THE EUROPEAN SCHOOL OF ANTENNAS

High-Frequency Scattering from Impedance Wedges and Finite Strip Gratings

Giuliano Manara

Dept. of Information Engineering
University of Pisa, Italy

g.manara@iet.unipi.it

February 21-23, 2005, Siena, Italy

Summary

- ✓ Ray technique extension to the scattering from electrically large finite composite surfaces
- ✓ Impedance Boundary Conditions (IBCs)
- ✓ Isotropic/anisotropic impedance wedge scattering
- ✓ Scattering from finite strip gratings
- ✓ Concluding remarks

Motivations

- ✓ Artificial surfaces (frequency/polarization selective surfaces, absorbers, or more general multi-layer and periodic surfaces) are often applied in antenna and microwave device technology

- ✓ Accurate numerical analysis (MoM, FEM, Mode Matching, hybrid methods) of reflection and transmission properties is often performed by considering an *infinite* surface under plane wave or, more rarely, dipole excitation (Floquet theorem reduces the analysis to the periodicity cell)

- ✓ Need for an efficient evaluation of high-frequency EM scattering from edges in electrically *large finite* composite surfaces (PO, GTD, UTD), as numerical methods become highly inefficient

The Asymptotic High-Frequency Approach

- ✓ GTD and its uniform extension, UTD, are very powerful and physically appealing techniques

- ✓ Analytical dyadic diffraction coefficients come out from the definition of a canonical diffraction problem

- ✓ Impenetrable surfaces and semi-transparent slabs can be accurately modeled* by means of proper homogeneous impedance or transition boundary conditions (BCs) as for instance: first and higher order BCs, anisotropic BCs

(*) **S. Tretyakov**, *Analytical Modeling in Applied Electromagnetics*, Artech House, 2003
T.B.A. Senior and J.L. Volakis, *Approximate boundary conditions in electromagnetics*, IEE Electromagnetic Wave Series, London, UK, 1995.
D.J. Hoppe and Y. Rahmat-Samii, *Impedance boundary conditions in electromagnetics*, Taylor & Francis, London, UK, 1995.

The Asymptotic High-Frequency Approach (cont'd)

- ✓ Introduction of approximate BCs allows us to define a canonical diffraction problem: EM scattering of an arbitrarily polarized plane wave illuminating at oblique incidence a geometrical/electrical discontinuity

- ✓ The Wiener-Hopf technique and the Sommerfeld-Maliuzhinets method are the most applied techniques to derive an analytical spectral solution

- ✓ UTD diffraction coefficients can be derived from the spectral solution by means of standard asymptotics

Osipov, A. V., and A. N. Norris, The Maliuzhinets theory for scattering from wedge boundaries: a review, Wave Motion, 29(4), 313-340, 1999.
Daniele, V.G., On the solution of vector Wiener-Hopf equations occuring in scattering problems, Radio Science, 19(5), 1173-1178, 1984

Closed Form Analytical Solutions

- ✓ Exact analytical solutions for the above diffraction canonical problems do exist only for specific electrical and geometrical configurations
- ✓ Exact analytical solutions for a class of canonical problems have been used as a starting point to derive perturbative solutions valid for more general configurations
- ✓ Closed form approximate analytical solutions have been constructed by a proper modification of the above exact solutions

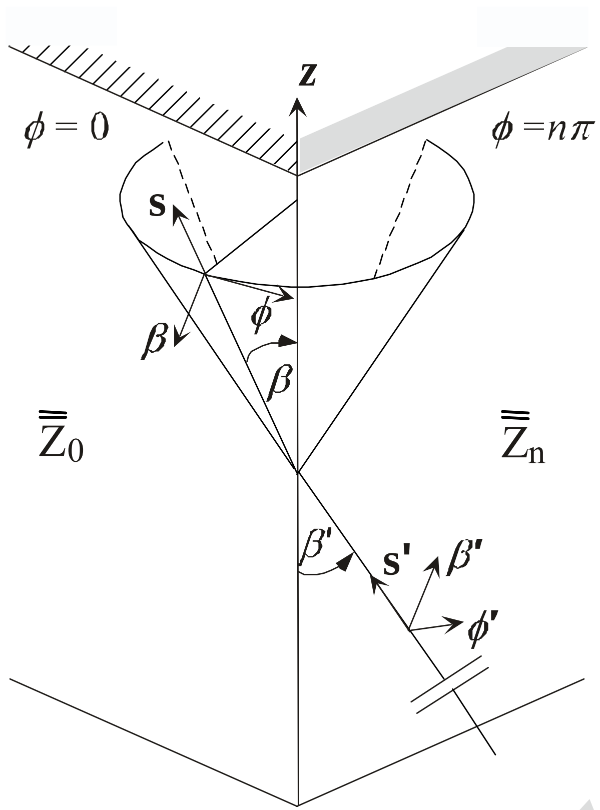
Impedance/Transition Boundary Conditions (BCs)

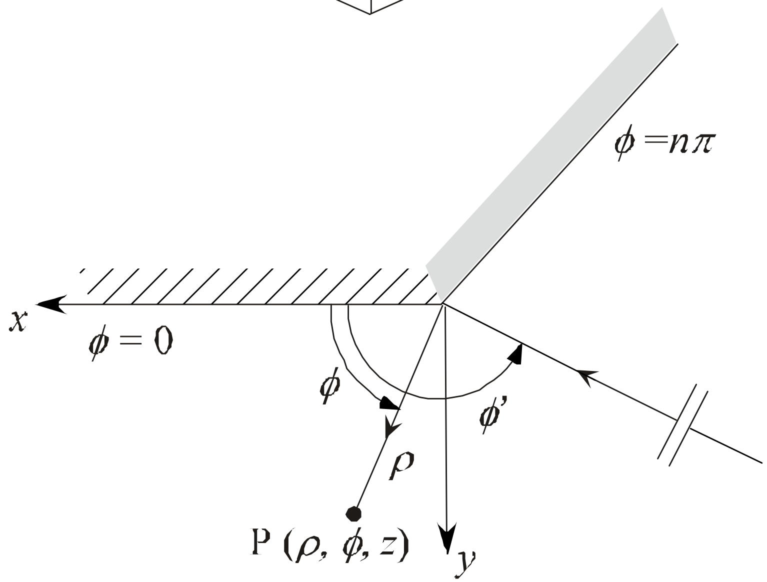
- Impedance BCs have been used for modeling metal backed dielectric slabs, rough surfaces, non-perfectly conducting surfaces
- Anisotropic impedance BCs have been applied to evaluate the scattering from artificially hard and soft surfaces (corrugated surfaces, strip loaded grounded dielectric slabs)
- Transition Conditions can be applied to strip gratings, wire grids, periodic surfaces

↓

Formulation of a Canonical Problem

The Wedge Canonical Problem





Impedance Wedge Scattering

$$\hat{n}\times\underline{E}\times\hat{n}=\overline{\overline{Z}}\left(Z_0\underline{H}\times\hat{n}\right),\quad \overline{\overline{Z}}=tensor\ surface\ impedance$$

$$\begin{bmatrix}-\varepsilon_{0,n}E_\rho\\ E_z\end{bmatrix}=\begin{bmatrix}\eta_{\rho\rho}^{0,n} & \eta_{\rho z}^{0,n}\\ \eta_{z\rho}^{0,n} & \eta_{zz}^{0,n}\end{bmatrix}\begin{bmatrix}\varepsilon_{0,n}\zeta H_\rho\\ \zeta H_z\end{bmatrix}\qquad \phi=0,n\pi \qquad \begin{aligned}\varepsilon_0&=-1\\ \varepsilon_n&=1\end{aligned}$$

$$\frac{1}{\rho}\frac{\partial}{\partial\phi}(\zeta H_z)-\varepsilon_{0,n}jk_t\sin\beta'\left[-\frac{\eta_{\rho\rho}^{0,n}}{\eta_{z\rho}^{0,n}}E_z+\left(-\eta_{\rho z}^{0,n}+\frac{\eta_{\rho\rho}^{0,n}\eta_{zz}^{0,n}}{\eta_{z\rho}^{0,n}}\right)(\zeta H_z)\right]+\cos\beta'\frac{\partial E_z}{\partial\rho}=0$$

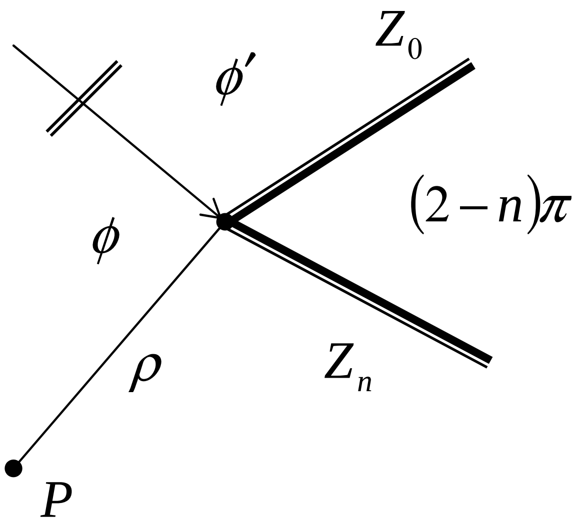
$$\frac{1}{\rho}\frac{\partial E_z}{\partial\phi}-\varepsilon_{0,n}jk_t\sin\beta'\left[-\frac{E_z}{\eta_{z\rho}^{0,n}}+\frac{\eta_{zz}^{0,n}}{\eta_{z\rho}^{0,n}}(\zeta H_z)\right]-\cos\beta'\frac{\partial}{\partial\rho}(\zeta H_z)=0$$

IBCs **result to be coupled** when expressed in terms of the longitudinal field components (potentials) !

The Isotropic Impedance Wedge

$$\underline{i}_n \times \underline{E} \times \underline{i}_n = \zeta\, Z_{0,n} \left(\underline{i}_n \times \underline{H} \right)$$

$$\begin{bmatrix} -\varepsilon_{0,n} E_\rho \\ E_z \end{bmatrix} = \begin{bmatrix} 0 & \eta_{0,n} \\ \eta_{0,n} & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{0,n} \zeta H_\rho \\ \zeta H_z \end{bmatrix}$$



$$\left[\frac{1}{\rho} \frac{\partial}{\partial \phi} + \varepsilon_{0,n}\, jk_t \frac{1}{\eta_{0,n}} sin\beta' \right] E_z - \cos\beta' \frac{\partial}{\partial \rho} \left(\zeta H_z \right) = 0$$

$$\left[\frac{1}{\rho} \frac{\partial}{\partial \phi} + \varepsilon_{0,n}\, jk_t \eta_{0,n} sin\beta' \right] \left(\zeta H_z \right) + \cos\beta' \frac{\partial E_z}{\partial \rho} = 0$$

IBC’s **are still coupled** when expressed in terms of the longitudinal field components !

Isotropic Impedance Wedge: normal incidence case, arbitrary interior wedge angle

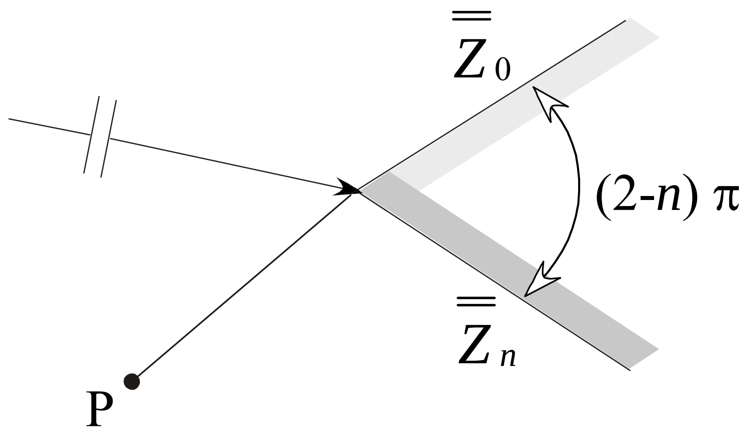
$$\left[\frac{1}{\rho}\frac{\partial}{\partial\phi} + \varepsilon_{0,n} jk \frac{1}{\eta_{0,n}}\right] E_z = 0 \qquad \left[\frac{1}{\rho}\frac{\partial}{\partial\phi} + \varepsilon_{0,n} jk \eta_{0,n}\right](\zeta H_z) = 0 \qquad \phi = 0, n\pi$$

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\phi} + \varepsilon_{0,n} jk \sin\vartheta_{0,n}\right] u = 0 \qquad \phi = 0, n\pi$$

$$\sin\vartheta_{0,n} = \sin\vartheta_e^{0,n} = 1/\eta_{0,n}$$ *TM polarization* ($u=E_z$)

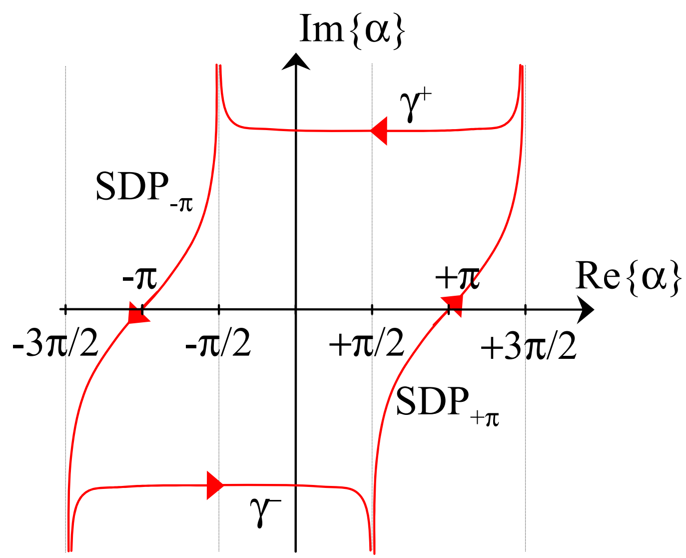
$$\sin\vartheta_{0,n} = \sin\vartheta_h^{0,n} = \eta_{0,n}$$ *TE polarization* ($u=H_z$)

The Sommerfeld-Maliuzhinets Technique



$$E_z(\rho,\phi,z)=\frac{1}{2\pi j}\int_{\gamma^+ +\gamma^-} s_e(\alpha+\phi)\, e^{jk\rho\cos\alpha}\, d\alpha$$

$$H_z(\rho,\phi,z)=\frac{1}{2\pi j}\int_{\gamma^+ +\gamma^-} s_h(\alpha+\phi)\, e^{jk\rho\cos\alpha}\, d\alpha$$



G.D. Maliuzhinets, “Excitation, reflection and emission of surface waves from a wedge with given face impedances,” *Sov. Phys. Dokl.*, 1959

The Generalized Reflection Method

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\phi}+\varepsilon_{0,n}\, jk\sin\vartheta_{0,n}\right]u=0 \qquad u(\rho,\phi)=\frac{1}{2\pi j}\int_{\gamma^{+}+\gamma^{-}} s(\alpha+\phi)\, e^{jk\rho\cos\alpha}d\alpha$$

$$\int_{\gamma^{+}+\gamma^{-}}\left(\sin(\alpha)-\sin\vartheta_{0,n}\right)s(\alpha+\phi)\, e^{jk\rho\cos\alpha}d\alpha=0$$

By applying the nullification theorem it comes out that the kernel of the integrand must be an even function of α, so obtaining a set of homogeneous first-order difference equations

$$s(\alpha\pm n\pi/2)\left(\sin(\alpha)-\sin\vartheta_{0,n}\right)+s(-\alpha\mp n\pi/2)\left(\sin(\alpha)+\sin\vartheta_{0,n}\right)=0$$

$$\boxed{s(-\alpha\pm n\pi/2)=-\frac{\left(\sin(\alpha)-\sin\vartheta_{0,n}\right)}{\left(\sin(\alpha)+\sin\vartheta_{0,n}\right)}s(\alpha\mp n\pi/2)}$$

G.D. Maliuzhinets, “Inversion formula for the Sommerfeld integral,” *Sov. Phys. Dokl.*, Vol. 3, 1958.

The Generalized Reflection Method

$$u(\rho,\phi)=\frac{1}{2\pi j}\int_{\gamma^{+}+\gamma^{-}} s(\alpha+\phi)\, e^{jk\rho\cos\alpha} d\alpha$$

$$\boxed{s(\alpha \pm n\pi/2)\left(\sin(\alpha)-\sin\vartheta_{0,n}\right)+s(-\alpha\mp n\pi/2)\left(\sin(\alpha)+\sin\vartheta_{0,n}\right)=0}$$

The Rigorous Spectral Solution

$$s(\alpha)=\frac{1}{n}\frac{\sin(\phi'/n)}{\sin(\alpha/n)+\cos(\phi'/n)}\Psi(\alpha-n\pi/2,\vartheta_0)\Psi(\alpha+n\pi/2,\vartheta_n)$$

$$\Psi(\alpha,\vartheta_{0,n})=\psi_n\left(\alpha+\pi/2-\vartheta_{0,n}\right)\psi_n\left(\alpha-\pi/2+\vartheta_{0,n}\right)$$

$$\psi_n(\alpha)=\exp\left[\frac{j}{4n\pi}\int_0^{\alpha} du \int_{-j\infty}^{j\infty} tg\,\frac{v}{2n}\frac{dv}{\cos(u-v)}\right]$$ is the Maliuzhinets special function

A.V. Osipov and A.N. Norris, “The Maliuzhinets theory for scattering from wedges boundaries: a review,” *Wave Motion*, May 1999

Isotropic Impedance Wedge: the **skew** incidence case

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\phi} - jk\eta_0\right]E_y = 0 \qquad \left[\frac{1}{\rho}\frac{\partial}{\partial\phi} - jk\frac{1}{\eta_0}\right]\left(\zeta H_y\right) = 0 \qquad \phi = 0$$

IBC's **decouple** if expressed in terms of the field components perpendicular to the isotropic IBC surface!

