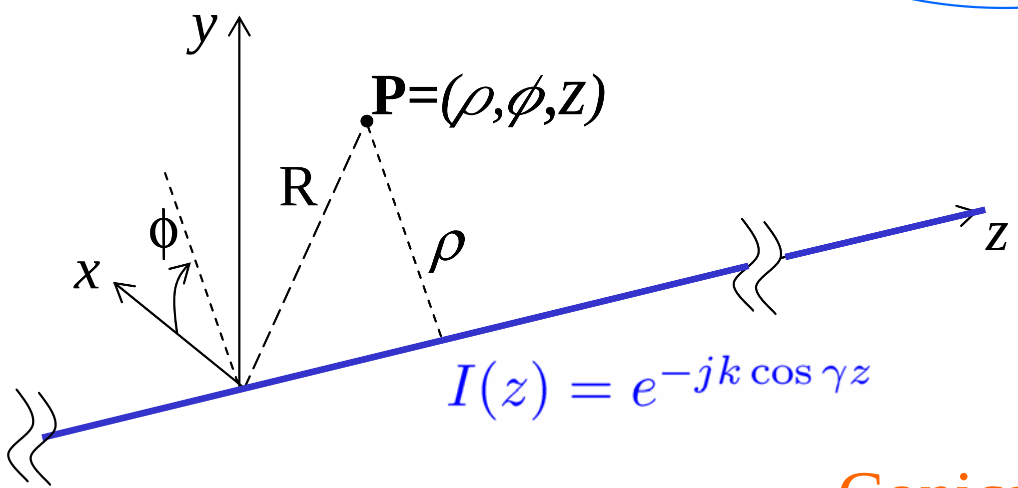
ρ=const

BACKGROUND: INFINITE CURRENT LINE

Linearly phased current

$$A_z(P) = \frac{1}{8\pi j}\int_{-\infty}^{\infty} H_0^{(2)}(k_\rho \rho)\, e^{-jk_z z}\int_{\infty}^{\infty} e^{j(k_z - k\cos\gamma)z'}\,dz'\,dk_z = \frac{1}{4j} H_0^{(2)}(k\sin\gamma\rho)\, e^{-jk\cos\gamma z}$$

$$2\pi\delta(k_z - k\cos\gamma)$$

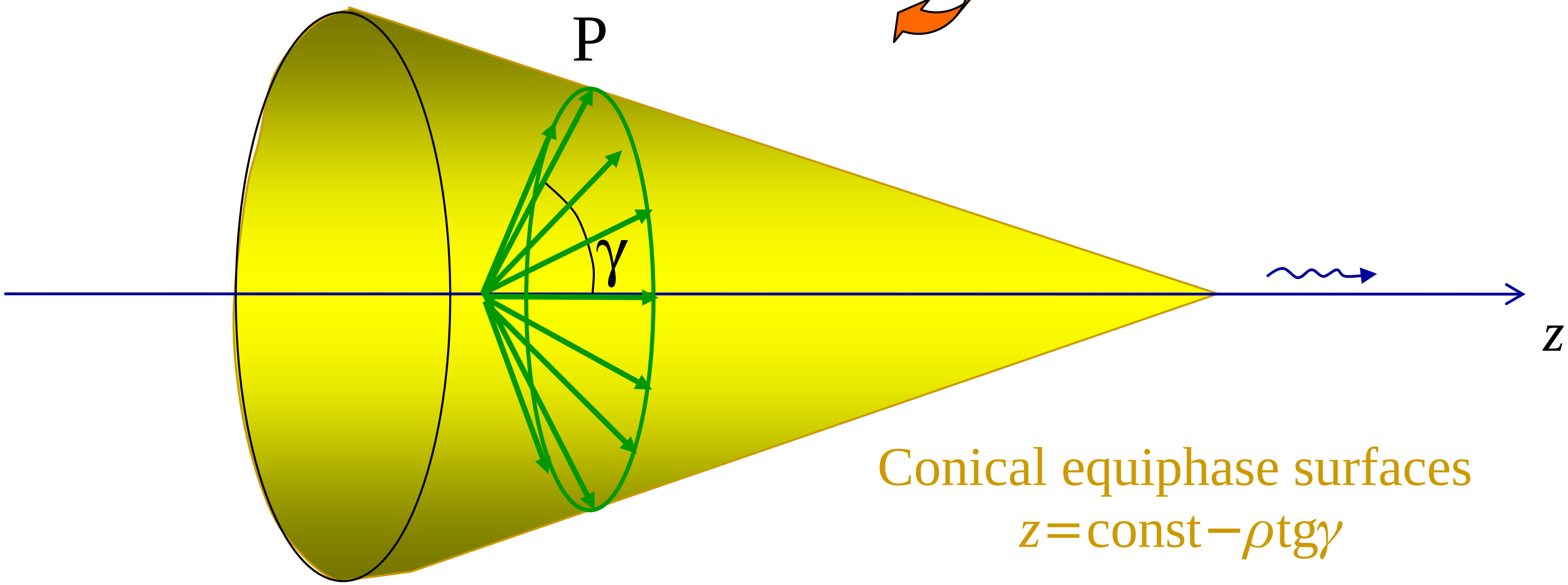


$$I(z) = e^{-jk\cos\gamma z}$$

At large distances ($k\rho \gg 1$)

$$A_z(P) \simeq \frac{1}{2\sqrt{2\pi j}}\,\frac{e^{-jk(\sin\gamma\rho + \cos\gamma z)}}{\sqrt{k\sin\gamma\rho}}$$

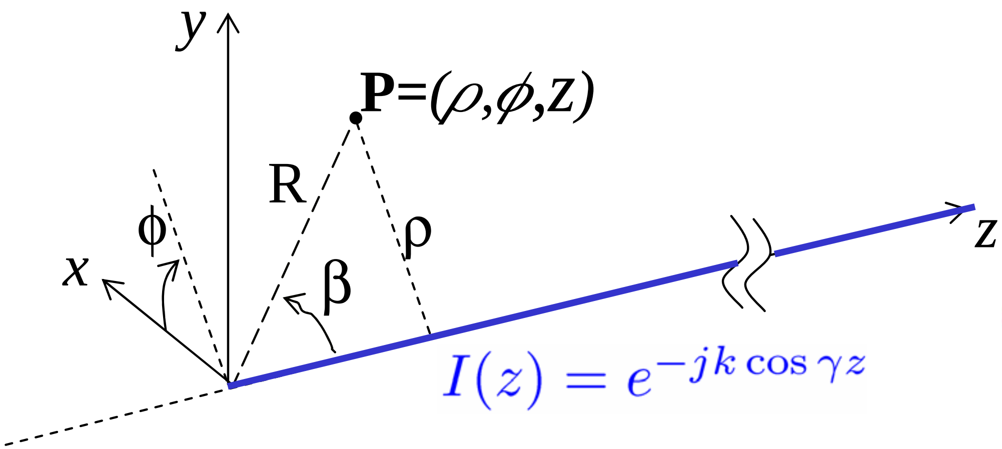
Conical wave



Conical equiphase surfaces

$z = \text{const} - \rho\,\text{tg}\gamma$

FIRST APPROACH: SPECTRAL DOMAIN INTEGRAL



After introducing the approximation of the Hankel function for large argument:

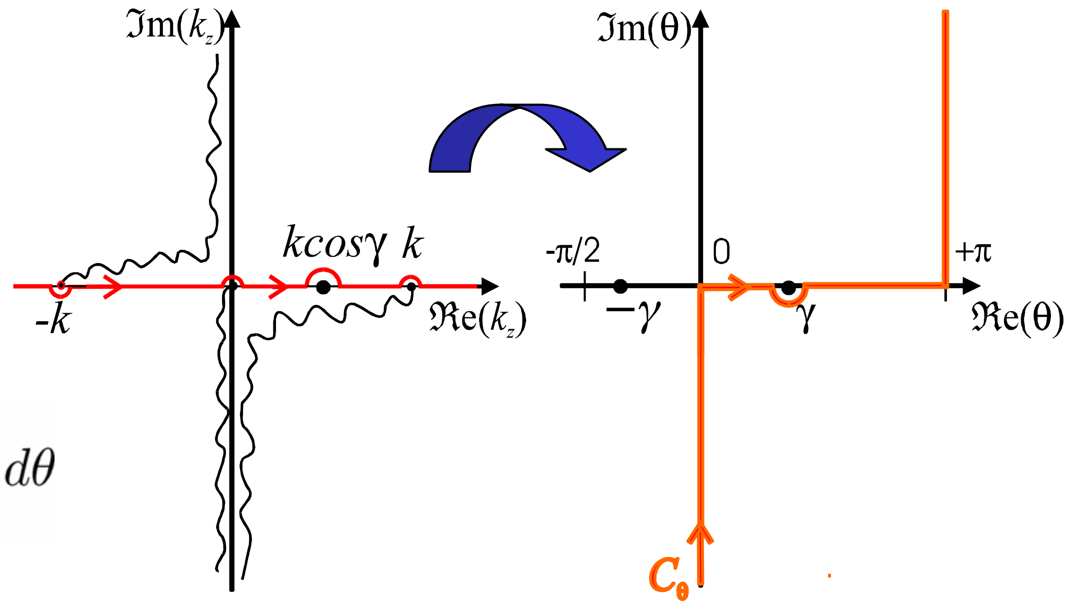
$$A_z(P) \simeq \frac{1}{8\pi}\sqrt{\frac{2j}{\pi\rho}}\int_{-\infty}^{\infty}\frac{e^{-j(k_\rho\rho+k_z z)}}{\sqrt{k_\rho}(k_z - k\cos\gamma)}dk_z$$

A spherical coordinate system is introduced both in the spatial and in the spectral domain

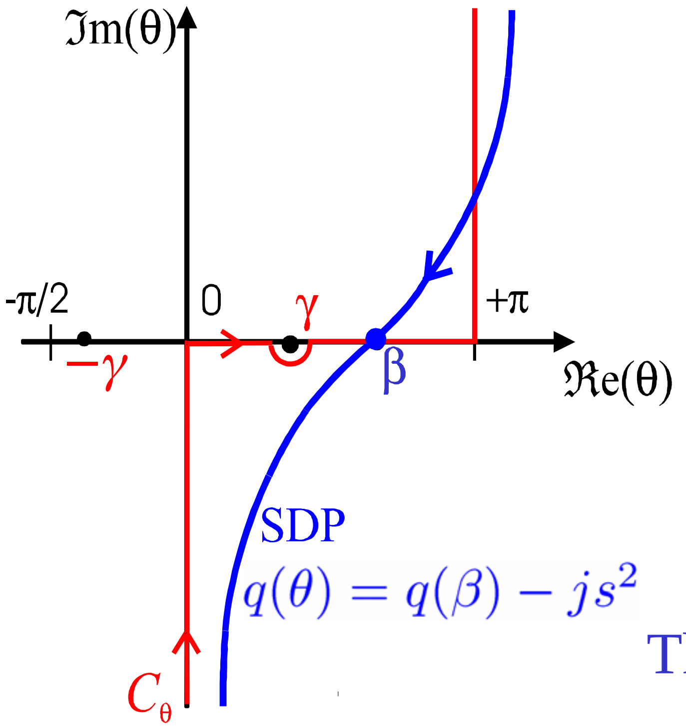
$$\begin{cases}\rho = R\sin\beta \\ z = R\cos\beta\end{cases} \qquad \begin{cases}k_\rho = k\sin\theta \\ k_z = k\cos\theta\end{cases}$$

$$A_z(P) \simeq \frac{1}{8\pi}\sqrt{\frac{2j}{\pi}}\int_{C_\theta}\frac{\sqrt{\sin\theta}}{k\rho}\frac{e^{-jkR\cos(\theta-\beta)}}{(\cos\theta - \cos\gamma)}d\theta$$

simple poles for $\theta = \pm\gamma$



ASYMPTOTIC EVALUATION OF THE SPECTRAL INTEGRAL



$$I_\theta = \int_{C_\theta} \sqrt{\sin\theta}\, \frac{e^{-jkR\cos(\theta-\beta)}}{\sqrt{k\rho}(\cos\theta - \cos\gamma)} d\theta = \int_{C_\theta} f(\theta) e^{-j\Omega q(\theta)} d\theta$$

$$\begin{cases} \Omega = kR \\ q(\theta) = \cos(\theta - \beta) \\ f(\theta) = \dfrac{\sqrt{\sin\theta}}{\sqrt{k\rho}(\cos\theta - \cos\gamma)} \end{cases}$$

$\Rightarrow$ first order saddle point at $\theta=\beta$

The integral can be asymptotically evaluated through the STEEPEST DESCENT METHOD

The original contour is deformed onto the steepest descent path through the saddle point

$$SDP: \begin{cases} u(s) = \Re e\{q(\beta)\} = 1 \\ v(s) = \Im m\{q(\beta)\} - js^2 = -js^2 \end{cases}$$

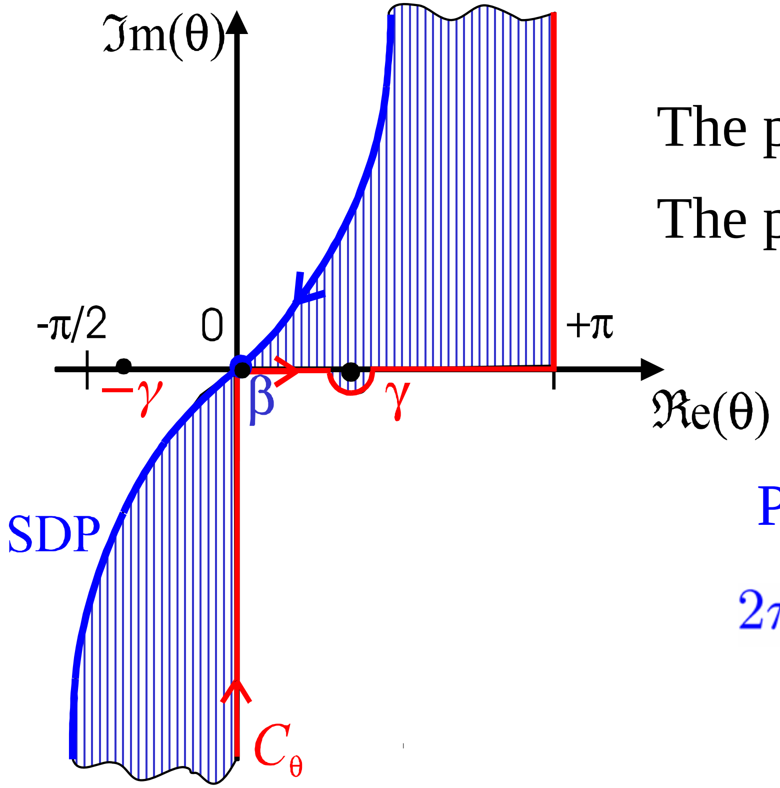
$\Rightarrow$ $\cos\left(\Re e\{\theta\} - \beta\right) \cosh\left(\Im m\{\theta\}\right) = 1$

The residues at the poles intercepted during path deformation must be included

$$I_\theta = \int_{SDP} \sqrt{\sin\theta}\, \frac{e^{-jkR\cos(\theta-\beta)}}{\sqrt{k\rho}(\cos\theta - \cos\gamma)} d\theta + 2\pi j \sum \mathrm{Res}\{\text{intercepted poles}\}$$