Scattering from the edge of a half plane (or full plane impedance junction)

O.M. Bucci and G. Franceschetti, "Electromagnetic scattering by a half-plane with two face impedances," *Radio Science*, Vol. 11, No. 1, pp. 49-59, January 1976.

R. Tiberio, A. Toccafondi, "Revisiting the exact solution for edges in planar impedance surfaces at skew incidence," IEEE AP-S Symposium, June 2003.

- Scattering from the edge of a right angled wedge with a PEC face

V.G. Vaccaro, "Electromagnetic diffraction from a right angled wedge with soft conditions on one face," Optica Acta, Vol. 28, No. 3, pp. 293-311, 1981.

The impedance half-plane case

$$E_y = \frac{1}{2\pi j}\int_\gamma t_e\left(\alpha+\phi-n\pi/2\right)e^{jk_t\rho\cos\alpha}d\alpha \qquad \zeta H_y = \frac{1}{2\pi j}\int_\gamma t_h\left(\alpha+\phi-n\pi/2\right)e^{jk_t\rho\cos\alpha}d\alpha$$

$$\begin{bmatrix} s_e(\alpha) \\ s_h(\alpha) \end{bmatrix} = \frac{sin\beta'}{\Delta(\alpha)}\begin{bmatrix} \cos\beta'\, sin\left(\alpha+n\pi/2\right) & \cos\left(\alpha+n\pi/2\right) \\ -\cos\left(\alpha+n\pi/2\right) & \cos\beta'\, sin\left(\alpha+n\pi/2\right) \end{bmatrix}\begin{bmatrix} t_e(\alpha) \\ t_h(\alpha) \end{bmatrix}$$

$$\Delta(\alpha) = 1 - sin^2\beta'\, sin^2(\alpha+n\pi/2)$$

$$\left(sin\alpha+\varepsilon_{0,n}sin\vartheta_h^{0,n}\right)t_e\left(a+\varepsilon_{0,n}n\pi/2\right)+\left(sin\alpha-\varepsilon_{0,n}sin\vartheta_h^{0,n}\right)t_e\left(-a+\varepsilon_{0,n}n\pi/2\right)=0$$

$$\left(sin\alpha+\varepsilon_{0,n}sin\vartheta_e^{0,n}\right)t_h\left(a+\varepsilon_{0,n}n\pi/2\right)+\left(sin\alpha-\varepsilon_{0,n}sin\vartheta_e^{0,n}\right)t_h\left(-a+\varepsilon_{0,n}n\pi/2\right)=0$$

The exact spectral solution

$$t_e(\alpha)=\left[e_y\,\sigma(\alpha)+c_e+c_e'\,\sin(\alpha/n)\right]\frac{\psi\left(\alpha-n\pi/2,\vartheta_h^0\right)\psi\left(\alpha+n\pi/2,\vartheta_h^n\right)}{\psi\left(\phi'-n\pi,\vartheta_h^0\right)\psi\left(\phi',\vartheta_h^n\right)}$$

$$t_h(\alpha)=\left[h_y\,\sigma(\alpha)+c_h+c_h'\,\sin(\alpha/n)\right]\frac{\psi\left(\alpha-n\pi/2,\vartheta_e^0\right)\psi\left(\alpha+n\pi/2,\vartheta_e^n\right)}{\psi\left(\phi'-n\pi,\vartheta_e^0\right)\psi\left(\phi',\vartheta_e^n\right)}$$

$$\sigma(\alpha)=\frac{1}{n}\frac{\sin\left(\phi'/n\right)}{\sin\left(\alpha/n\right)+\cos\left(\phi'/n\right)}$$

$$\psi_\pi(\alpha)=\exp\left[\frac{1}{8\pi}\int_0^\alpha\frac{\pi\sin u-2\sqrt{2}\pi\sin(u/2)-2u}{\cos u}du\right]$$

$$\sin\vartheta_e^{0,n}=1/\left(\sin\beta'\,\eta_{0,n}\right)\qquad \sin\vartheta_h^{0,n}=\eta_{0,n}/\sin\beta'$$

Anisotropic Surface Modeling

- ✓ Anisotropic Impedance Boundary Conditions (IBCs)

$$\hat{n}\times\underline{E}\times\hat{n}=\overline{\overline{Z}}\left(Z_0\underline{H}\times\hat{n}\right),\quad \overline{\overline{Z}}=tensor\ surface\ impedance$$

- ✓ Anisotropic Transition Boundary Conditions (semi-transparent surfaces)

Senior, T.B.A., and J.L. Volakis, Approximate Boundary Conditions in Electromagnetics, IEE Electromagn. Waves Ser., vol. 41, Inst. of Electr. Eng., London, 1995.

S. Tretyakov, Analytical Modeling in Applied Electromagnetics, Artech House, 2003.

Anisotropic surfaces

Anisotropic materials and surfaces are finding a wide application in the design of microwave devices

- ✓ Artificially soft and hard surfaces (polarization selective surfaces)
- ✓ 2-D periodic surfaces (frequency selective surfaces)
- ✓ Metallic ground planes covered by artificial material slabs

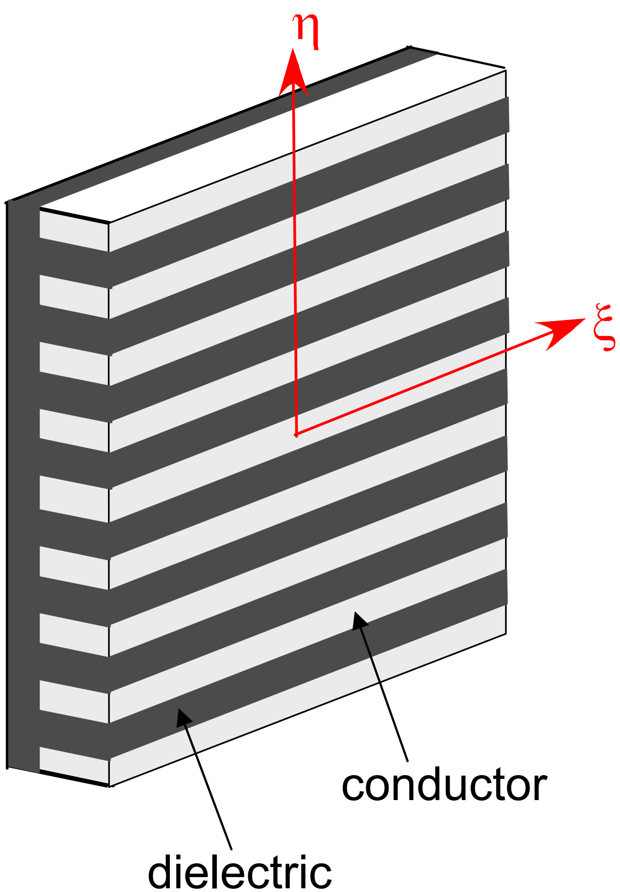
P.-S. Kildal, A. Kishk, S. Maci Eds., Special Issue on "Artificial Magnetic Conductors, Soft/Hard Surfaces, and other Complex Surfaces", IEEE Trans. Antennas Propagat., 2004.

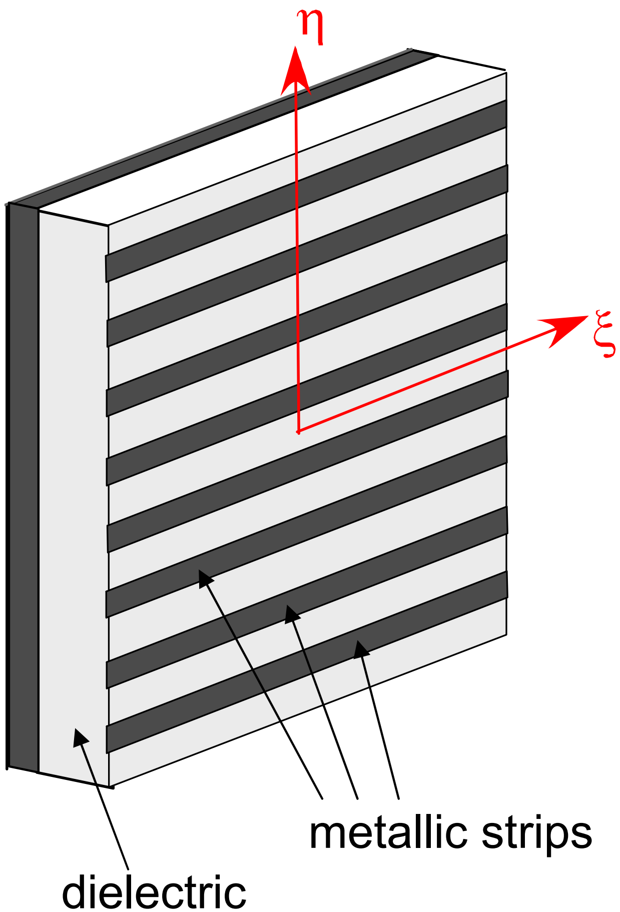
R.W. Ziolkowski and N. Engheta, Eds., *Special Issue* on "Metamaterials", *IEEE Trans. Antennas Propagat.*, AP-51, n. 10, October 2003.

B.A. Munk, Frequency Selective Surfaces: Theory and Design, John Wiley, 2000.

Artificially Hard and Soft Surfaces

Corrugated /strip loaded surfaces

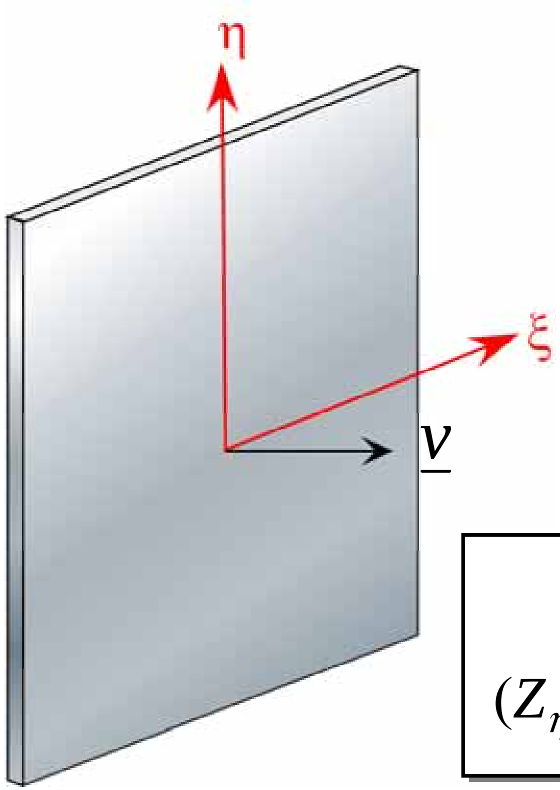




P. S. Kildal, "Artificially soft and hard surfaces in electromagnetics", *IEEE Trans. Antennas Propagat.*, Vol. 38, No. 10, Oct.1990.

BCs for hard/soft surfaces

- Anisotropic impedance BCs have been applied to evaluate the scattering from artificially hard and soft surfaces



$$\underline{\nu} \times \underline{E} \times \underline{\nu} = \overline{\overline{Z}}\left(\underline{\nu} \times \underline{H}\right), \qquad \overline{\overline{Z}} = \begin{bmatrix} 0 & Z_\xi \\ Z_\eta & 0 \end{bmatrix}$$

$$Z_\xi = 0, \quad Z_\eta = \infty$$

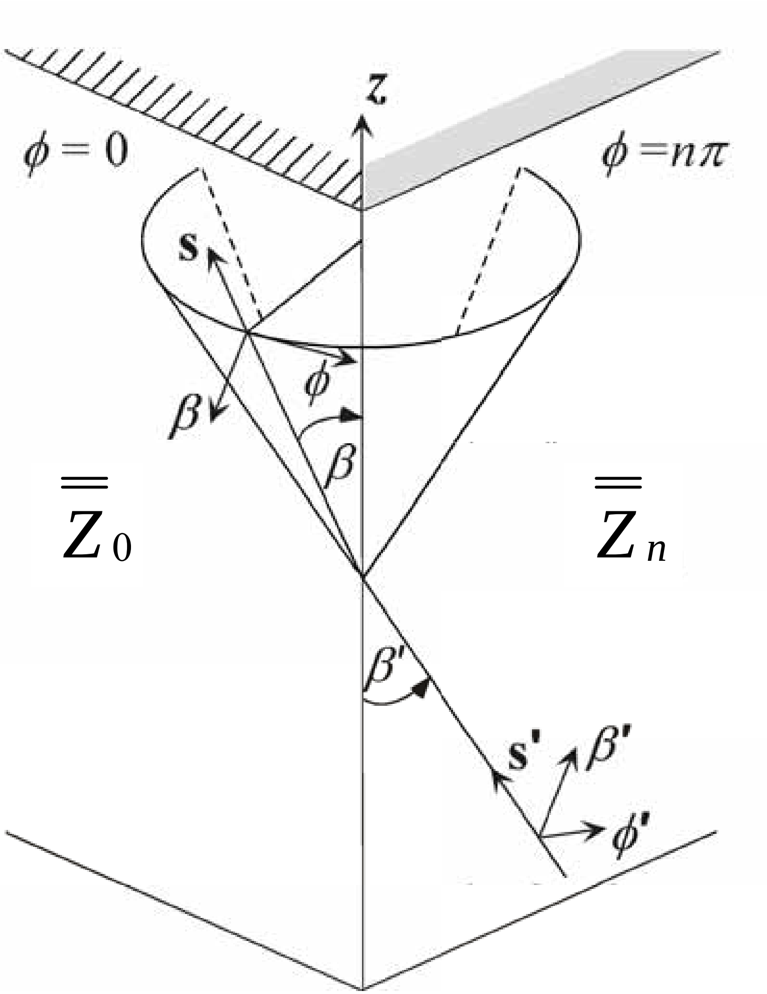
Asymptotic Boundary Conditions (p/λ<<1)

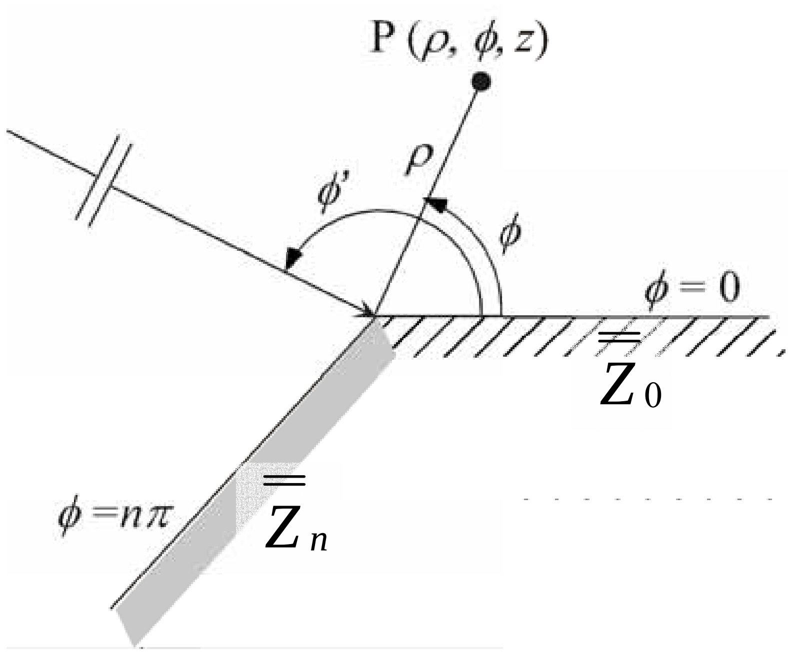
A more accurate model: $Z_\xi = 0, \quad Z_\eta \neq 0$

(Z_η can be related to the electrical and geometrical parameters)

Formulation of a Canonical Problem

Wedge Scattering Problem





- ✓Oblique incidence
- ✓Arbitrary polarization
- ✓Corrugations/strips arbitrarily oriented with respect to the edge

Available Solutions

- NUMERICAL SOLUTIONS
  (Hybrid PO-MoM, parabolic equation method)

- APPROXIMATE ANALYTICAL SOLUTIONS
  (Perturbative approach, Uniform Asymptotic PO)

- RIGOROUS ANALYTICAL SOLUTIONS
  (Wiener-Hopf technique, Maliuzhinets technique)

Scattering from wedges with anisotropic impedance BCs: Wiener-Hopf technique

- Hurd R. A., and E. Luneburg, Diffraction by an anisotropic impedance half plane, Canadian J. Phys., 63, 1135-1140, 1985
- Serbest, A.H., A. Büyükaksoy, and G. Uzgören, Diffraction at a discontinuity formed by two anisotropic impedance half-planes, IEICE Trans., E74(5), pp. 1283-1287, 1991.
- Serbest, A.H., and E. Lüneburg, Scattering of plane waves at the junction of two corrugated half-planes, Microwave Signature '92, Innsbruck, Austria, 1-3 July, 1992.
- Büyükaksoy, A., A.H. Serbest and A. Kara, Diffraction coefficient for a half-plane with anisotropic conductivity, IEE Proc.-Sci. Meas. Technol., 143(6), 384-388, 1996
- Sendag, R., and A.H. Serbest, Scattering at the junction formed by a PEC half-plane and a half-plane with anisotropic conductivity, Electromagnetics, 21, pp. 415-434, 2001.

Scattering from wedges with anisotropic impedance BCs: Maliuzhinets method (1)

- Legault, S.R., and T.B.A. Senior, Solution of a second order difference equation using the bilinear relations of Riemann, J. of Math. Physics, 43 (3), pp. 1598-1621, 2002 .
- Senior, T.B.A., and S.R. Legault, Second-order difference equations in diffraction theory, Radio Science, 35(3), 683-690, 2000.
- Senior, T.B.A., S.R. Legault, and J.L. Volakis, A novel technique for the solution of second-order difference equations, IEEE Trans. Antennas Propag., AP-49(12), 1612-1617, 2001.
- Senior, T.B.A., and E. Topsakal, Diffraction by an anisotropic impedance half plane, J. of Electromagn. Waves and Appl., 16 (7), pp. 889-906, 2002

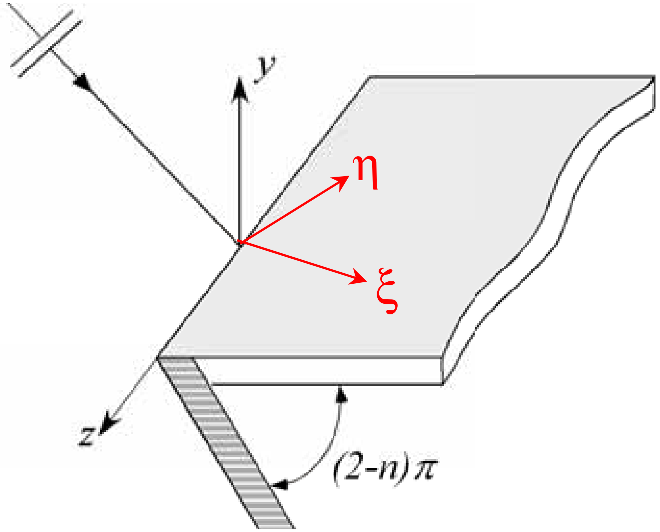
- Bernard J.M.L., Diffraction at skew incidence by an anisotropic impedance wedge in electromagnetism theory: a new class of canonical cases, J. Phys. A.: Math. Gen., 31, 1998
- Lyalinov, M. A., and N.Y. Zhu, Diffraction of a skewly incident plane wave by an anisotropic impedance wedge – a class of exactly solvable cases," Wave Motion, 30(3), 275-288, 1999 .

- Lyalinov, M. A., and N.Y. Zhu, Exact solution to diffraction problem by wedges with a class of anisotropic impedance faces: oblique incidence of a plane electromagnetic wave, IEEE Trans. Antennas Propag., AP-51(6), 2003

Scattering from wedges with anisotropic impedance BCs: Maliuzhinets method (2)

- G. Manara, P. Nepa, “Electromagnetic diffraction of an obliquely incident plane wave by a right-angled anisotropic impedance wedge with a perfectly conducting face,” IEEE Trans. Ant. Prop., April 2000.
- G. Manara, P. Nepa, G. Pelosi, "High-frequency EM scattering by edges in artificially hard and soft surfaces illuminated at oblique incidence," IEEE Trans. Antennas Propagat., May 2000.
- P. Nepa, G. Manara, and A. Armogida, "Electromagnetic scattering by anisotropic impedance half and full planes illuminated at oblique incidence," IEEE Trans. Antennas Propagat., Jan. 2001
- G. Manara, P. Nepa, G. Pelosi and A. Vallecchi, “Skew Incidence Diffraction by an Anisotropic Impedance Half Plane with a PEC Face and Arbitrarily Oriented Anisotropy Axes,” IEEE Trans. Antennas and Propagation, March 2004
- G. Manara, P. Nepa, G. Pelosi and A. Vallecchi, “An approximate solution for skew incidence diffraction by an interior right-angled anisotropic impedance wedge,” Progress in Electromagnetics Research (PIER), vol. 45, Ed. J.A. Kong, EMW Publishing, 2004.

Normal Incidence



- ✓ Corrugations/strips parallel or perpendicular to the edge

  A rigorous spectral solution can be found for an arbitrary interior wedge angle

G.D. Maliuzhinets, "Excitation, reflection and emission of surface waves from a wedge with given face impedances," *Sov. Phys. Dokl.*, 1959.

S. Maci, R. Tiberio, A. Toccafondi, "Diffraction coefficients at edges in artificially soft and hard surfaces," Electronics Letters, vol. 30, n. 3, February 1994.

- ✓ Corrugations/strips arbitrarily oriented with respect to the edge

  A rigorous spectral solution can be found only for the half plane (n=2) and the full plane (n=1) configurations

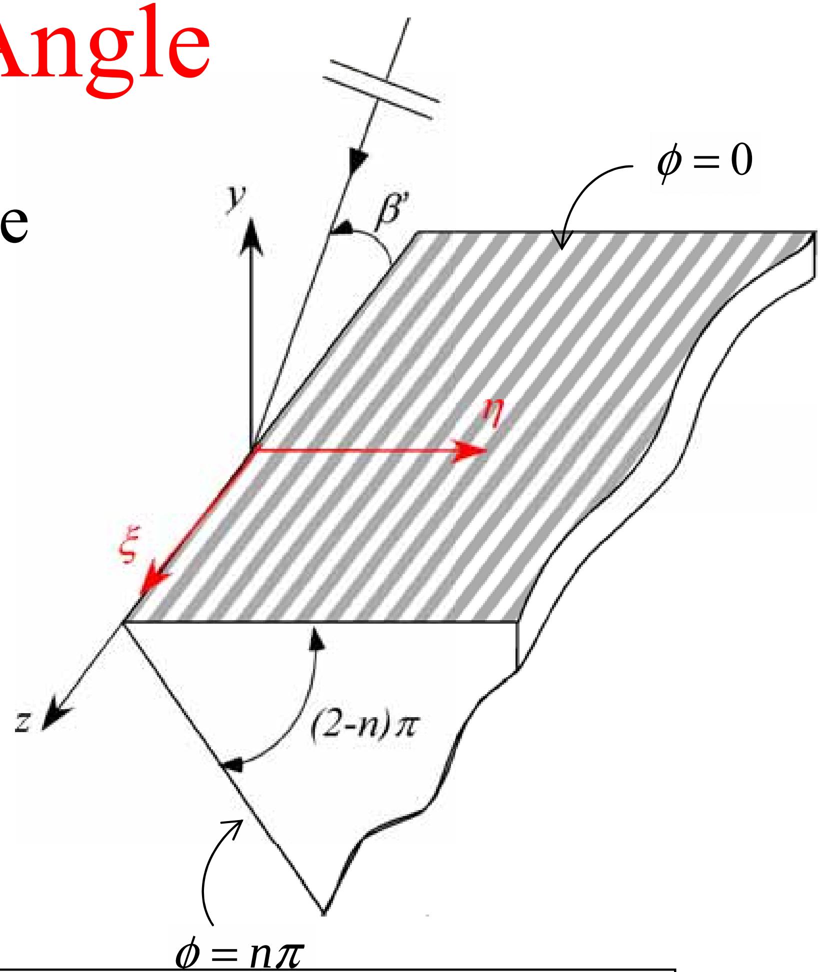
P. Nepa, G. Manara and A. Armogida, "EM scattering by metallic half plane loaded by anisotropic face with arbitrarily oriented anisotropy axes," *Electronic Letters,* Vol. 35, No. 21,1999.

$$Z_\xi=0\,,\; Z_\eta\neq 0$$

(Z_η can assume an arbitrary finite value, in the limit case $Z_\eta=0 \rightarrow$ PEC)

Oblique Incidence
Arbitrary Interior Wedge Angle

✓ Corrugations/strips parallel to the edge (on both faces)



$$E_z = 0$$

$$\left[\frac{1}{\rho}\frac{\partial}{\partial \phi} + \varepsilon_{0,n}\, jk_t \eta_\eta^{0,n} \sin\beta'\right](\zeta H_z) = 0$$

$Z_\xi = 0$, $Z_\eta \neq 0$

(Z_η can assume an arbitrary finite value, in the limit case $Z_\eta = 0 \rightarrow$ PEC)

G. Manara, P. Nepa, G. Pelosi, "High-frequency EM scattering by edges in artificially hard and soft surfaces illuminated at oblique incidence," *IEEE Trans. AP*, Vol. 48, No. 5, 2000.

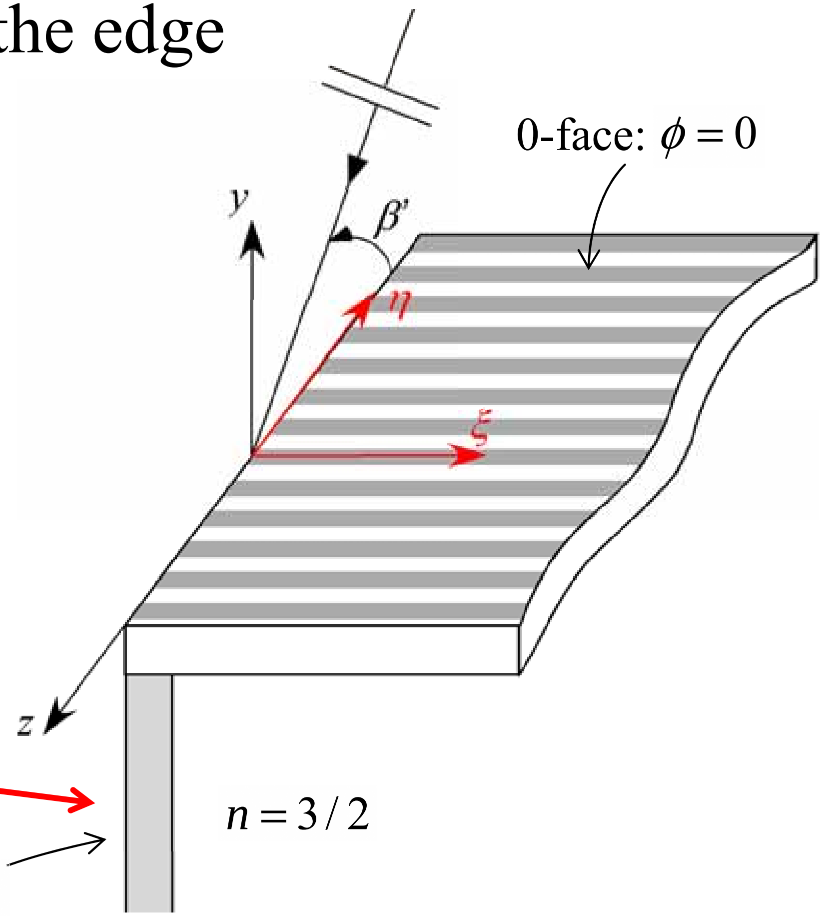
Right-Angled Wedge

- ✓ Corrugations / strips perpendicular to the edge on the 0-face (*x-direction*)

$$E_x = 0$$

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\phi} + jk\eta_\eta^n \left(1 + \frac{1}{k^2}\frac{\partial^2}{\partial\rho^2}\right)\right](\zeta H_x) = 0$$

ISOTROPIC Impedance BCs



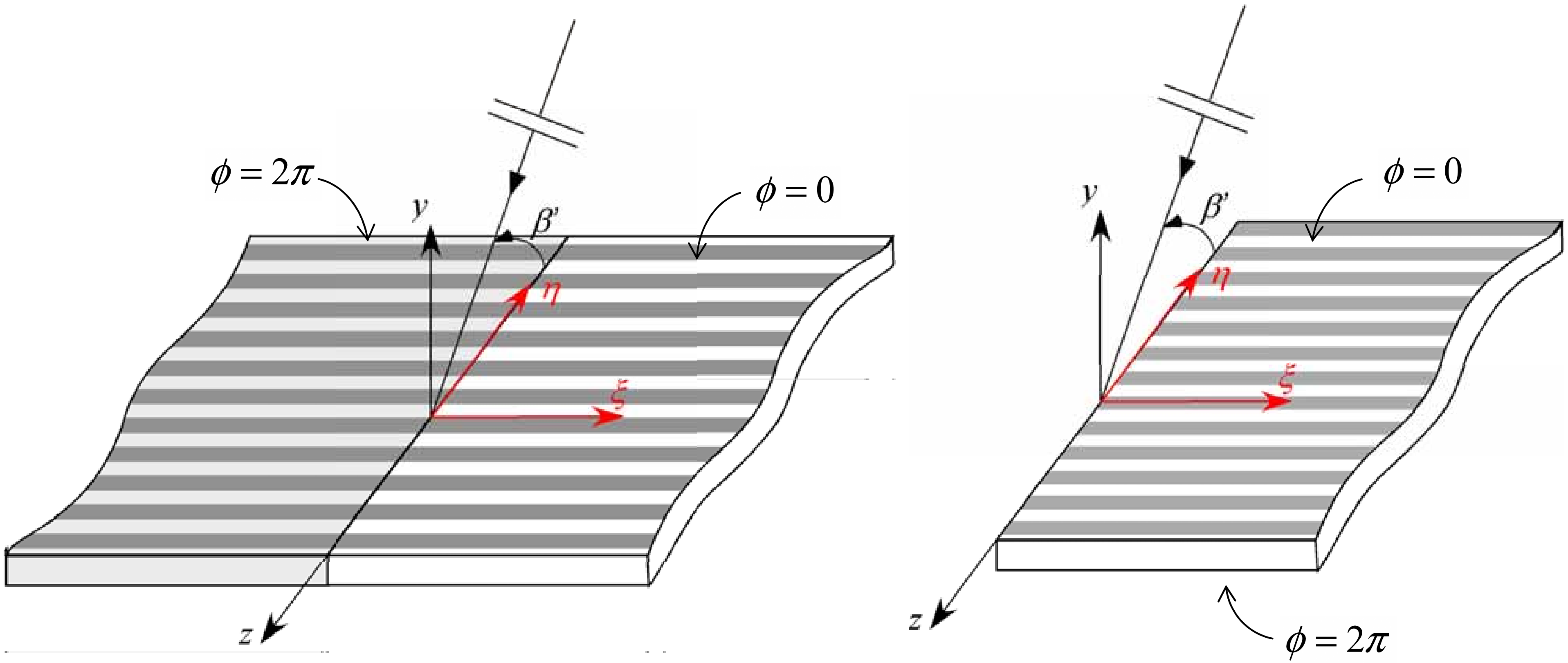
Z_ξ=0 , $Z_\eta \neq 0$

(Z_η can assume an arbitrary finite value, in the limit case Z_η =0 $\rightarrow$ PEC)

G. Manara, P. Nepa, G. Pelosi, "Electromagnetic scattering by a right-angled anisotropic impedance wedge," *Electronics Letters*, Vol. 32, n. 10, June 1996.

Half and Full Planes

✓ Corrugations / strips perpendicular to the edge on both faces



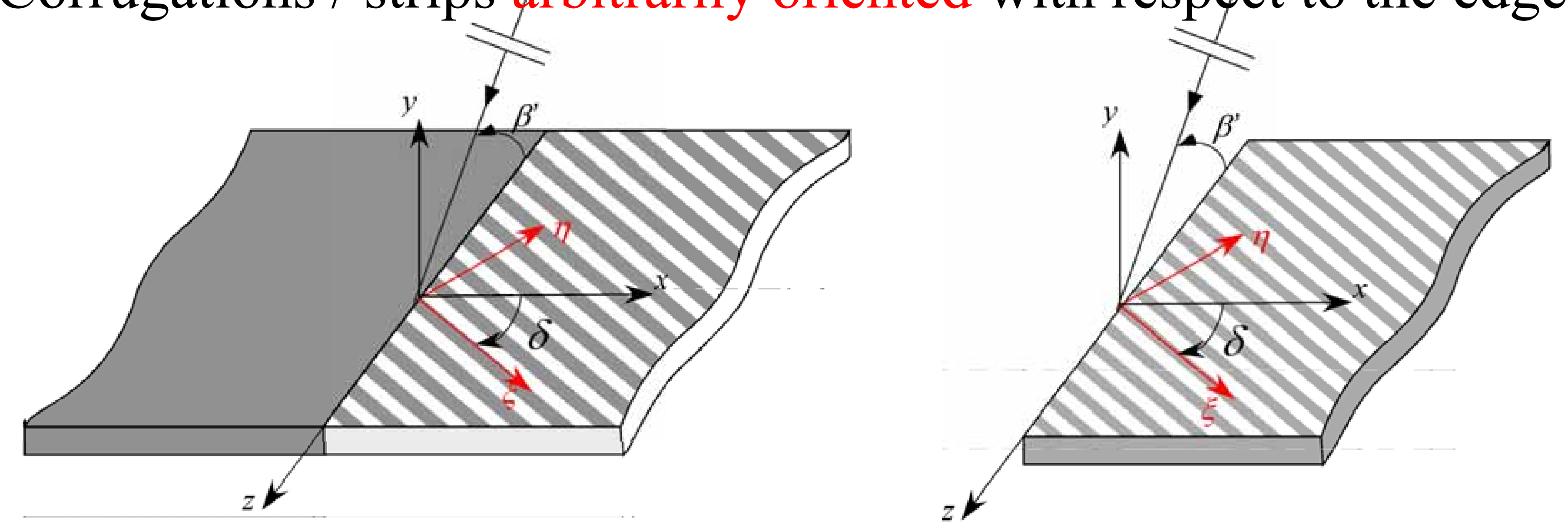
$Z_\xi = 0$, $Z_\eta \neq 0$

(Z_η can assume an arbitrary finite value, in the limit case $Z_\eta = 0 \rightarrow$ PEC)

G. Manara, P. Nepa, G. Pelosi, "High-frequency EM scattering by edges in artificially hard and soft surfaces illuminated at oblique incidence," *IEEE Trans. AP*, Vol. 48, No. 5, May 2000.

Half and Full Planes with a PEC face

- ✓ Corrugations / strips arbitrarily oriented with respect to the edge



$Z_\eta = \infty$: spectra contain only trigonometric functions

P. Nepa, G. Manara, and A. Armogida, "Electromagnetic scattering by anisotropic impedance half and full planes illuminated at oblique incidence," *IEEE Trans. AP,* Vol. 49, No. 1, 2001.