ASYMPTOTIC EVALUATION OF THE SPECTRAL INTEGRAL

POLE CONTRIBUTION

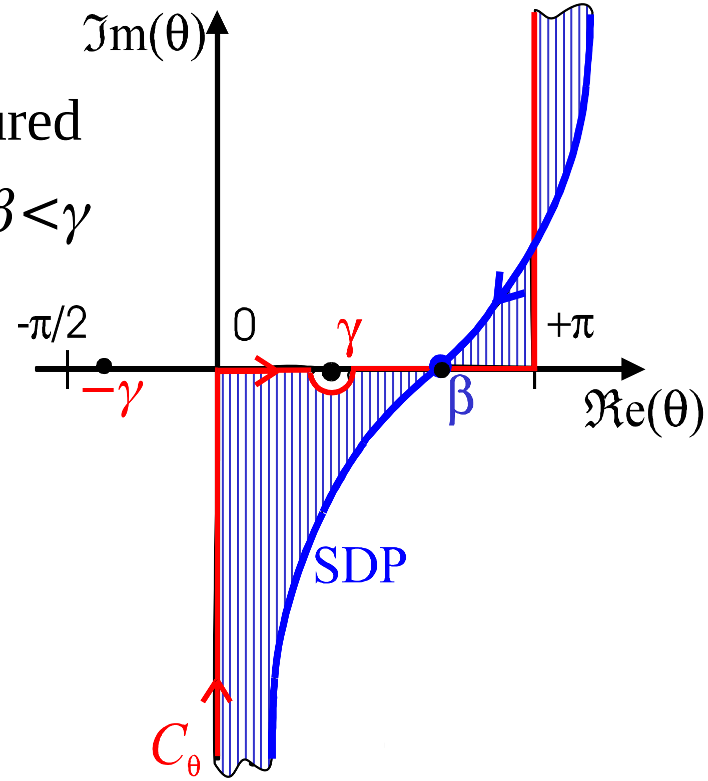


The pole $\theta = -\gamma$ is never captured

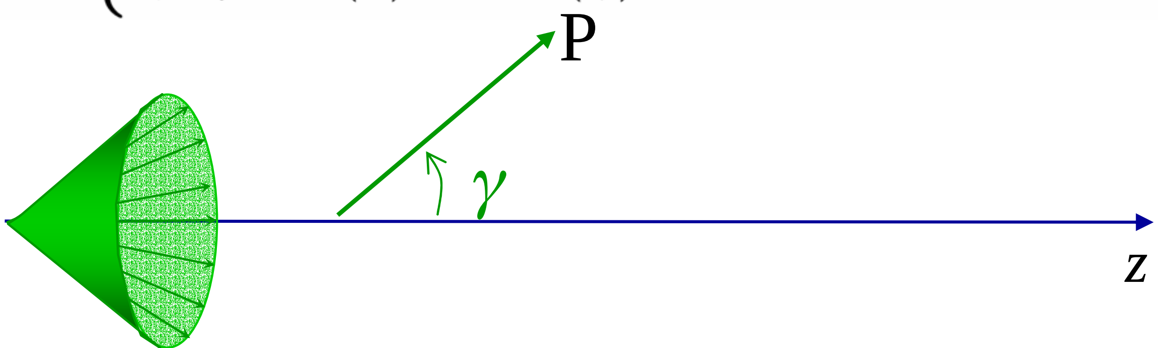
The pole $\theta = \gamma$ is captured for $\beta < \gamma$

Poles contribution to I_θ:

$$2\pi j \mathrm{Res}(\theta = \gamma) U(\gamma - \beta)$$



$$\mathrm{Res}\left\{\frac{\sqrt{\sin\theta}}{\sqrt{k\rho}\cos(\theta)-\cos(\gamma)}e^{-jkR\cos(\theta-\beta)}, \theta = \gamma\right\} = -\frac{e^{-jkR\cos(\gamma-\beta)}}{\sqrt{k\rho\sin\gamma}}$$



Conical wave with a conical shadow boundary

UNIFORM EVALUATION OF DIFFRACTION INTEGRAL

- **MULTIPLICATIVE (PAULI-CLEMMOW) APPROACH**

$$\int_{SDP} f(\theta)e^{-j\Omega q(\theta)}d\theta \sim f(\beta)e^{-j\Omega q(\beta)}\sqrt{\frac{2\pi}{j\Omega q''(\beta)}}F(\Omega b)$$

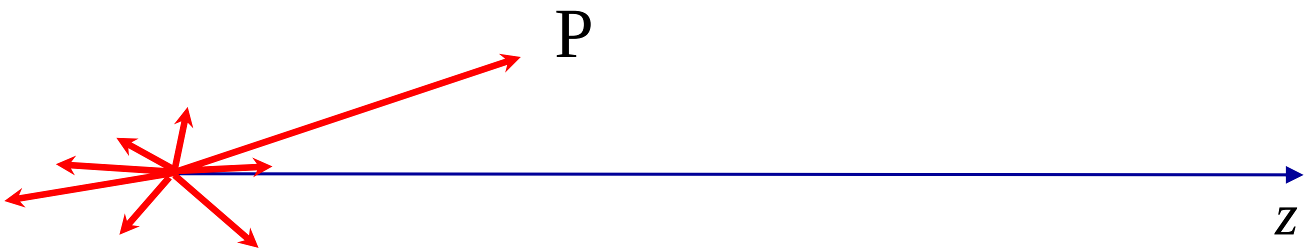
first order saddle point for $\theta=\beta$

$$b = q(\beta) - q(\gamma)$$

simple pole for $\theta = \gamma$

$$\begin{cases} \Omega = kR \\ q(\theta) = \cos(\theta-\beta) \\ f(\theta) = \dfrac{\sqrt{\sin\theta}}{\sqrt{k\rho}(\cos\theta - \cos\gamma)} \end{cases} \Rightarrow \begin{cases} q(\beta) = 1 \\ b = 2\sin^2\left(\frac{\gamma-\beta}{2}\right) \\ q''(\beta) = -1 \end{cases}$$

$$\int_{SDP} \sqrt{\sin\theta}\frac{e^{-jkR\cos(\theta-\beta)}}{\sqrt{k\rho}(\cos\theta-\cos\gamma)}d\theta \sim \sqrt{2\pi j}\frac{e^{-jkR}}{kR(\cos\beta-\cos\gamma)}F\left[2kR\sin^2\left(\frac{\gamma-\beta}{2}\right)\right]$$



Spherical wave from the line tip with uniform compensation

UNIFORM EVALUATION OF DIFFRACTION INTEGRAL

- **MULTIPLICATIVE (PAULI-CLEMMOW) APPROACH**

Finally, the high-frequency representation of the magnetic potential for the semi-infinite line is:

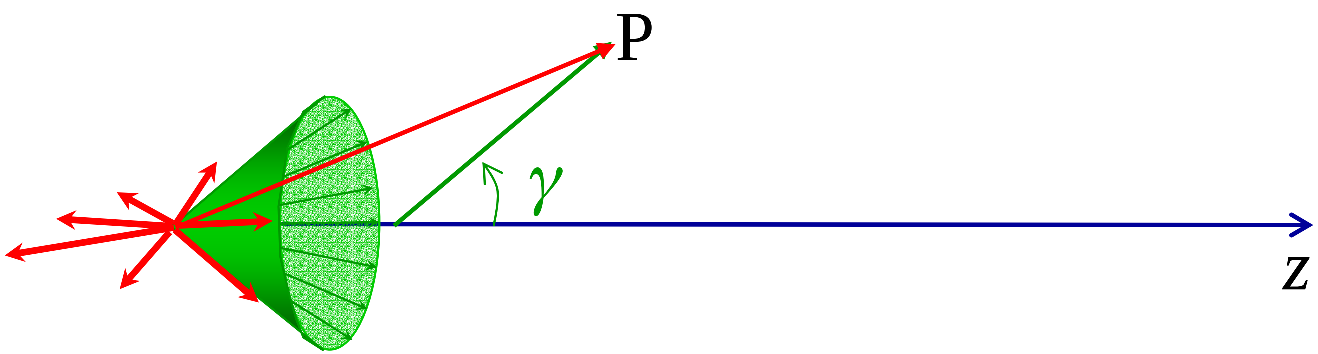
$$A_z(P) = A^{\infty}(P)U(\gamma - \beta) + A^{U}D(P)$$

$$\propto \frac{e^{-jkr\cos(\beta-\gamma)}}{\sqrt{k\rho}}$$

truncated FW

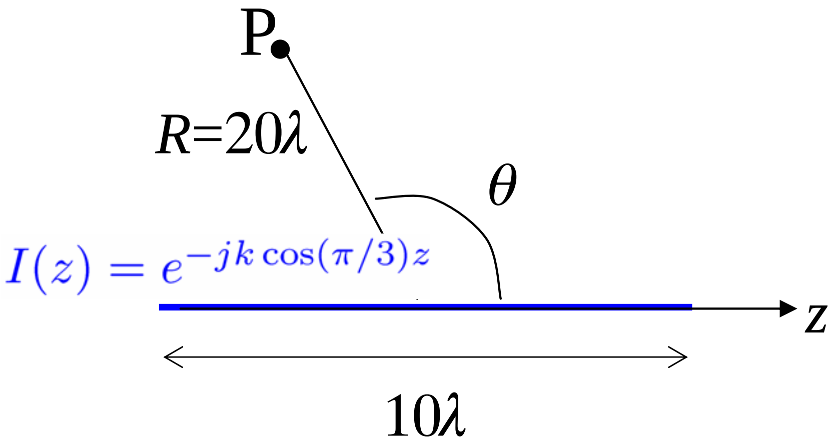
$$\propto \frac{e^{-jkR}}{kR}\frac{1}{(\cos\beta - \cos\gamma)}F\left[2kR\sin^2\left(\frac{\beta-\gamma}{2}\right)\right]$$

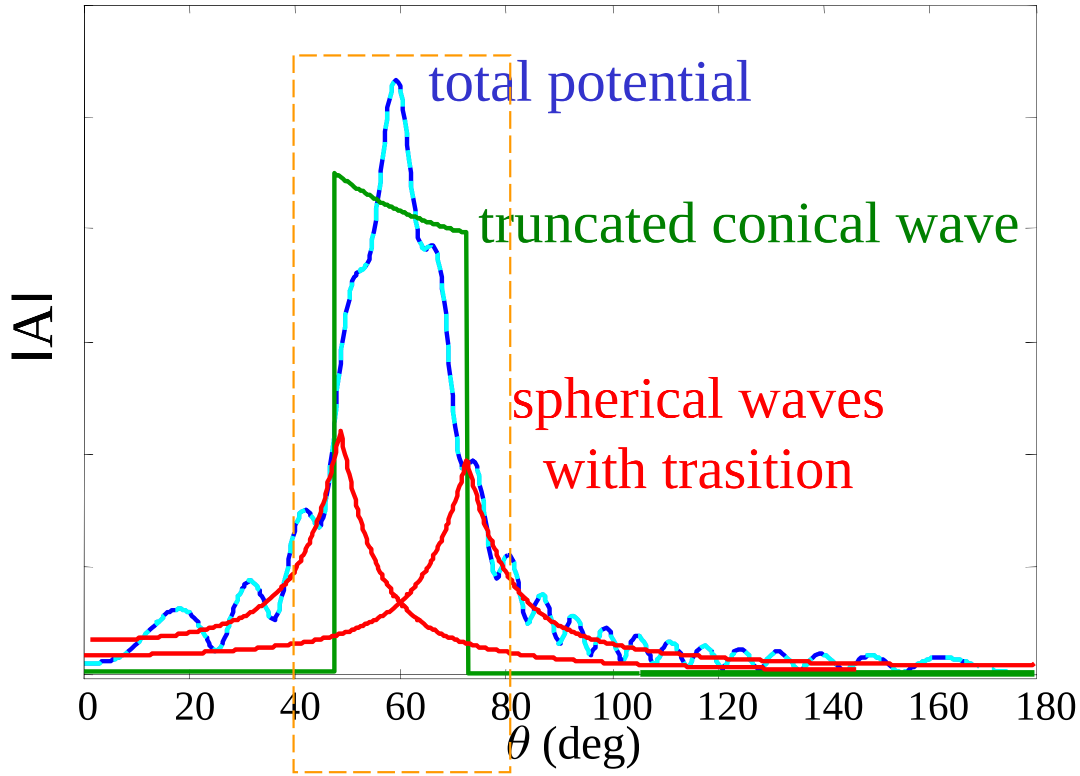
spherical diffracted wave with uniform compensation

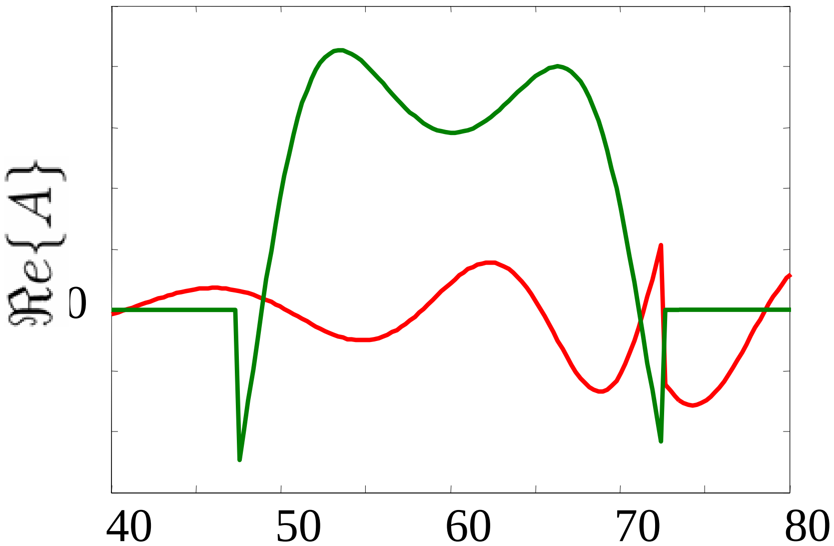


UNIFORM EVALUATION OF DIFFRACTION INTEGRAL

MULTIPLICATIVE APPROACH: NUMERICAL EXAMPLE







UNIFORM EVALUATION OF DIFFRACTION INTEGRAL

ADDITIVE (VAN DER WAERDEN) APPROACH

$$\int_{SDP} f(\theta)e^{-j\Omega q(\theta)}d\theta \sim e^{-j\Omega q(\beta)}\sqrt{\frac{\pi}{\Omega}}\frac{a}{\sqrt{-jb}}\left[1 - F\left(\Omega b\right)\right] + f(\beta)e^{-j\Omega q(\beta)}\sqrt{\frac{2\pi}{j\Omega q''(\beta)}}$$

first order saddle point at $\theta=\beta$

simple pole at $\theta = \gamma$

$$a = \lim_{\theta\to\gamma} f(\theta)(\theta-\gamma)$$

$$b = q(\beta) - q(\gamma)$$

$$\begin{cases} \Omega = kR \\ q(\theta) = \cos(\theta-\beta) \\ f(\theta) = \dfrac{\sqrt{\sin\theta}}{\sqrt{k\rho}(\cos\theta - \cos\gamma)} \end{cases} \Rightarrow \begin{cases} q(\beta) = 1 \\ b = 2\sin^2\left(\frac{\gamma-\beta}{2}\right) \\ q''(\beta) = -1 \\ a = -\dfrac{1}{\sqrt{k\rho}\sqrt{\sin\gamma}} \end{cases}$$