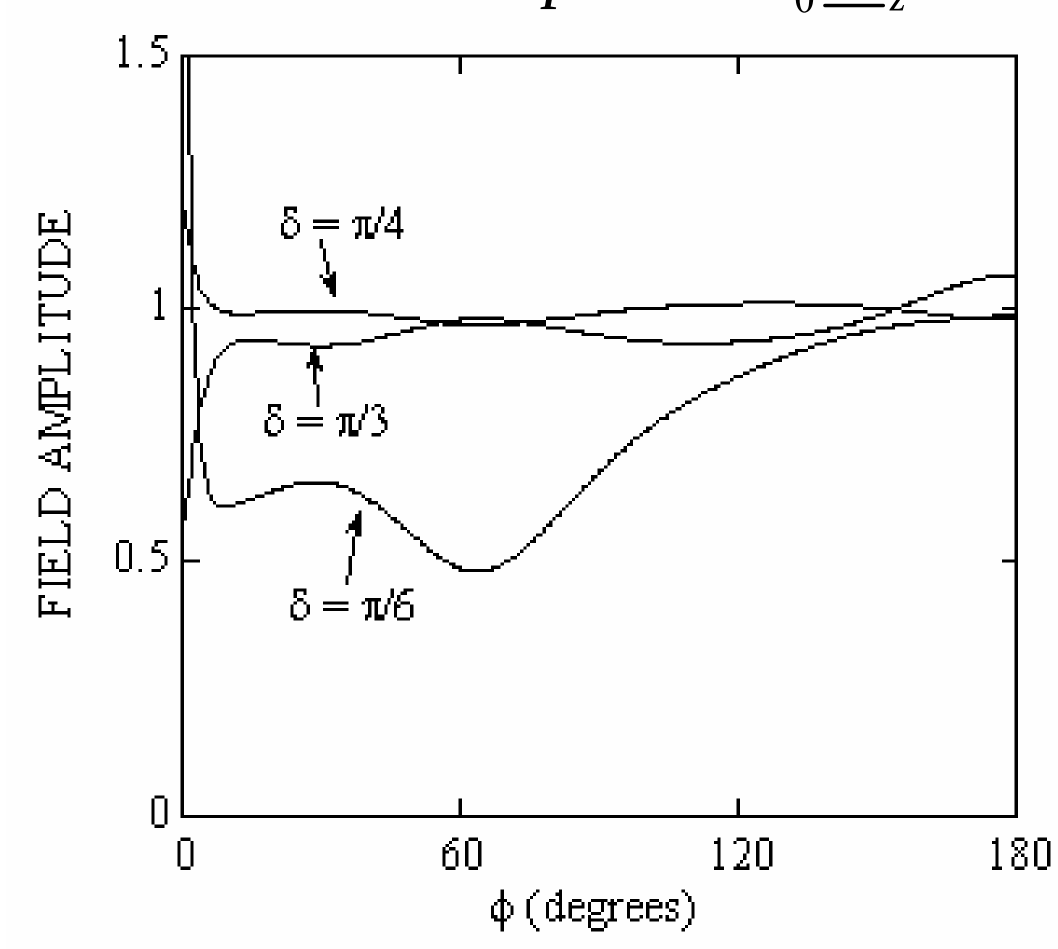
$Z_\eta \neq 0$: spectra contain an extension of Maliuzhinets' special function

G. Manara, P. Nepa, G. Pelosi and A. Vallecchi, "Skew incidence diffraction by an anisotropic impedance half plane with a PEC face and arbitrarily oriented anisotropy axes," IEEE Trans. Antennas Propagat., January 2005.

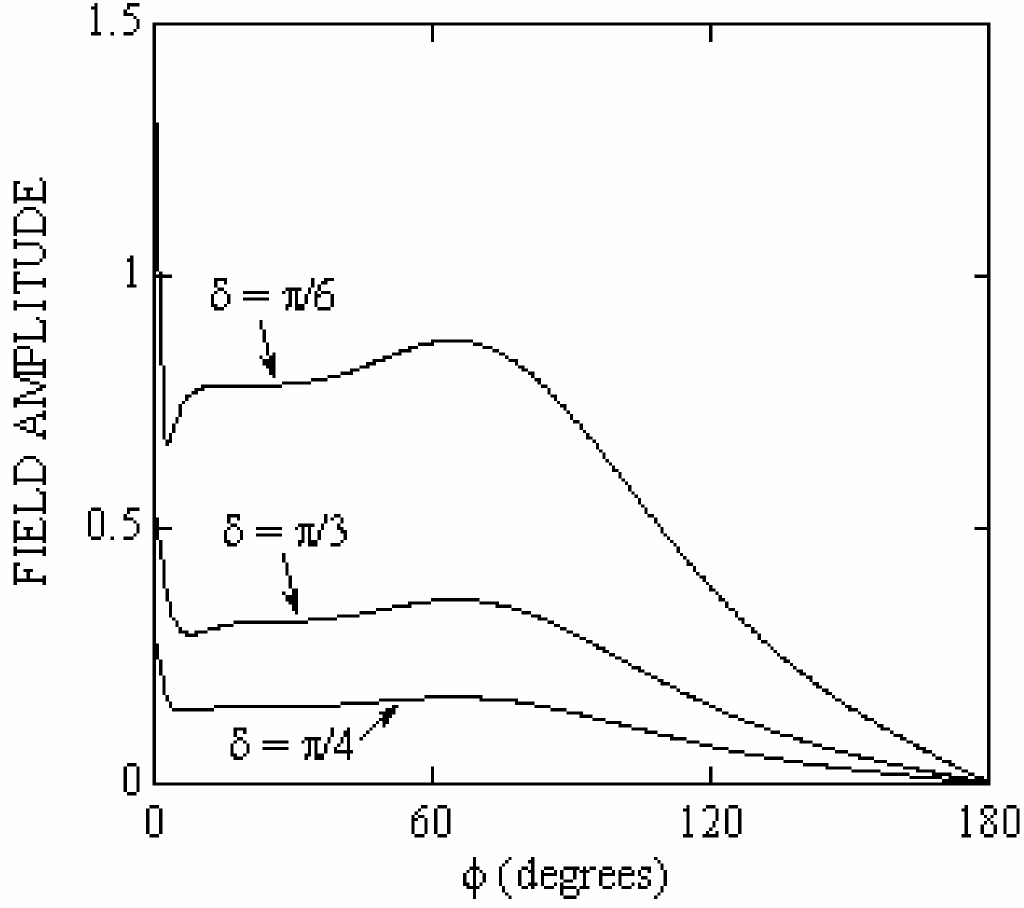
Numerical Results (full-plane)

$\boxed{Z_\xi = 0, Z_\eta = \infty}$ $\quad \beta' = \pi/6, \phi' = \pi/3, k_t\rho = 5, \quad TE_z$ *polarization*

$co-polar : Z_0 \underline{H}_z$

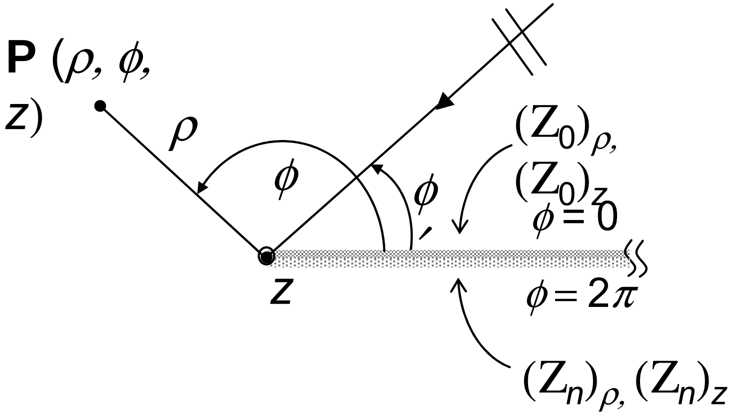


$cross-polar : \underline{E}_z$



δ denotes the direction of corrugations/strips with respect to the edge

Anisotropic Half-Plane Diffraction



$$\forall \quad (Z_0)_z, (Z_0)_\rho$$

$$(Z_n)_z = -(Z_0)_z$$

$$(Z_n)_\rho = -(Z_0)_\rho$$

$$(Z_n)_z = 1/(Z_0)_z$$

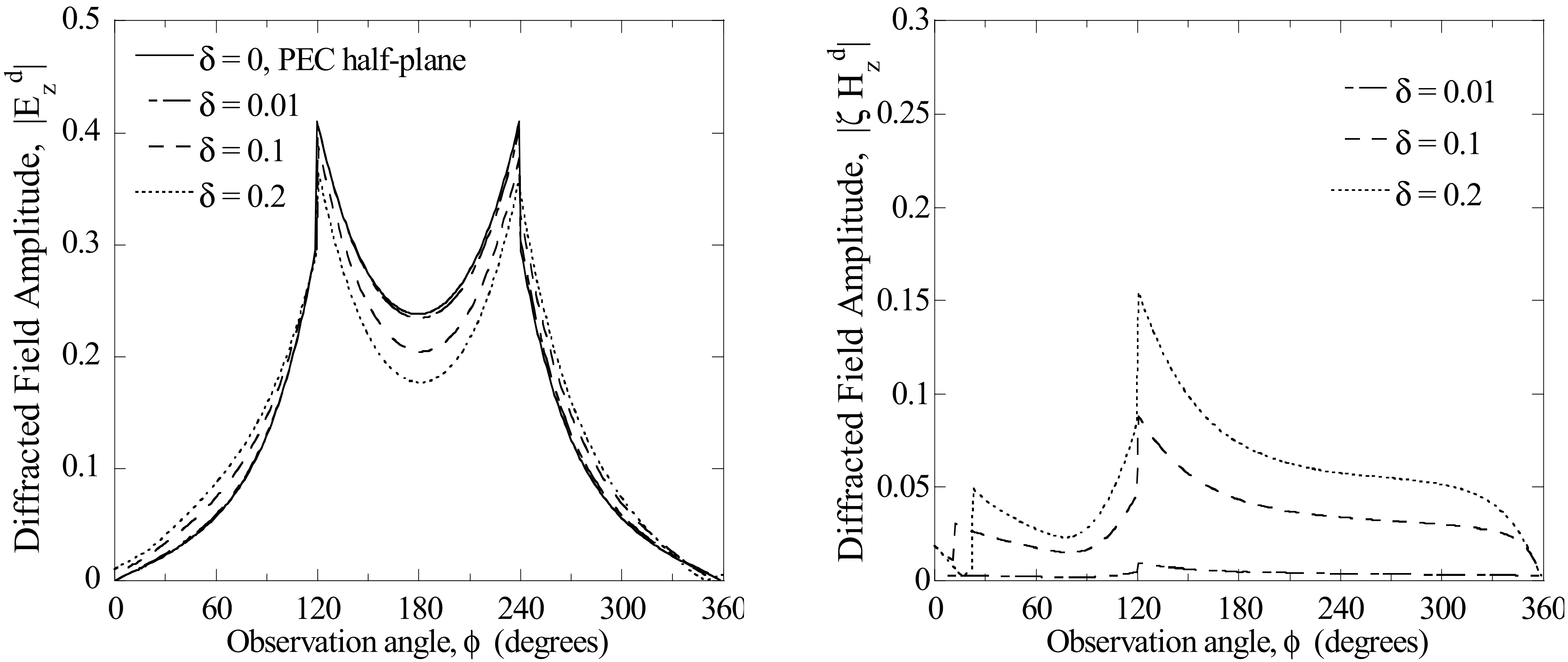
$$(Z_n)_\rho = 1/(Z_0)_\rho$$

P. Nepa, G. Manara, A. Armogida, “New Exact Analytical Solutions Benchmarking Skew Incidence Diffraction by Anisotropic Impedance Half and Full Planes,” submitted to Radio Science

Anisotropic Half-Plane Diffraction

$(Z_0)_\rho = j\delta \quad (Z_0)_z = j2\delta \qquad \delta = 0.01, 0.1, 0.2 \qquad k_t\rho = 5$

$(Z_n)_\rho = -(Z_0)_\rho \quad (Z_n)_z = -(Z_0)_z \qquad \phi' = \pi/3 \quad \beta' = \pi/4$ TM polarization

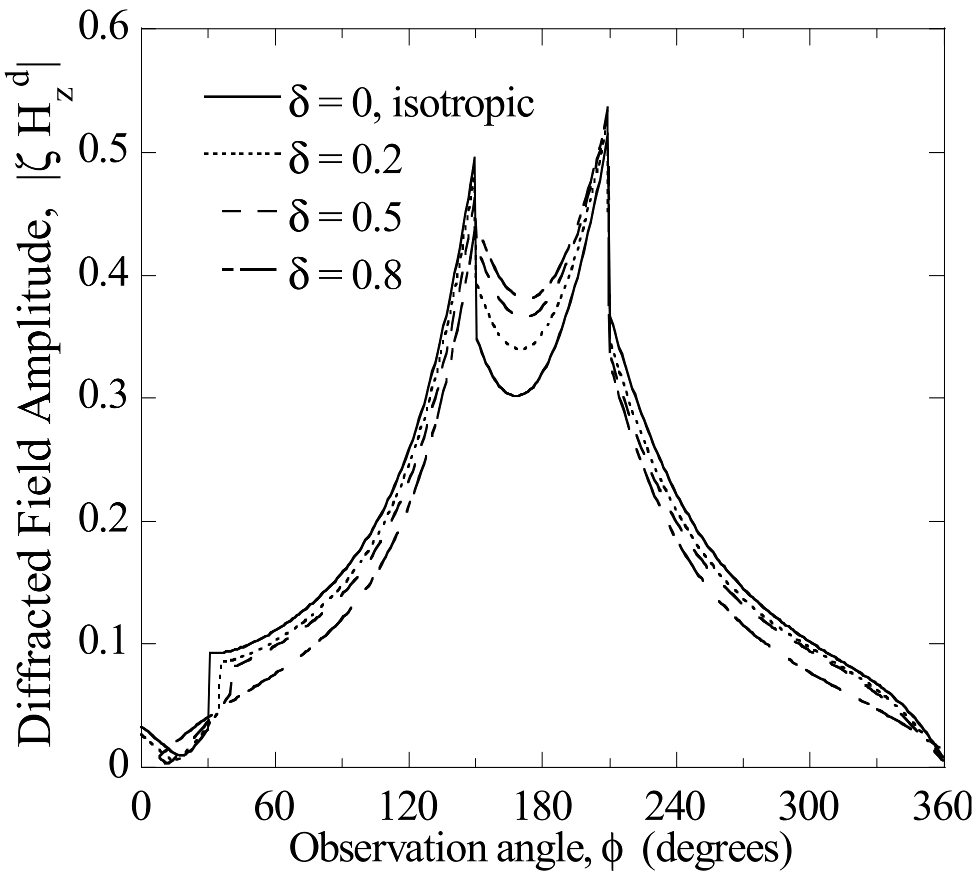


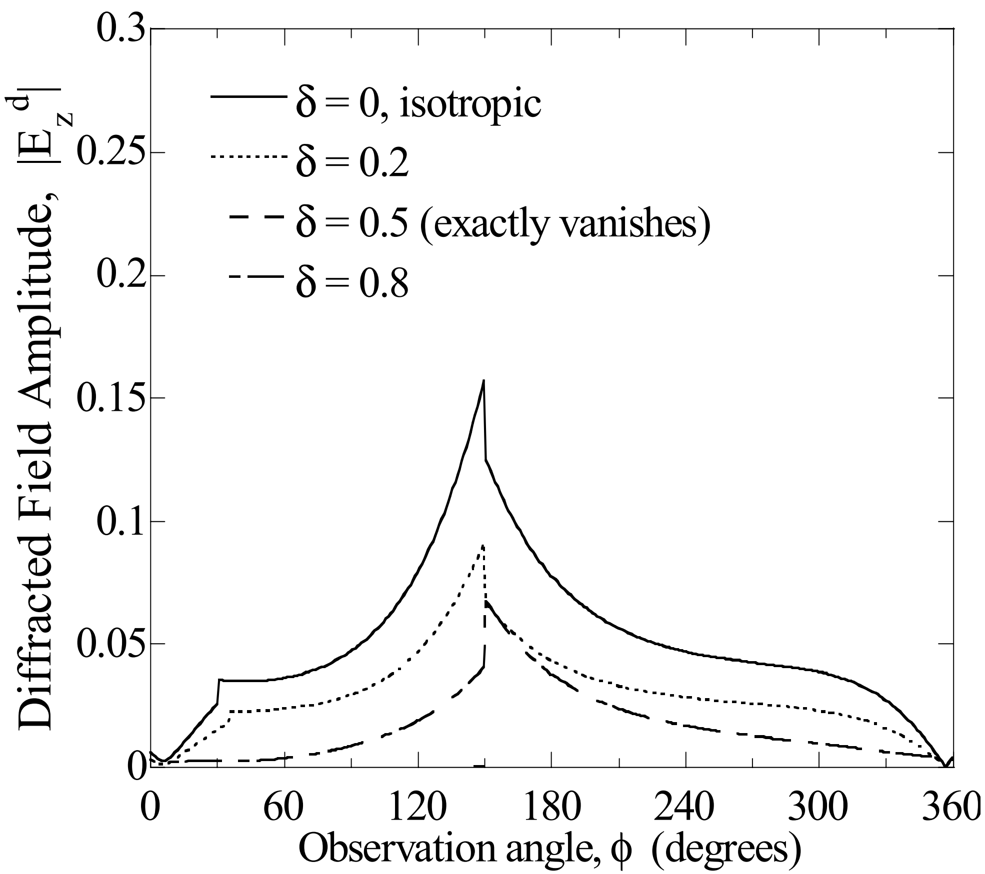
Anisotropic Half-Plane Diffraction

$$(Z_0)_\rho = j(0.5+\delta) \quad (Z_0)_z = j(0.5-\delta) \quad \delta = 0.2, 0.5, 0.8 \qquad k_t\rho = 5$$

$$(Z_n)_\rho = -(Z_0)_\rho \quad (Z_n)_z = -(Z_0)_z \quad \phi' = \pi/6 \quad \beta' = \pi/3$$