$$\int_{SDP} f(\theta)e^{-j\Omega q(\theta)}d\theta \sim \frac{e^{-jkR}}{kR}\frac{\sqrt{2\pi j}}{(\cos\beta-\cos\gamma)} - \frac{e^{-jkR}}{kR}\frac{\sqrt{\pi}}{\sqrt{-2j\sin\gamma\sin\beta}\sin\left(\frac{\gamma-\beta}{2}\right)}\left[F\left(\delta^2\right)-1\right]$$

$$\delta = \sqrt{2kR}\sin\left(\frac{\gamma-\beta}{2}\right)$$

Spherical wave from the line tip + uniform compensation term

UNIFORM EVALUATION OF DIFFRACTION INTEGRAL

ADDITIVE (VAN DER WAERDEN) APPROACH

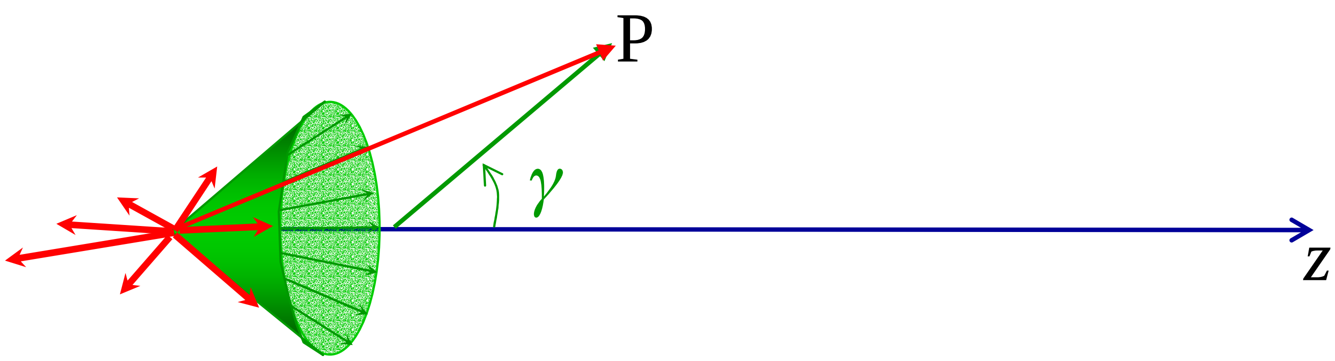
Finally, the high-frequency representation of the magnetic potential for the semi-infinite line is:

$$A(P) = A^{\infty}(P)U(\gamma-\beta) + A^{U}(P) + A^{D}(P)$$

$$\propto \frac{e^{-jkr\cos(\beta-\gamma)}}{\sqrt{k\rho}}$$ truncated FW

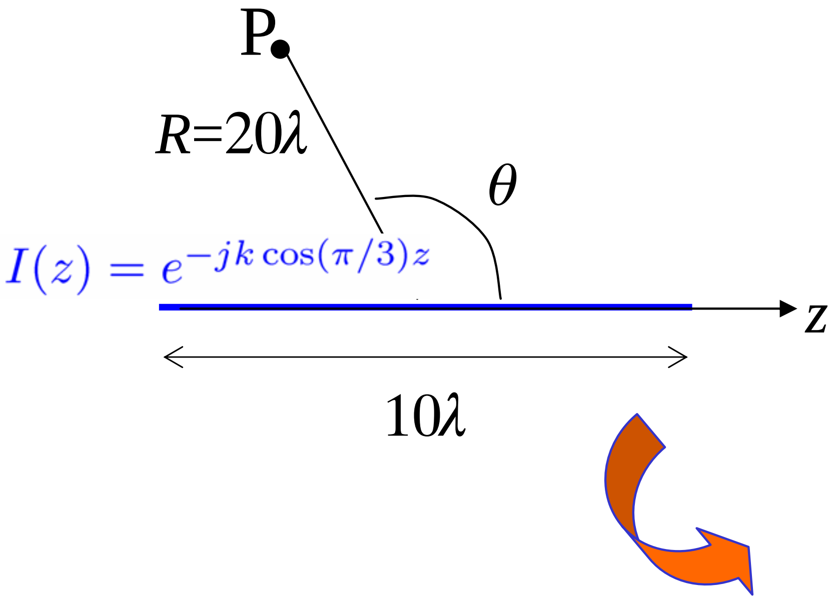
$$\propto \frac{e^{-jkR}}{kR}\frac{1}{(\cos\beta-\cos\gamma)}$$ spherical diffracted wave

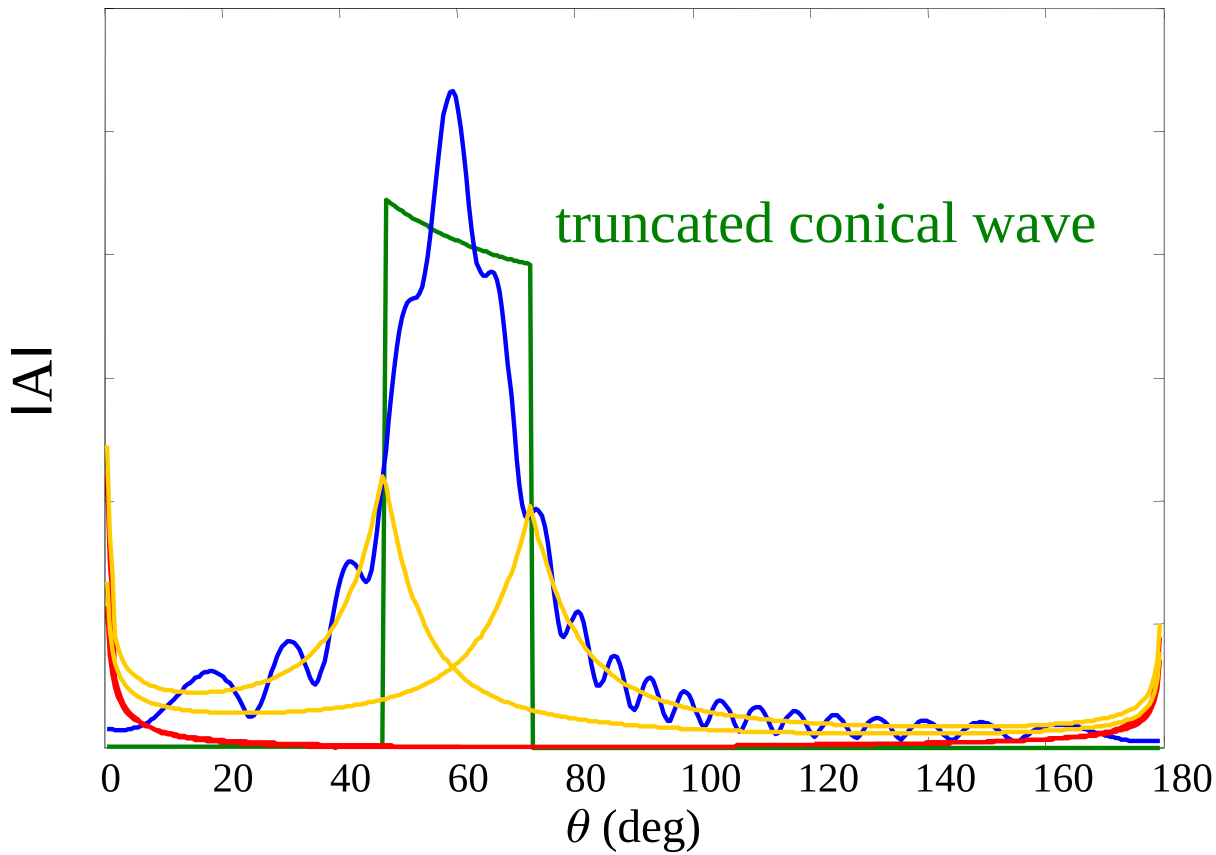
$$\propto \frac{e^{-jkR}}{kR}\frac{1}{\sqrt{\sin\beta}\sin\left(\dfrac{\beta-\gamma}{2}\right)}\left\{F\left[2kR\sin^2\left(\frac{\beta-\gamma}{2}\right)\right]-1\right\}$$ uniform contribution



UNIFORM EVALUATION OF DIFFRACTION INTEGRAL

- ## ADDITIVE APPROACH: NUMERICAL EXAMPLE



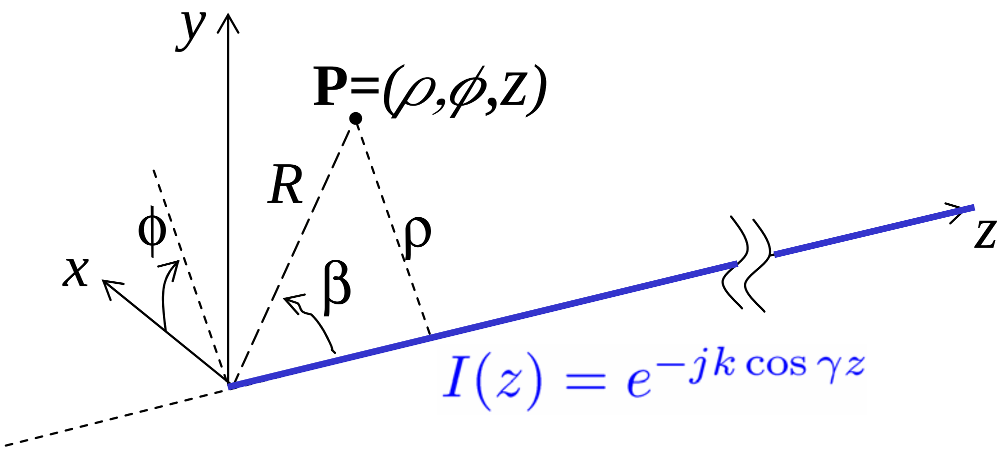


$$-\frac{e^{-jkR\cos(\gamma-\beta)}}{\sqrt{k\rho\sin\gamma}}$$

$$\frac{e^{-jkR}}{kR}\left\{\frac{\sqrt{2\pi j}}{(\cos\beta-\cos\gamma)}+\frac{\sqrt{\pi}}{\sqrt{-2j\sin\gamma\sin\beta\sin\left(\frac{\gamma-\beta}{2}\right)}}\right\}$$

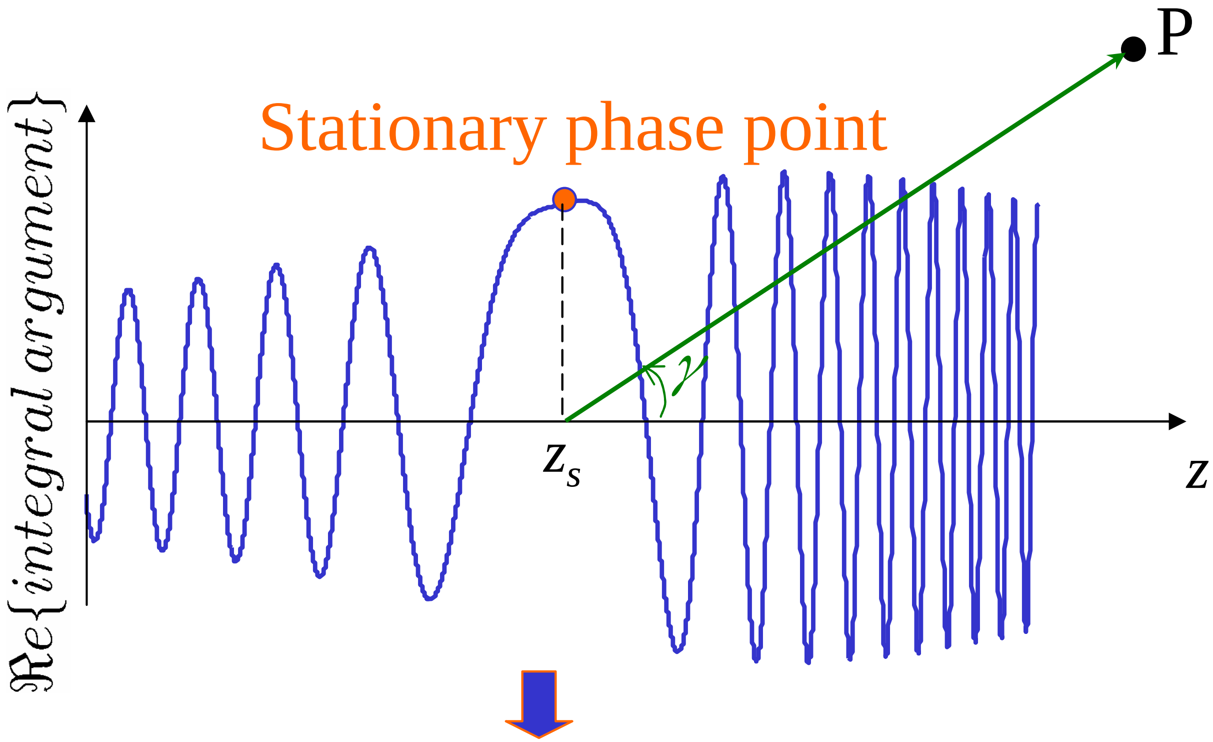
$$-\frac{e^{-jkR}}{kR}\frac{\sqrt{\pi}}{\sqrt{-2j\sin\gamma\sin\beta\sin\left(\frac{\gamma-\beta}{2}\right)}}F\left(\delta^2\right)$$

SECOND APPROACH: SPATIAL DOMAIN INTEGRAL



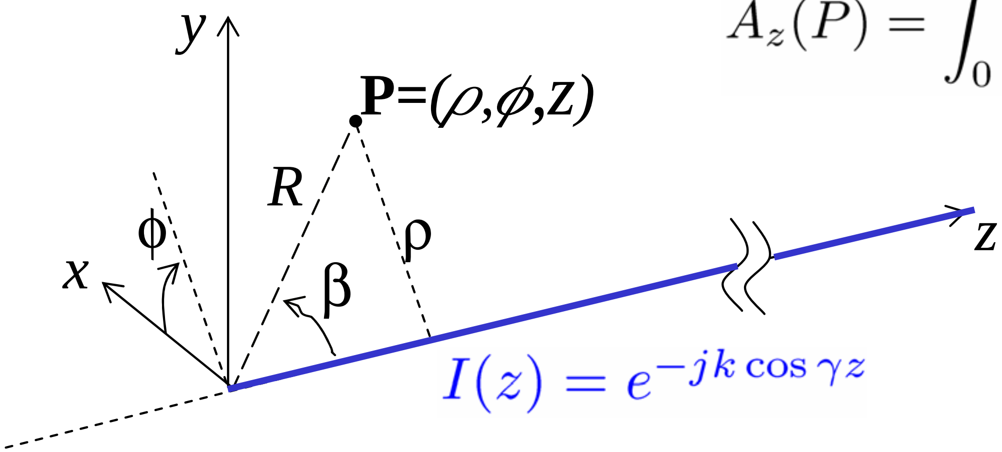
$$A_z(P) = \int_0^{\infty} \frac{e^{-jk(R' + \cos\gamma z')}}{4\pi R'} dz'$$