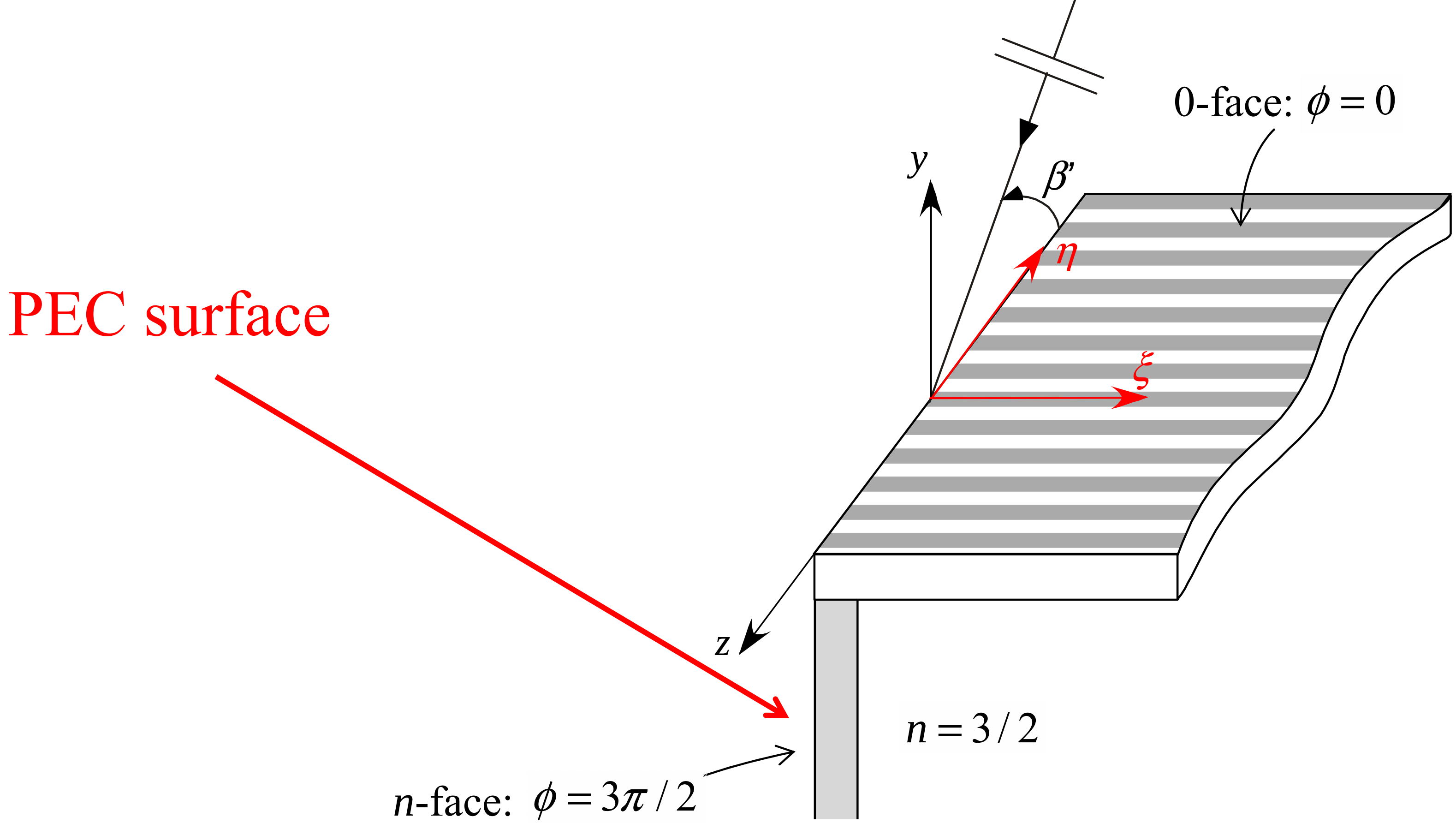
TE polarization





Right-Angled Wedge

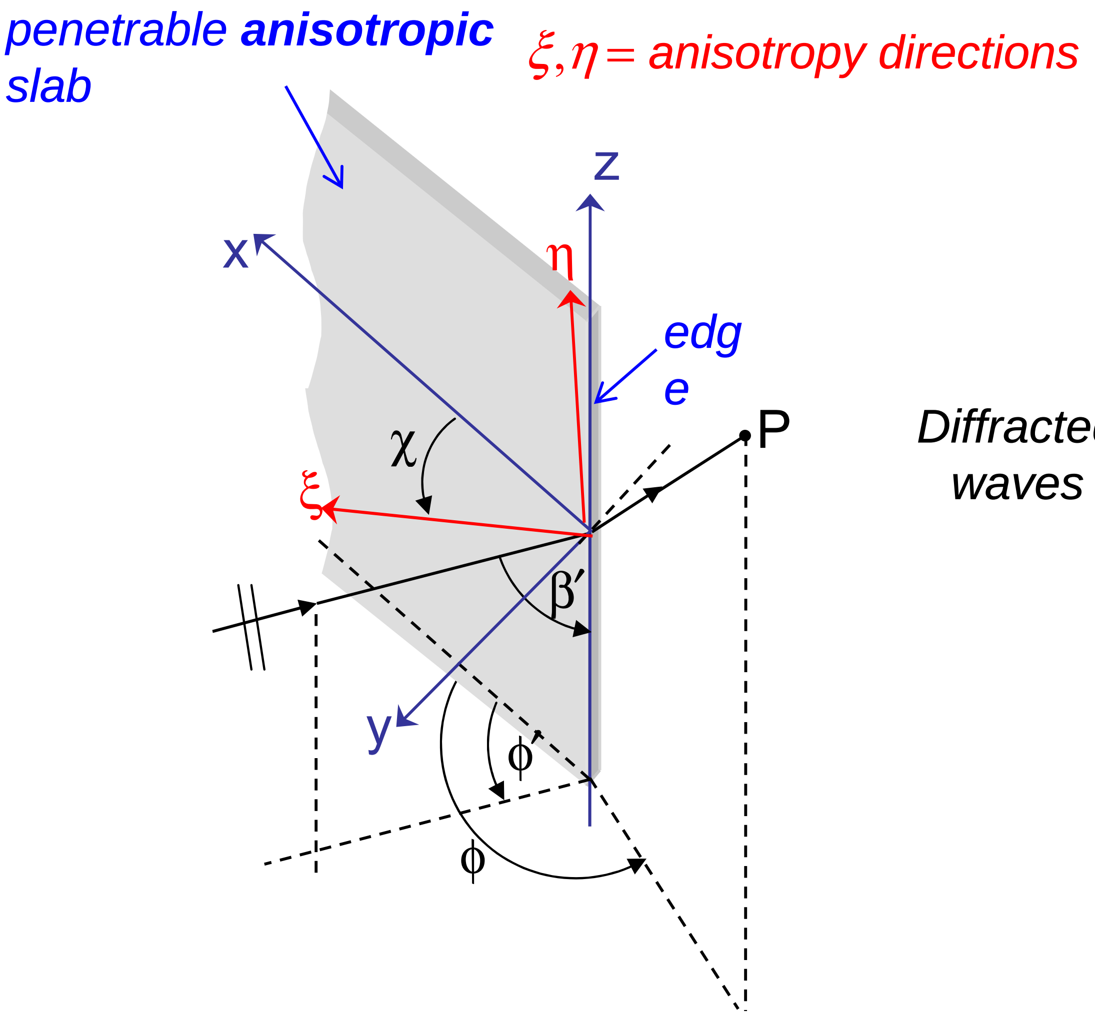
✓ Arbitrary tensor surface impedance at the face $\phi=0$

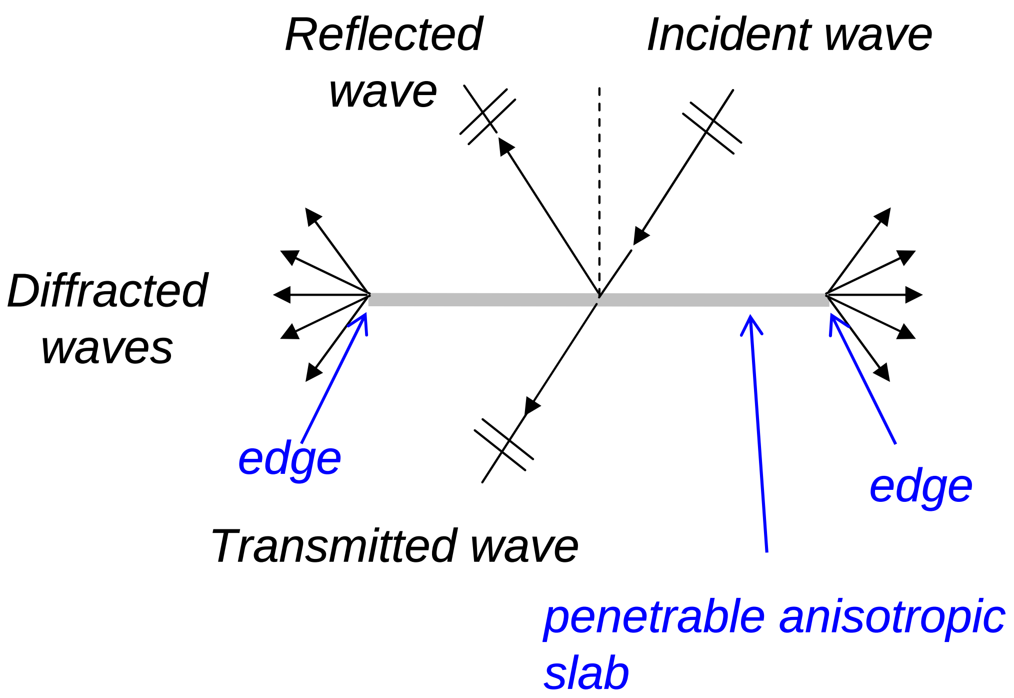


$$\mathbf{Z_\xi \neq 0\,,\ Z_\eta \neq 0}$$

G. Manara, P. Nepa, “Electromagnetic Diffraction of an Obliquely Incident Plane Wave by a Right-Angled Anisotropic Impedance Wedge with a Perfectly Conducting Face,” *IEEE Trans. Antennas and Propagation*, vol. 48, no. 4, pp. 547-555, April 2000 .

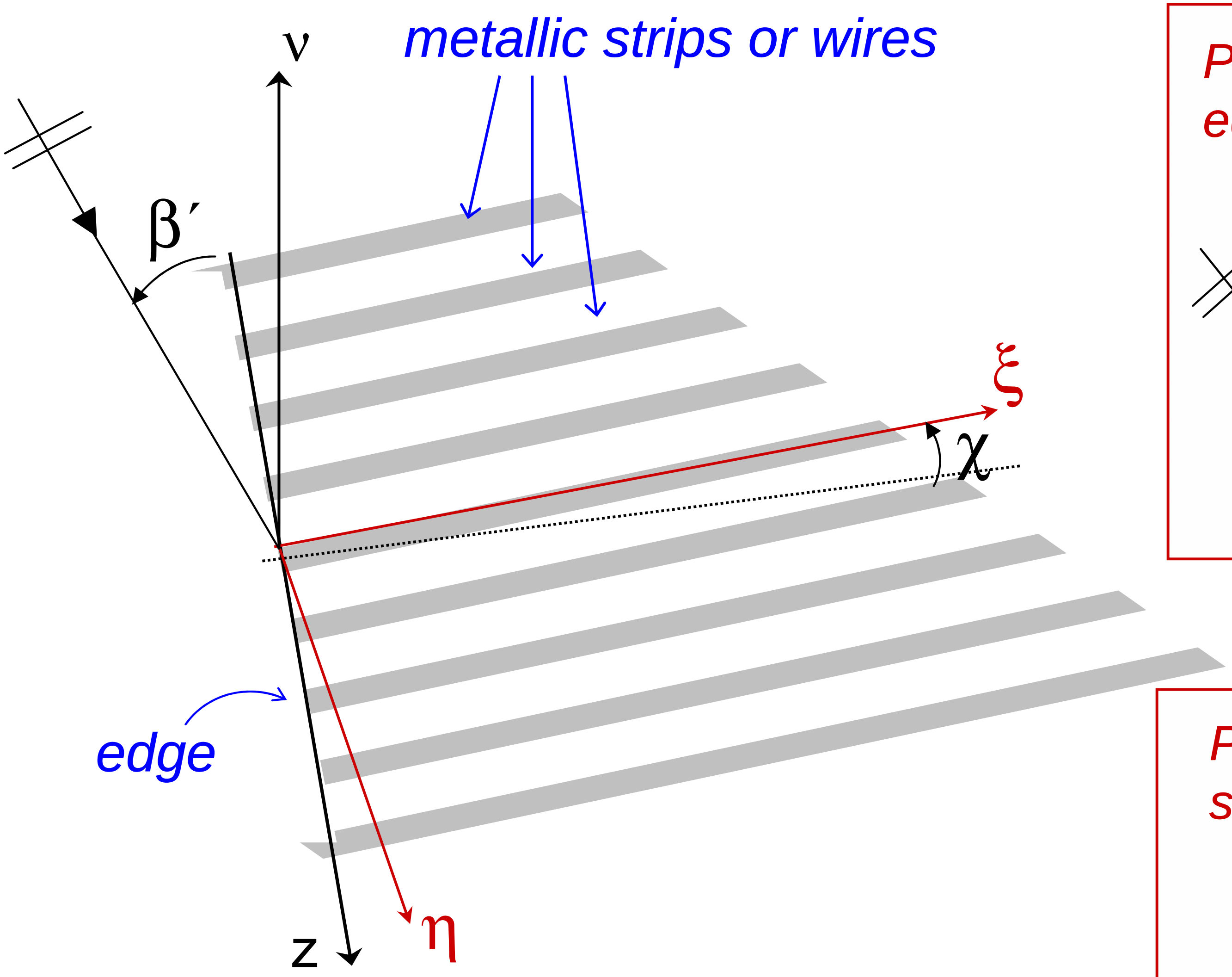
An equivalent canonical problem for a penetrable periodic surface

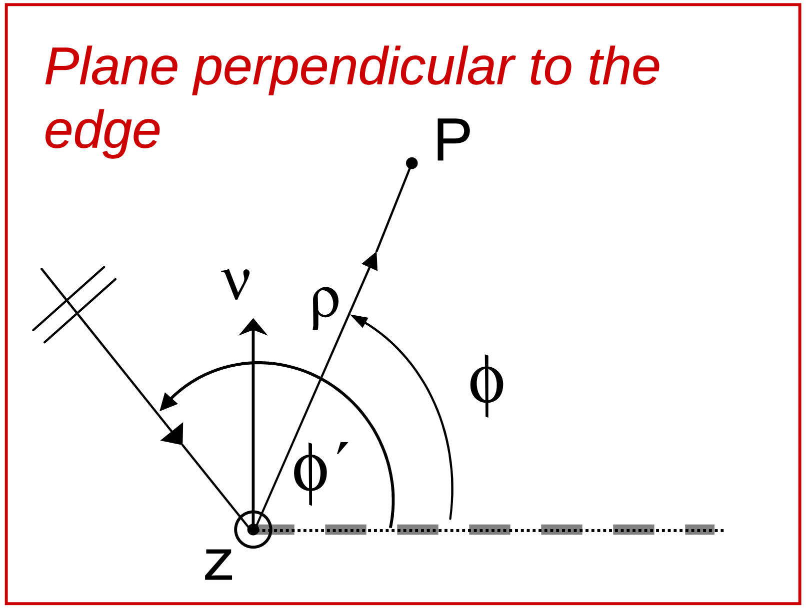


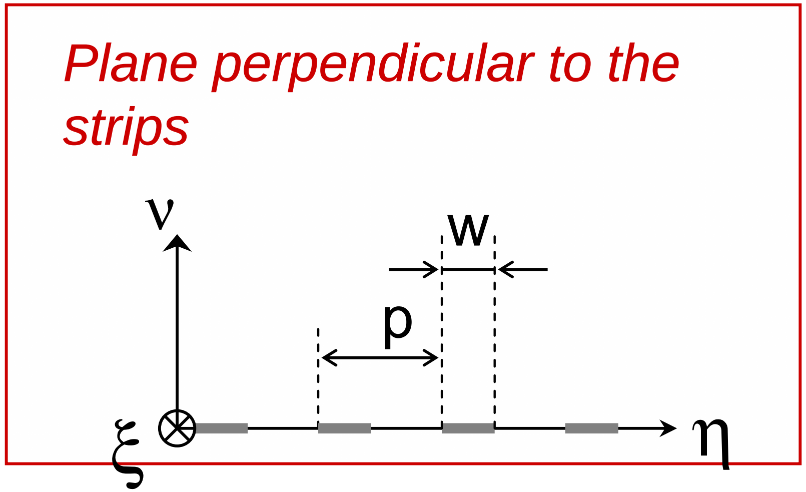


- ✓ Total field must satisfy suitable transition conditions on the half plane
- ✓ **Homogeneous** transition conditions can be assumed when periodicity is electrically small ($p \ll \lambda$)

Polarization selective screen







Scattering from edges in strip or wire gratings

- R.A. Hurd, ‘Diffraction by a unidirectionally conducting half-plane,’ *Can. J. Phys*., 38, 1960
- M. Ando, A. Kondo, K. Kagoshima, ‘Scattering of an arbitrary wave by a thin strip grating reflector,’ *IEE Proc.*, 133, 1986
- V.A. Kozov, S.A. Tretyakov, ‘Diffraction of plane electromagnetic waves by a semi-infinite grid made of parallel conductors arranged at an angle to the grid’s edge,’ *Radio Eng. Elect. Phys.*, 5, 1984
- P. Nepa, G. Manara and A. Armogida, “Plane Wave Scattering by Edges in Unidirectionally Conducting Screens,” Radio Science, Vol. 35, N. 6, pp. 1265-1278, Nov.-Dec. 2000
- K.W. Whites, R. Mittra, ‘An equivalent boundary-condition model for lossy planar periodic structures at low frequencies,’ *IEEE Trans. Antennas Propag.*, 44, 12, 1996.

Equivalent transition conditions

- Zero-th order model (with respect to p/λ):

  Unidirectionally Electric Conducting (UEC) screen

$$E_\xi^+ = E_\xi^- = 0, \quad E_\eta^+ - E_\eta^- = 0, \quad H_\xi^+ - H_\xi^- = 0$$

- First order model (with respect to p/λ)

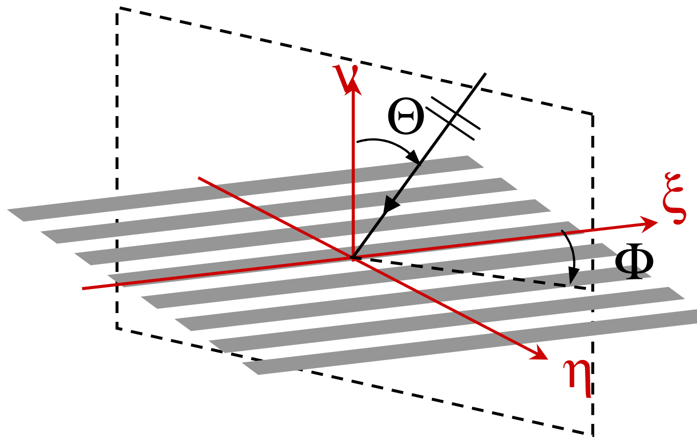
$$E_\xi = \frac{l_2}{2}\left[j\omega\mu_0\left(H_\xi^+ - H_\xi^-\right) + \frac{\partial}{\partial\xi}\left(E_y^+ - E_y^-\right)\right]$$

$$H_\xi^+ - H_\xi^- = 2\, l_1\left(-j\omega\mu_0 E_\eta + \frac{\partial H_y}{\partial \xi}\right)$$

$$E_\xi^+ = E_\xi^-, \qquad E_\eta^+ = E_\eta^-$$

dove $l_1 = \frac{p}{\pi}\ln\left(\sec\left(\frac{\pi w}{2p}\right)\right), \qquad l_2 = \frac{p}{\pi}\ln\left(\operatorname{cosec}\left(\frac{\pi w}{2p}\right)\right)$

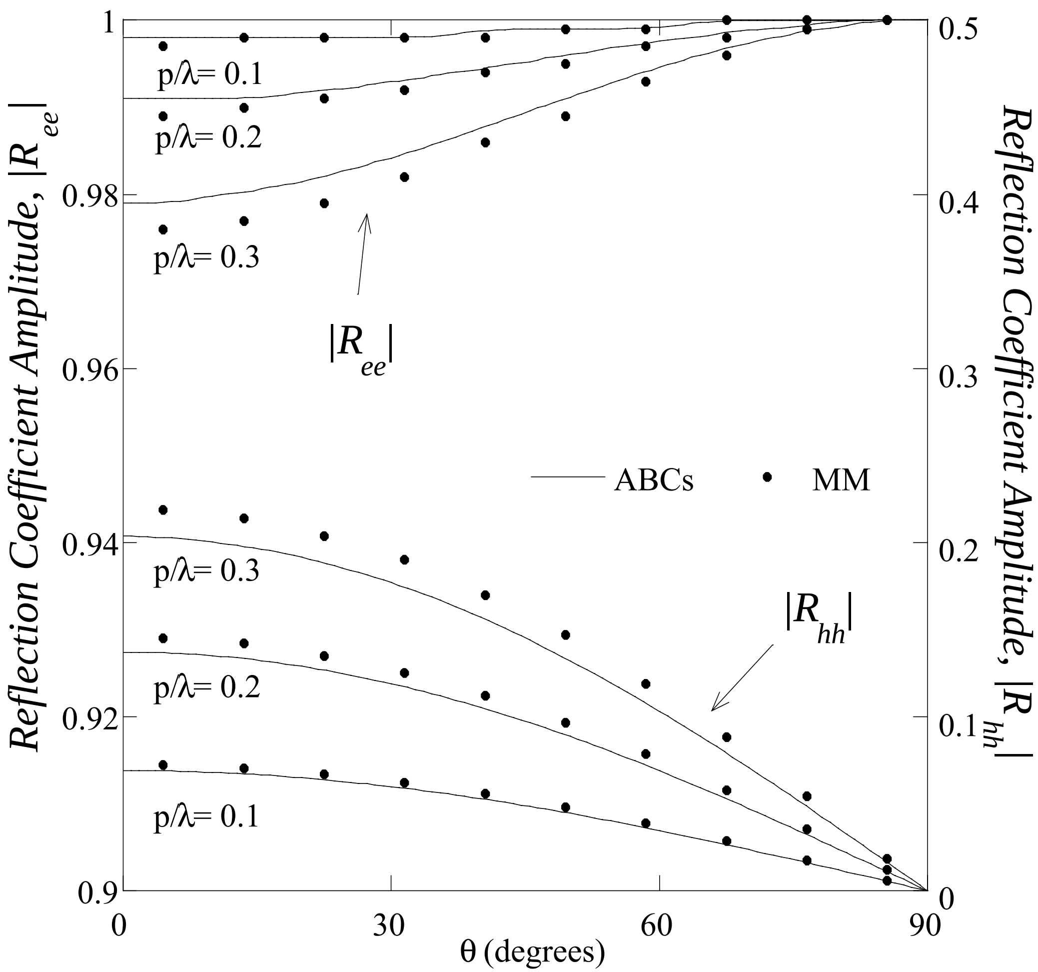
Validation of the Transition Conditions



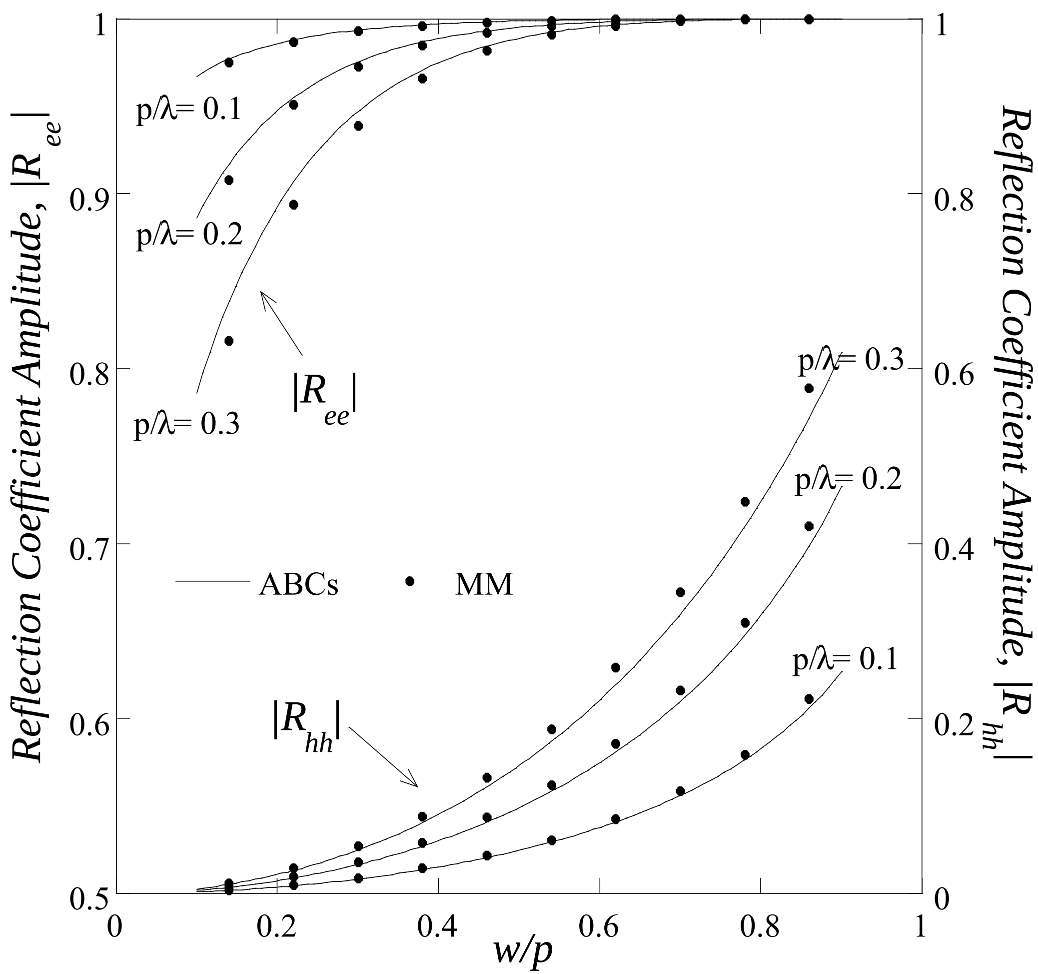
Reflection Coefficients:

$$R_{ee} = E_{\xi}^{r} / E_{\xi}^{i}, \quad R_{hh} = ZH_{\xi}^{r} / ZH_{\xi}^{i}$$

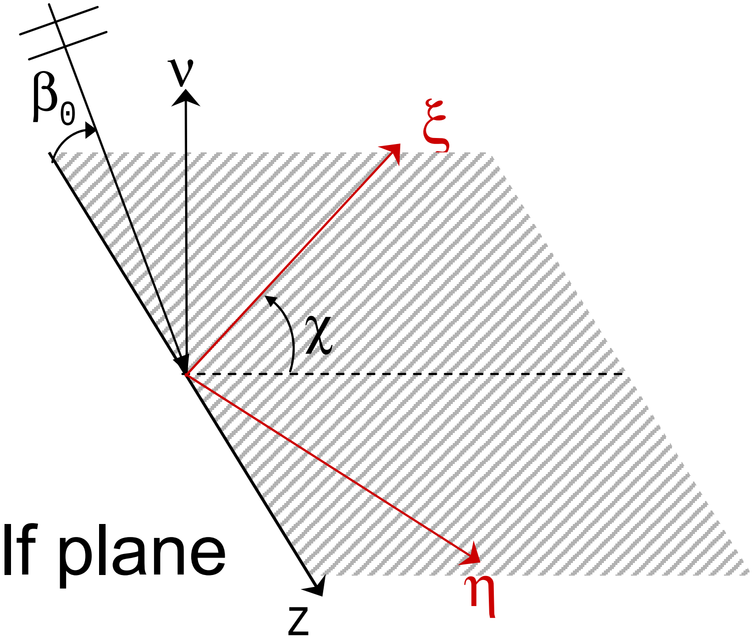
w/p = 0.5, Φ = 45°



Θ = 45°, Φ = 45°



UEC model $\sigma_\xi = \infty, \sigma_\eta = 0$



- ✓ 3D scattering by the edge of a UEC half plane
- ✓ 3D scattering by the edge of a UEC-PEC junction

⬇

Closed form exact solution

(contains Fresnel integral)

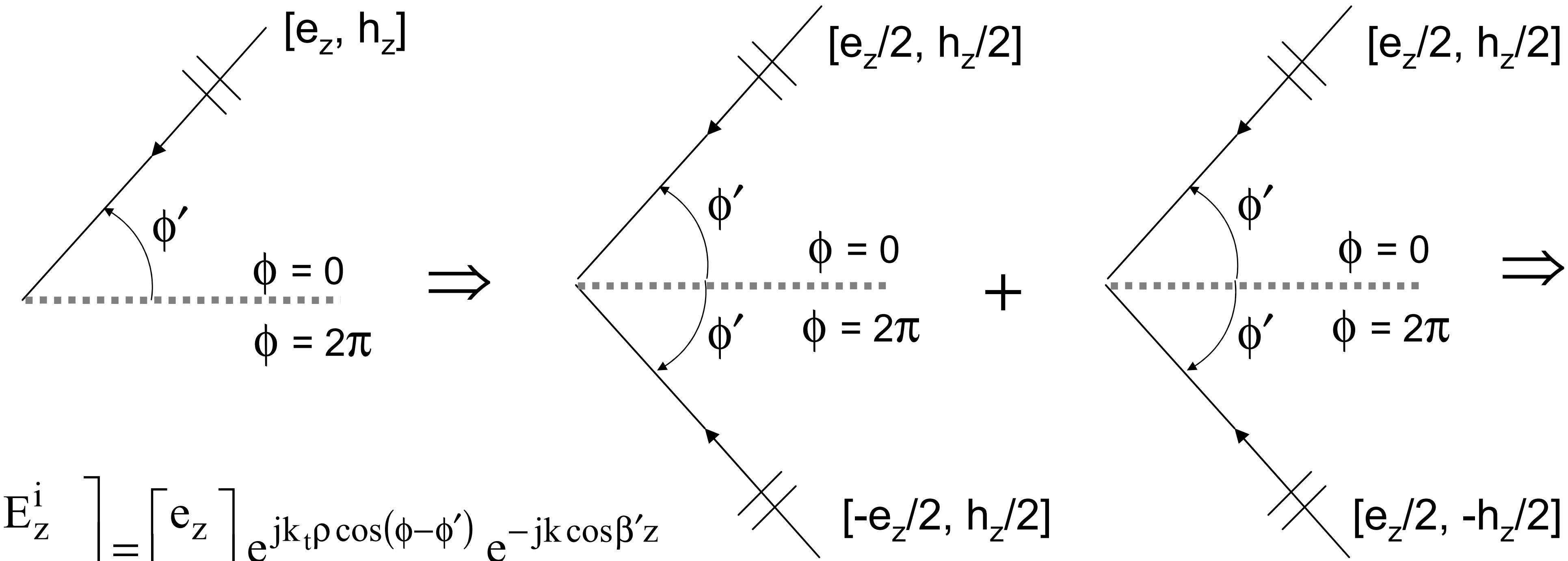
P. Nepa, G. Manara and A. Armogida, "Plane Wave Scattering by Edges in Unidirectionally Conducting Screens," *Radio Science*, vol. 35, No. 6, Nov.-Dec. 2000.

Solution Procedure

- ✓ Reduction to a couple of two-part boundary value problems: bisection method
- ✓ Exact spectral solution: Sommerfeld-Maliuzhinets method
- ✓ Uniform high-frequency expressions for the scattered field: saddle point technique

R.G. Kouyoumjian, G. Manara, P. Nepa, and B.J.E. Taute, "A UTD Solution for the Diffraction of an Inhomogeneous Plane Wave by a Wedge," *Radio Science*, vol. 31, no. 6, pp. 1387-1397, Nov.-Dec. 1996

Bisection Method



$$\begin{bmatrix} E_z^i \\ Z H_z^i \end{bmatrix} = \begin{bmatrix} e_z \\ h_z \end{bmatrix} e^{jk_t \rho \cos(\phi - \phi')}\, e^{-jk\cos\beta' z}$$

where $\quad k_t = k \sin\beta'$

ODD symmetric excitation

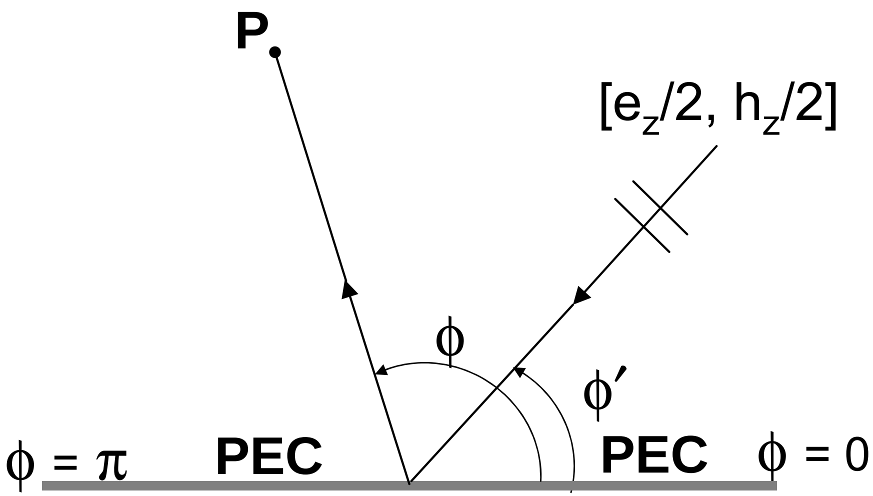
EVEN symmetric excitation

H.C. Ly, R.G. Rojas, P.H. Pathak, “EM Plane Wave Diffraction by a Planar Function of Two Thin Material Half Planes - Oblique Incidence”, *IEEE Trans. Antenna Propag.*, 41, 4, 1997

Bisection Method (cont'd)

ODD symmetric excitation:

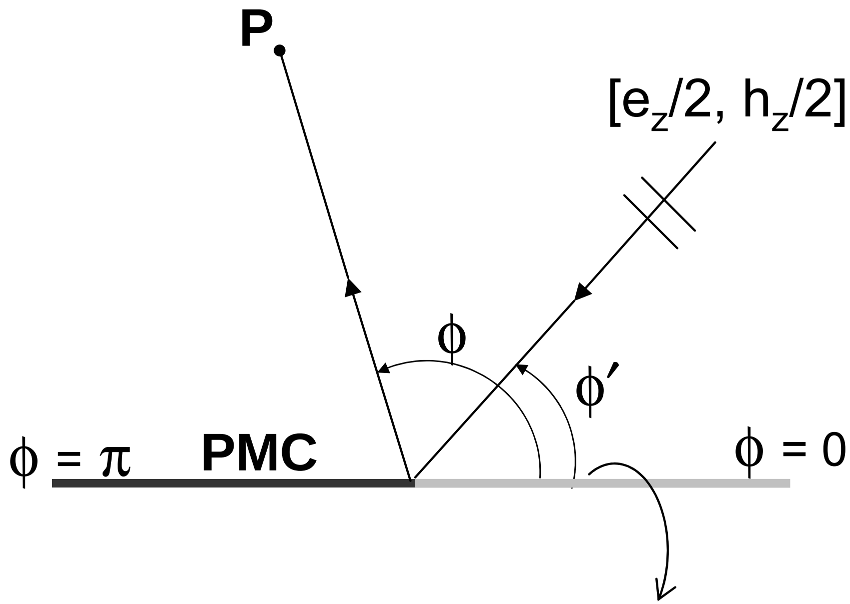
Plane wave reflection by an infinite PEC screen: $0 \le \phi \le \pi$



+

EVEN symmetric excitation:

Two-part boundary value problem: $0 \le \phi \le \pi$



$$\frac{1}{\rho}\frac{\partial E_\xi}{\partial \phi} - jk_t \sin\theta_2\, E_\xi = 0, \quad \frac{1}{\rho}\frac{\partial H_\xi}{\partial \phi} - jk_t \sin\theta_1\, H_\xi = 0, \quad \sin\theta_{1,2} = \left(jk_t\, l_{1,2}\right)^{-1}$$

The Sommerfeld-Maliuzhinets Method

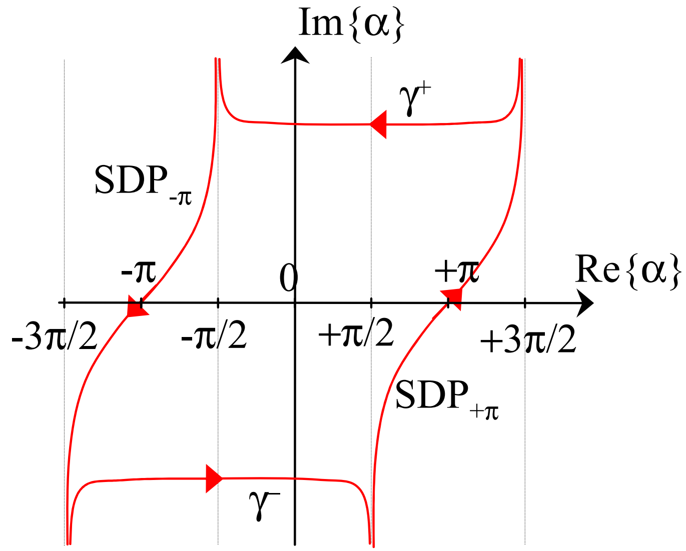
G. D. Maliuzhinets, “Excitation, Reflection and Emission of Surface Waves from a Wedge with Given Face Impedances”, *Sov. Phys. Dokl.*, 3, 1958.