The phase of the integrand is highly oscillatory along the integration path



The integral can be asymptotically evaluated through the STATIONARY PHASE METHOD

SECOND APPROACH: SPATIAL DOMAIN INTEGRAL



$$A_z(P) = \int_0^{\infty} \frac{e^{-jk[R'+\cos\gamma z']}}{4\pi R'} dz' = \int_{z_a}^{\infty} f(z') e^{\Omega q(z')} dz'$$

$$\begin{cases} z_a = 0 \\ \Omega = k\rho \\ f(z') = \dfrac{1}{4\pi R'} \\ q(z') = \dfrac{-j}{\rho}(R' + z'\cos\gamma) \end{cases}$$

$$R' = \sqrt{\rho^2 + (z - z')^2}$$

$$q'(z') = \frac{j}{\rho}\left(\frac{z - z'}{R} - \cos\gamma\right)$$

first order saddle point at

$z_s = z - \rho\cot\gamma$

$$I(\Omega) \simeq G(0)\frac{e^{\Omega q(z_s)}}{\sqrt{\Omega}}\sqrt{\pi}U\left(-\Re\{\sqrt{\Omega}s_a\}\right)\frac{1}{2\Omega}\frac{G(s_a)}{s_a}e^{\Omega q(z_a)} + G(0)\frac{e^{\Omega q(z_a)}}{2\Omega s_a}\left[F(-j\Omega s_a^2) - 1\right]$$

$$s_a = \sqrt{q(z_s) - q(z_a)} = \sqrt{\frac{2j}{\sin\beta}}\sin\left(\frac{\gamma-\beta}{2}\right) \qquad G(0) = f(z_s)\sqrt{\frac{-2}{q''(z_s)}} = -j\frac{\sqrt{2j}}{4\pi\sqrt{\sin\gamma}}$$

$$\frac{G(s_a)}{s_a} = \frac{-2f(0)}{q'(0)} = \frac{1}{2\pi}\frac{1}{j(\cos\gamma - \cos\beta)}$$

SECOND APPROACH: SPATIAL DOMAIN INTEGRAL

- Stationary phase point contribution

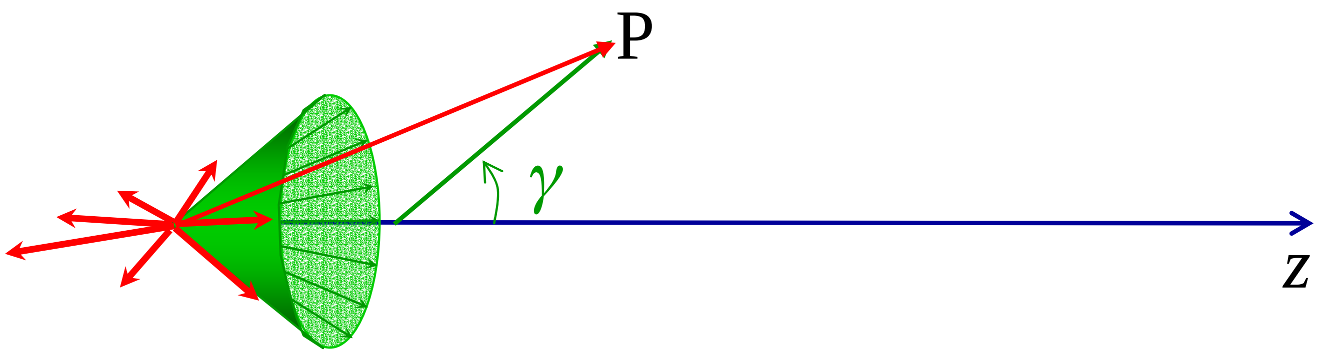
$$G(0)\frac{e^{\Omega q(z_s)}}{\sqrt{\Omega}}\sqrt{\pi}U\left(-\Re\{\sqrt{\Omega}s_a\}\right) = -j\frac{\sqrt{2j}}{4\sqrt{\pi}}\frac{e^{-jkR\cos(\beta-\gamma)}}{\sqrt{k\rho\sin\gamma}}U\left(\gamma-\beta\right)$$

- End-point contribution

$$\frac{1}{2\Omega}\frac{G(s_a)}{s_a}e^{\Omega q(z_a)} = \frac{j}{4\pi kR}\frac{e^{-jkR}}{(\cos\beta-\cos\gamma)}$$

- Uniform compensation term

$$G(0)\frac{e^{\Omega q(z_a)}}{2\Omega s_a}\left[F(-j\Omega s_a^2)-1\right] = \frac{j}{8kR\pi\sqrt{\sin\gamma\sin\beta}}\frac{e^{-jkR}}{\sin\left(\frac{\beta-\gamma}{2}\right)}\left\{F\left[2kR\sin^2\left(\frac{\beta-\gamma}{2}\right)\right]-1\right\}$$



EXERCISE 2: SLOPE UTD TRANSITION FUNCTION

UTD Transition function $F(x)$: $$F(x) = 2j\sqrt{x}e^{jx}\int_{\sqrt{x}}^{\infty} e^{-jt^2}dt, \quad -\frac{3}{2}\pi < \arg(x) < -\frac{\pi}{2}$$

Slope UTD Transition function $F_s(x)$: $$F_s(x) = 2jx\left[F(x) - 1\right]$$

Using the following equality

$$-\sqrt{\frac{\pi}{\Omega}}\frac{F(j\Omega s_0^2)}{s_0} = \int_{-\infty}^{\infty}\frac{e^{-\Omega s^2}}{s - s_0}ds$$

Saddle point + simple pole singularity

find the integrals

1) $\int_{-\infty}^{\infty}\frac{e^{-\Omega s^2}}{(s - s_0)^2}ds = ?$

saddle point + double pole

2) $\int_{-\infty}^{\infty}\frac{s\,e^{-\Omega s^2}}{s - s_0}ds = ?$

saddle point + simple zero + simple pole

3) $\int_{-\infty}^{\infty}\frac{s^2 e^{-\Omega s^2}}{s - s_0}ds = ?$

saddle point + double zero+ simple pole

4) $\int_{-\infty}^{\infty}\frac{e^{-\Omega s^2}}{(s - s_0)(s - s_1)}ds = ?$

saddle point + 2 simple poles

EXERCISE 2: SLOPE UTD TRANSITION FUNCTION

Proof

1) $$\int_{-\infty}^{\infty} \frac{e^{-\Omega s^2}}{(s-s_0)^2} ds = \frac{d}{ds_0}\left[\int_{-\infty}^{\infty} \frac{e^{-\Omega s^2}}{s-s_0} ds\right] = -\sqrt{\frac{\pi}{\Omega}} \frac{d}{ds_0}\left[\frac{F(j\Omega s_0^2)}{s_0}\right]$$

$$\frac{d}{dx}\left[\frac{F(x^2)}{x}\right] = \frac{d}{dx}\left[2je^{jx^2}\int_x^{\infty} e^{-jt^2}dt\right] = 2j\left[F(x^2)-1\right] = \frac{F_S(x^2)}{x^2}$$

$$\frac{d}{dx}\int_x^{\infty} e^{-jt^2}dt = -e^{-jx^2}$$

$$\int_{-\infty}^{\infty} \frac{e^{-\Omega s^2}}{(s-s_0)^2} ds = -\sqrt{\frac{\pi}{\Omega}} \frac{d}{ds_0}\left[\frac{F(j\Omega s_0^2)}{s_0}\right] = \sqrt{\frac{\pi}{\Omega}} \frac{F_s(j\Omega s_0^2)}{s_0^2}$$

$$\boxed{\int_{-\infty}^{\infty} \frac{e^{-\Omega s^2}}{(s-s_0)^2} ds = \sqrt{\frac{\pi}{\Omega}} \frac{F_s(j\Omega s_0^2)}{s_0^2}}$$

Saddle point + double pole singularity $\Longleftrightarrow$ Slope UTD transition function F_s