⇨ Spectral Representations:

$$\left[E_\xi, ZH_\xi\right] = \frac{1}{2\pi j}\int_\gamma \left[t_e(\alpha+\phi-\pi/2), t_h(\alpha+\phi-\pi/2)\right] e^{jk_t\rho\cos\alpha} d\alpha$$

$$\left[E_z, ZH_z\right] = \frac{1}{2\pi j}\int_\gamma \left[s_e(\alpha+\phi-\pi/2), s_h(\alpha+\phi-\pi/2)\right] e^{jk_t\rho\cos\alpha} d\alpha$$

γ is the Sommerfeld integration path



Rigorous Spectral Solution for the Even Symmetric Excitation Case

$$\begin{bmatrix} s_e(\alpha) \\ s_h(\alpha) \end{bmatrix} = \frac{\sin\beta'}{\Delta(\alpha)} \overline{\overline{M}}(\alpha) \begin{bmatrix} t_e(\alpha) \\ t_h(\alpha) \end{bmatrix}$$

where:

$$\overline{\overline{M}}(\alpha) = \begin{bmatrix} -\cos\chi\cos\beta'\sin\alpha - \sin\chi\sin\beta' & -\cos\chi\cos\alpha \\ \cos\chi\cos\alpha & -\cos\chi\cos\beta'\sin\alpha - \sin\chi\sin\beta' \end{bmatrix}$$

$$\Delta(\alpha) = (\cos\chi\cos\beta'\sin\alpha + \sin\chi\sin\beta')^2 + (\cos\chi\cos\alpha)^2 =$$
$$= -(\cos\chi\sin\beta')^2(\sin\alpha - \sin\vartheta^+)(\sin\alpha - \sin\vartheta^-)$$

with $\sin\vartheta^\pm = \dfrac{\sin\chi\cos\beta' \pm 1}{\cos\chi\sin\beta'}$ and $\vartheta^+ = \pi/2 + j\vartheta_i^+,\ \vartheta^- = -\pi/2 + j\vartheta_i^-$

$$\vartheta_i^\pm = \cosh^{-1}\left(\pm\sin\vartheta^\pm\right) \geq 0$$

Rigorous Spectral Solution

$$t_e(\alpha) = -\frac{e_\xi}{2} \cdot \frac{\Psi(\alpha, \vartheta_2)}{\Psi(\phi' - \pi/2, \vartheta_2)} \cdot \frac{\sin\alpha - \sin\vartheta^-}{\cos\phi' + \sin\vartheta^-} \cdot \frac{\sin\phi'}{\sin\alpha + \cos\phi'} \cdot \frac{\cos(\alpha/2 + \pi/4)}{\cos(\phi'/2)}$$

$$t_h(\alpha) = -\frac{\Psi(\alpha, \vartheta_1)}{\Psi(\phi' - \pi/2, \vartheta_1)} \cdot \frac{\sin\alpha - \sin\vartheta^-}{\cos\phi' + \sin\vartheta^-} \cdot \left(\frac{h_\xi}{2} \cdot \frac{\sin\phi'}{\sin\alpha + \cos\phi'} + C_h \right)$$

$\Psi(\alpha, \vartheta) = \Psi_{\pi/2}(\alpha + \vartheta - \pi)\, \Psi_{\pi/2}(\alpha - \vartheta)$, $\Psi_{\pi/2}$ is the standard Maliuzhinets special function

Non-physical Pole Singularities:

$$\alpha_p = \pi/2 + j\vartheta_i^+, -\pi/2 - j\vartheta_i^-, -\pi/2 + j\vartheta_i^-$$

Surface Wave Pole Singularity:

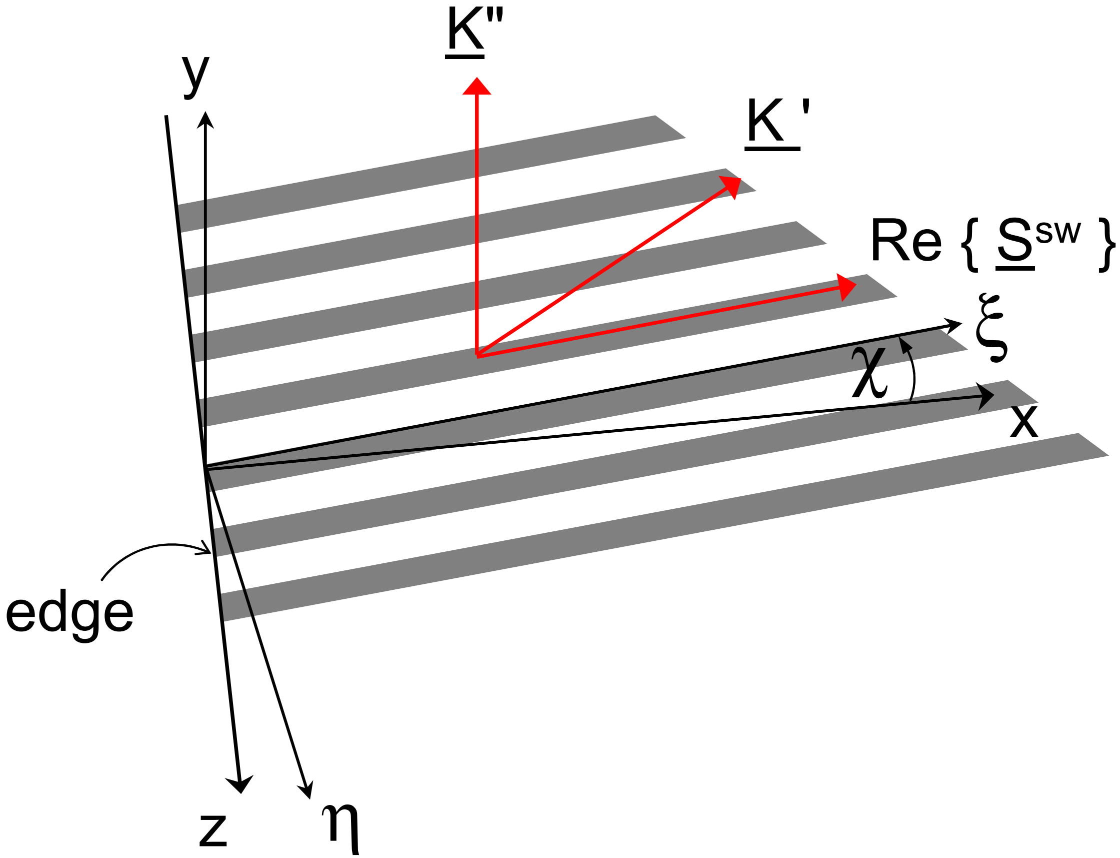
$$\alpha_p^{sw} = -\pi - \vartheta^+ = -3\pi/2 - j\vartheta_i^+$$

Surface Waves

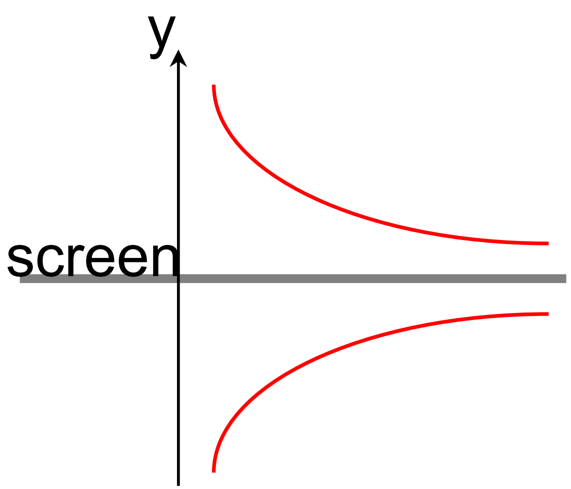
$$\begin{bmatrix} E_z^{sw} \\ Z_0 H_z^{sw} \end{bmatrix} = \begin{bmatrix} A_e \\ A_h \end{bmatrix} e^{-jk_t\rho\cos(\phi + j\vartheta_i^+)} U\left(\phi^{sw} - \phi\right) + \begin{bmatrix} A_e \\ -A_h \end{bmatrix} e^{-jk_t\rho\cos(-\phi + j\vartheta_i^+)} U\left(\phi - 2\pi + \phi^{sw}\right)$$

$$\underline{K}^{sw} = k\left(\sin\beta' \cosh\vartheta_i^+ \underline{i}_x + \cos\beta' \underline{i}_z\right) \mp jk\sin\beta' \sinh\vartheta_i^+ \underline{i}_y = \underline{K}' \mp j\underline{K}''$$

$$\boxed{\underline{K}^{sw} \cdot \underline{i}_\xi = k}$$



$$\boxed{E_\xi^{sw} = 0, \quad H_\xi^{sw} = 0}$$

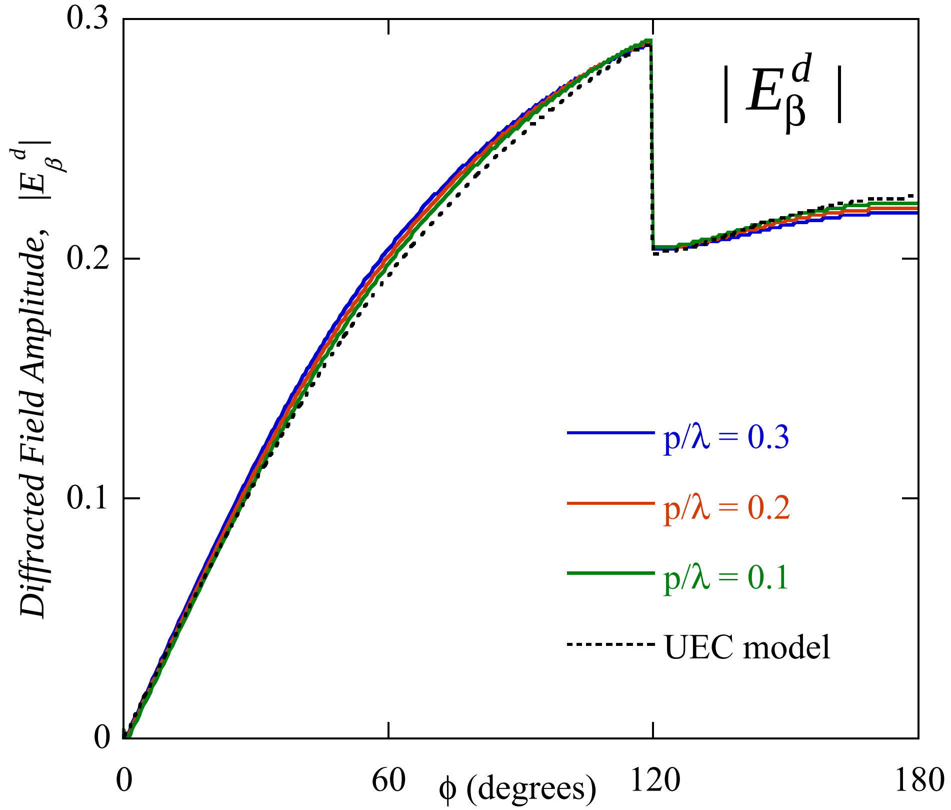


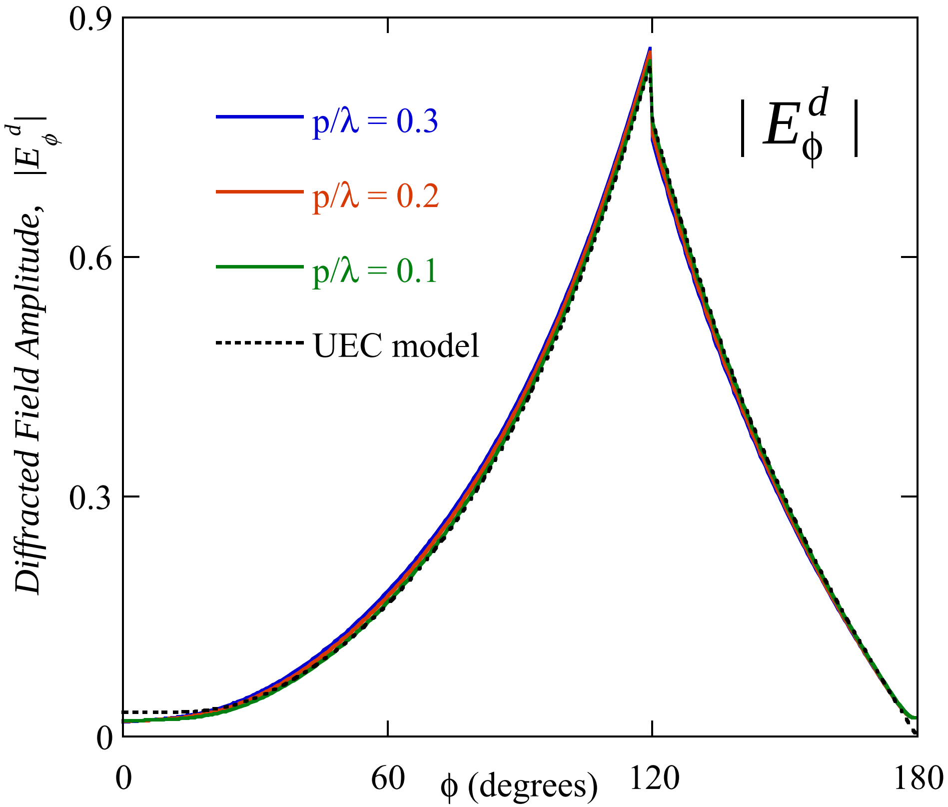
TM$_\xi$ polarization: $e_\xi = 1, \quad h_\xi = 0$