EXERCISE 2: SLOPE UTD TRANSITION FUNCTION

2) $$\int_{-\infty}^{\infty} \frac{s\, e^{-\Omega s^2}}{s-s_0} ds = \int_{-\infty}^{\infty} \frac{(s-s_0)\, e^{-\Omega s^2}}{s-s_0} ds + s_0 \int_{-\infty}^{\infty} \frac{e^{-\Omega s^2}}{s-s_0} ds = \sqrt{\frac{\pi}{\Omega}} - \sqrt{\frac{\pi}{\Omega}} F(j\Omega s_0^2) = -\frac{1}{2\Omega}\sqrt{\frac{\pi}{\Omega}} \frac{F_s(j\Omega s_0^2)}{s_0^2}$$

$$\boxed{\int_{-\infty}^{\infty} \frac{s\, e^{-\Omega s^2}}{s-s_0} ds = -\frac{1}{2\Omega}\sqrt{\frac{\pi}{\Omega}} \frac{F_s(j\Omega s_0^2)}{s_0^2}}$$

3) $$\int_{-\infty}^{\infty} \frac{s^2 e^{-\Omega s^2}}{s-s_0} ds = \int_{-\infty}^{\infty} \frac{(s^2 - s_0^2)\, e^{-\Omega s^2}}{s-s_0} ds + s_0^2 \int_{-\infty}^{\infty} \frac{e^{-\Omega s^2}}{s-s_0} ds = \cancelto{0}{\int_{-\infty}^{\infty} s\, e^{-\Omega s^2} ds} + s_0 \int_{-\infty}^{\infty} e^{-\Omega s^2} ds +$$

$$+ s_0^2 \int_{-\infty}^{\infty} \frac{e^{-\Omega s^2}}{s-s_0} ds = s_0 \sqrt{\frac{\pi}{\Omega}} - s_0 \sqrt{\frac{\pi}{\Omega}} F(j\Omega s_0^2) = -\frac{1}{2\Omega}\sqrt{\frac{\pi}{\Omega}} \frac{F_s(j\Omega s_0^2)}{s_0}$$

$$\boxed{\int_{-\infty}^{\infty} \frac{s^2 e^{-\Omega s^2}}{s-s_0} ds = -\frac{1}{2\Omega}\sqrt{\frac{\pi}{\Omega}} \frac{F_s(j\Omega s_0^2)}{s_0}}$$

EXERCISE 2: SLOPE UTD TRANSITION FUNCTION

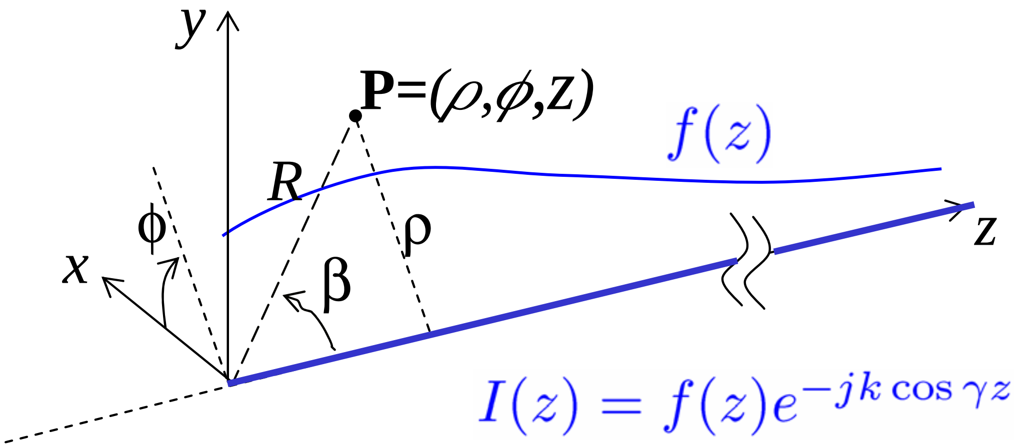
4) $$\frac{1}{(s-s_0)(s-s_1)} = \frac{A}{s-s_0} + \frac{B}{s-s_1}$$

$$\begin{cases} A+B=0 \\ -As_1 - Bs_0 = 1 \end{cases} \Rightarrow \begin{cases} A = -1/(s_1 - s_0) \\ B = 1/(s_1 - s_0) \end{cases}$$

$$\int_{-\infty}^{\infty} \frac{e^{-\Omega s^2}}{(s-s_0)(s-s_1)} ds = \frac{1}{(s_1 - s_0)} \left[-\int_{-\infty}^{\infty} \frac{e^{-\Omega s^2}}{s-s_0} ds + \int_{-\infty}^{\infty} \frac{e^{-\Omega s^2}}{s-s_1} ds \right] = \frac{1}{s_0 s_1} \sqrt{\frac{\pi}{\Omega}} \left[\frac{s_1 F(j\Omega s_0^2) - s_0 F(j\Omega s_1^2)}{s_1 - s_0} \right]$$

$$\boxed{\int_{-\infty}^{\infty} \frac{e^{-\Omega s^2}}{(s-s_0)(s-s_1)} ds = \frac{1}{s_0 s_1} \sqrt{\frac{\pi}{\Omega}} \left[\frac{s_1 F(j\Omega s_0^2) - s_0 F(j\Omega s_1^2)}{s_1 - s_0} \right]}$$

EXAMPLE: SEMI-INFINITE TAPERED LINE



$$A_z(P) = \frac{j}{8\pi}\int_{-\infty}^{\infty} H_0^{(2)}(k_\rho \rho)\, e^{-jk_z z}\int_0^{\infty} f(z')e^{j(k_z - k\cos\gamma)z'}\,dz'dk_z$$

$$\int_0^{\infty} f(z')e^{-j(k\cos\gamma - k_z)z'}dz' = \sum_{n=0}^{\infty}\frac{f^n(0)}{n!}\int_0^{\infty} e^{-j(k\cos\gamma - k_z)z'}z'^n dz = \sum_{n=0}^{\infty}\frac{(-1)^n f^n(0)}{\left[j\left(k_z - k\cos\gamma\right)\right]^{n+1}}$$

$$\Downarrow$$

$$\int_{\infty}^{\infty} H_0^{(2)}(k_\rho\rho)e^{-jk_z z}\int_0^{\infty} f(z)e^{-j(k\cos\gamma - k_z)z}dzdk_z = \sum_{n=0}^{\infty}\frac{(-1)^n f^n(0)}{j^{n+1}}\int_{\infty}^{\infty}\frac{H_0^{(2)}(k_\rho\rho)e^{-jk_z z}}{(k_z - k\cos\gamma)^{n+1}}dk_z$$

EXAMPLE: SEMI-INFINITE TAPERED LINE

In the θ-domain

$$A_z(P) \propto \sum_{n=0}^{\infty} (-1)^n f^n(0) \int_{\mathcal{C}_\theta} \frac{\sqrt{\sin\theta}}{\sqrt{\sin\beta}} \frac{e^{-jkr\cos(\beta-\theta)}}{\left[jk\left(\cos\beta - \cos\gamma\right)\right]^{n+1}} d\theta$$

- Lowest asymptotic order diffraction term (n = 0)

$$\sum_{n=0}^{\infty} f(0) \int_{SDP} \frac{\sqrt{\sin\theta}}{\sqrt{\sin\beta}} \frac{e^{-jkr\cos(\beta-\theta)}}{\left[jk\left(\cos\beta - \cos\gamma\right)\right]} d\theta$$

- First higher asymptotic order diffraction term (n = 1)

$$\sum_{n=0}^{\infty} -f'(0) \int_{SDP} \frac{\sqrt{\sin\theta}}{\sqrt{\sin\beta}} \frac{e^{-jkr\cos(\beta-\theta)}}{\left[jk\left(\cos\beta - \cos\gamma\right)\right]^{2}} d\theta$$

double pole at $\theta=\gamma$ $\Rightarrow$ Slope UTD transition function