Strip grating parameters:

w/p = 0.5, $\chi = \pi/4$

$\beta' = \pi/3, \quad \phi' = \pi/6, \quad k_t\rho = 5$



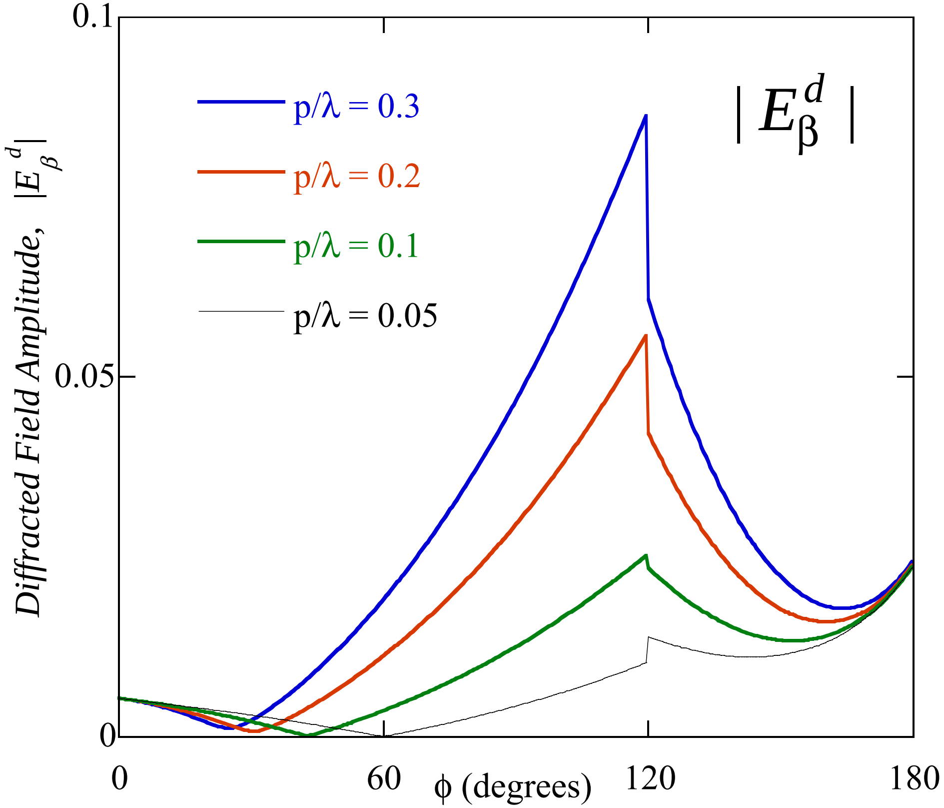


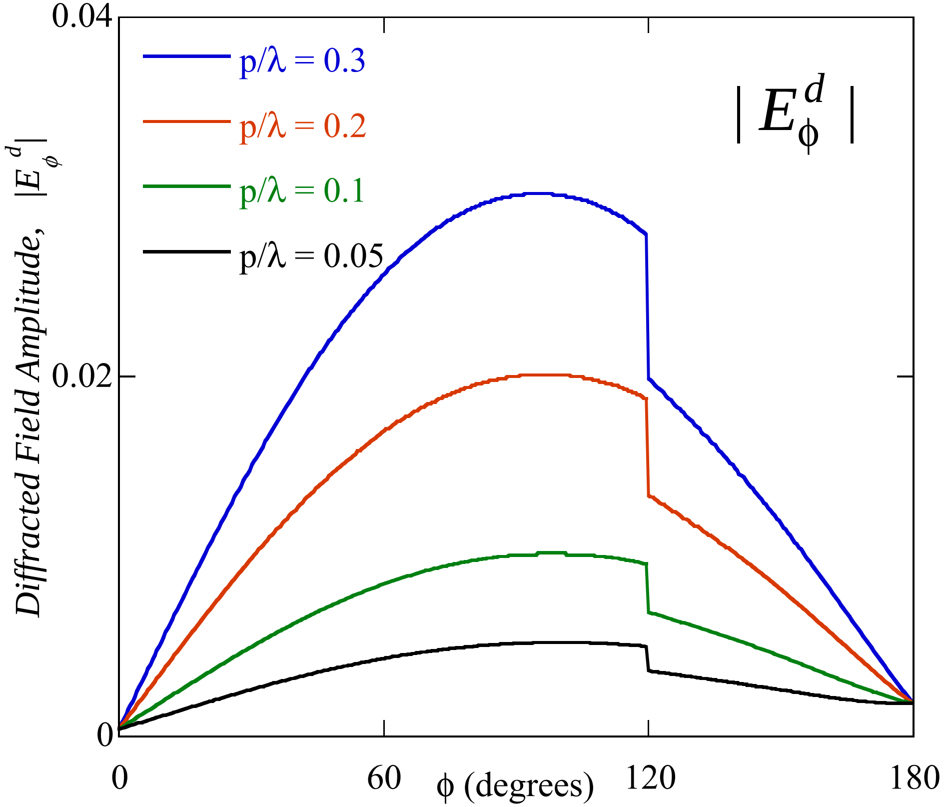
TE$_\xi$ polarization: $e_\xi = 0, \quad h_\xi = 1$

Strip grating parameters:

w/p = 0.5, $\chi = \pi / 4$

$\beta' = \pi / 3, \quad \phi' = \pi / 6, \quad k_t\rho = 5 \quad \chi = \pi / 4$



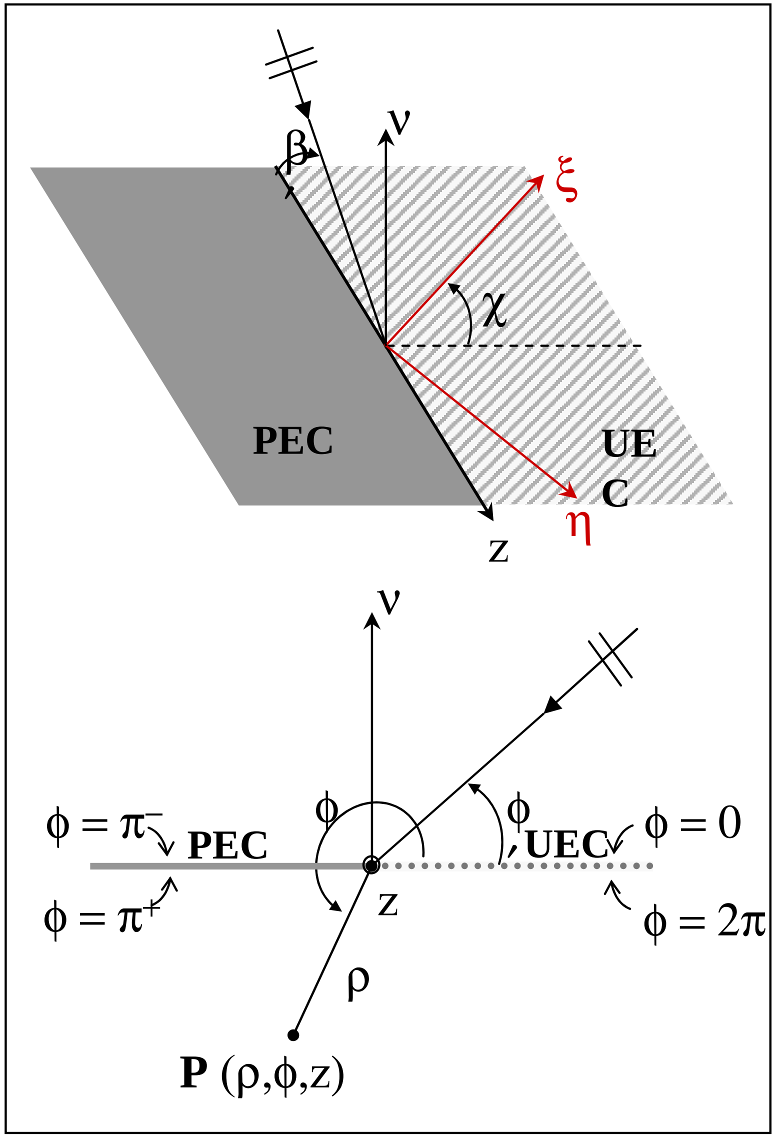


UEC model: $\left|E_\beta^d\right| = \left|E_\phi^d\right| = 0$

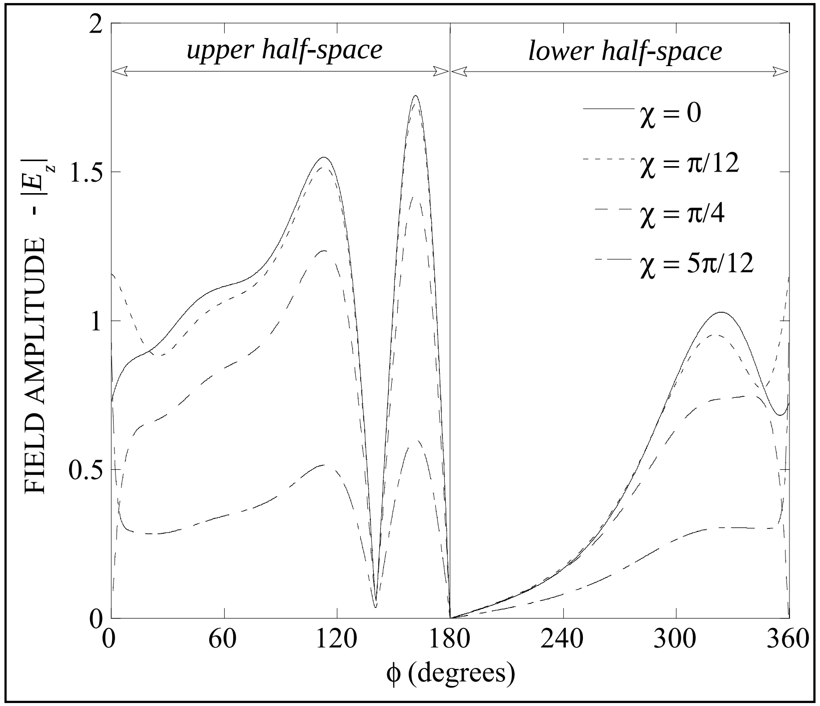
UEC-PEC junction

TE$_\xi$ polarization: $(e_\xi = 0, h_\xi = 1)$

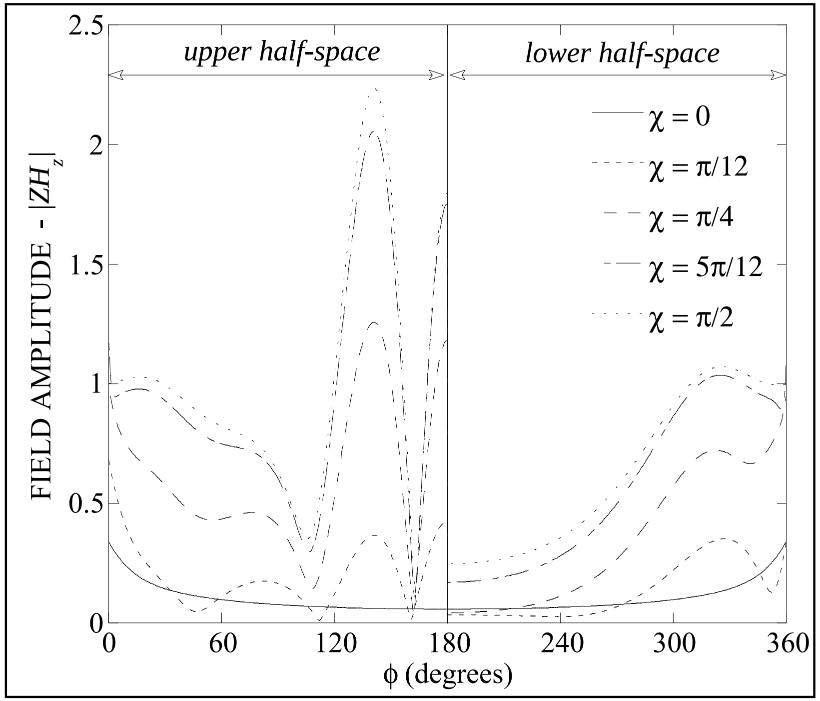
$k_t\rho = 5, \ \beta' = \pi/3, \quad \phi' = \pi/2$



Total Electric field along the Edge



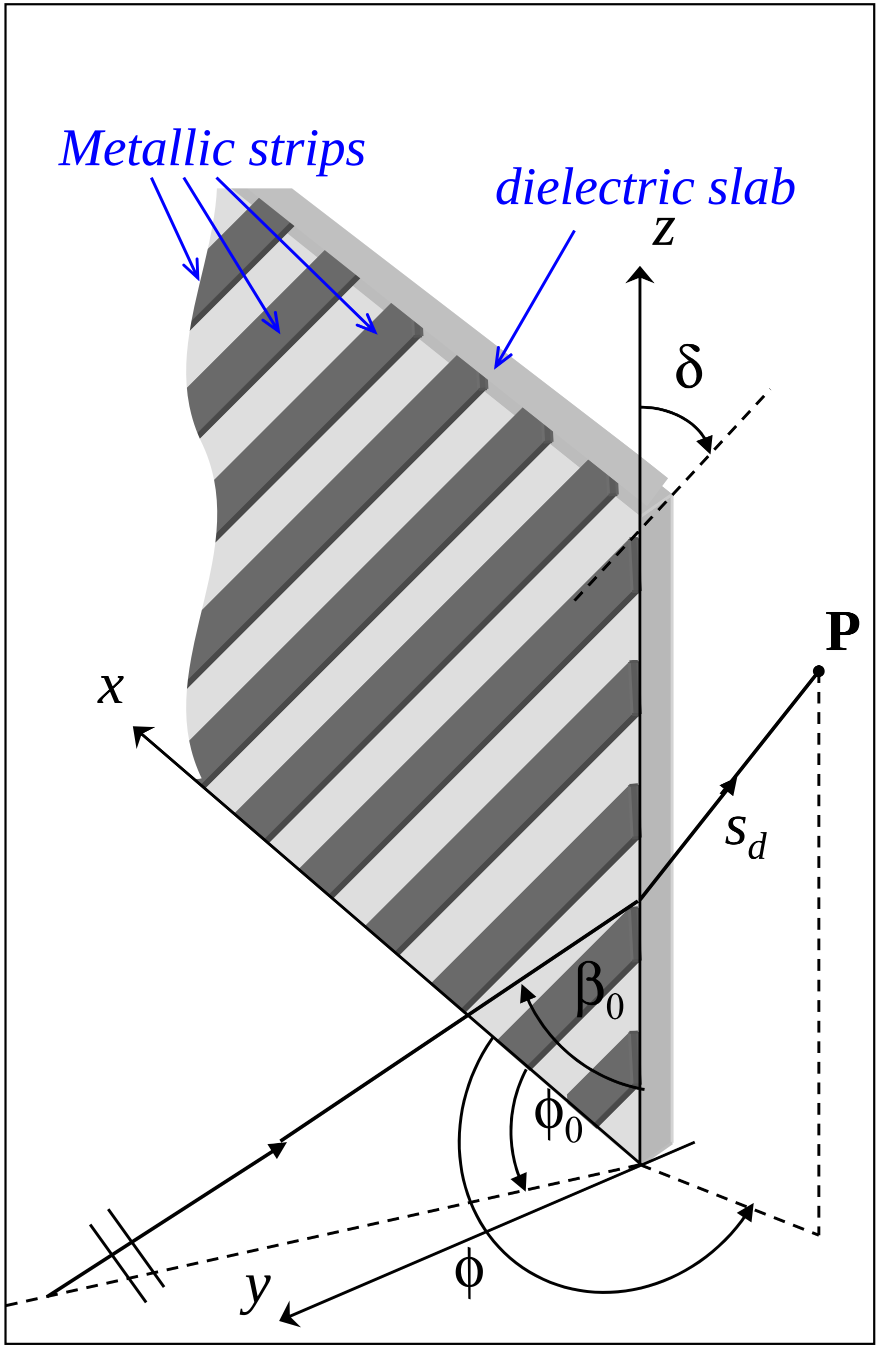
Total Magnetic field along the edge



Dielectric slab loaded by a *strip grating*

Heuristic solutions

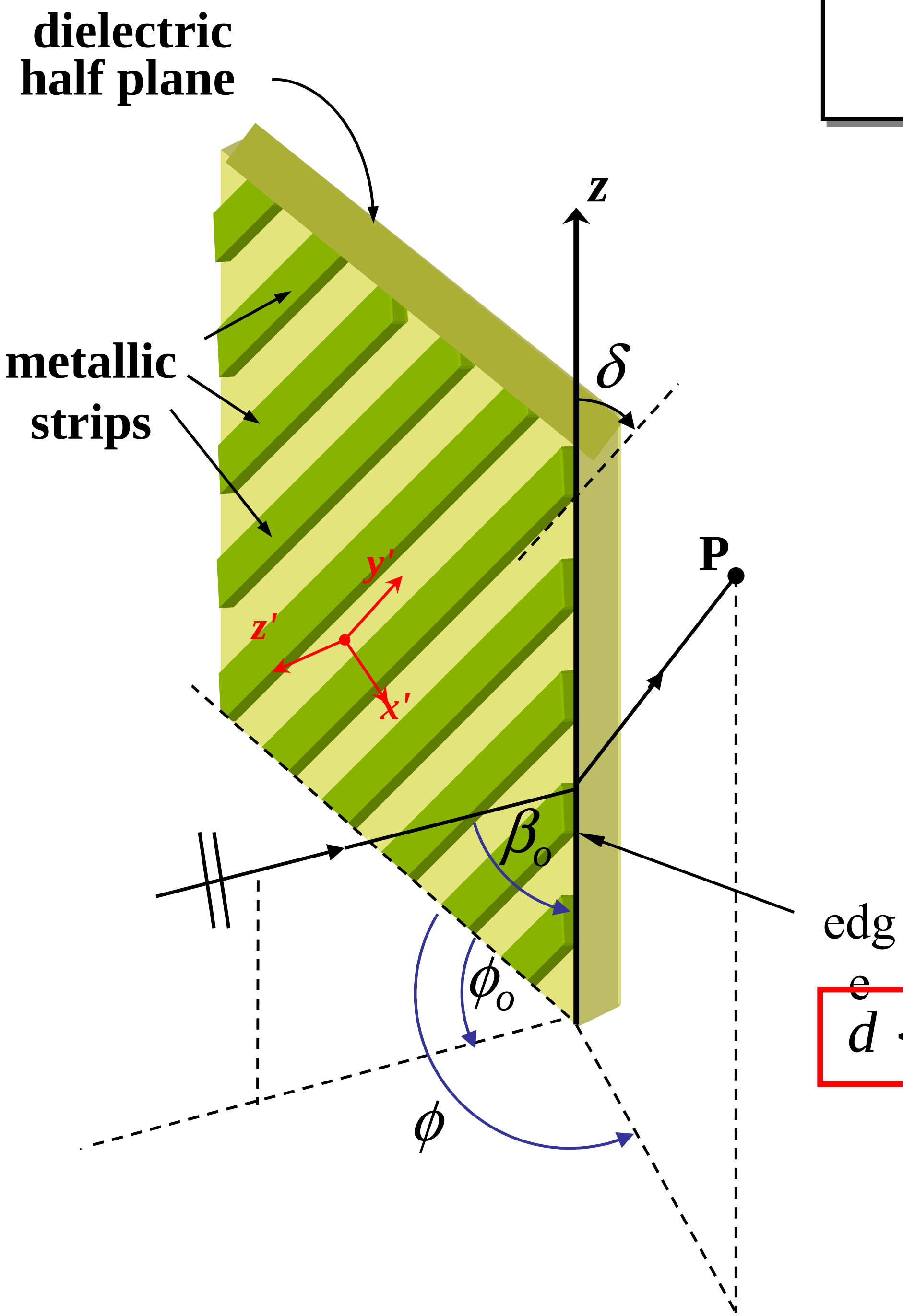
- ✓ compact and computationally efficient
- ✓ guarantees total field continuity at the shadow boundaries

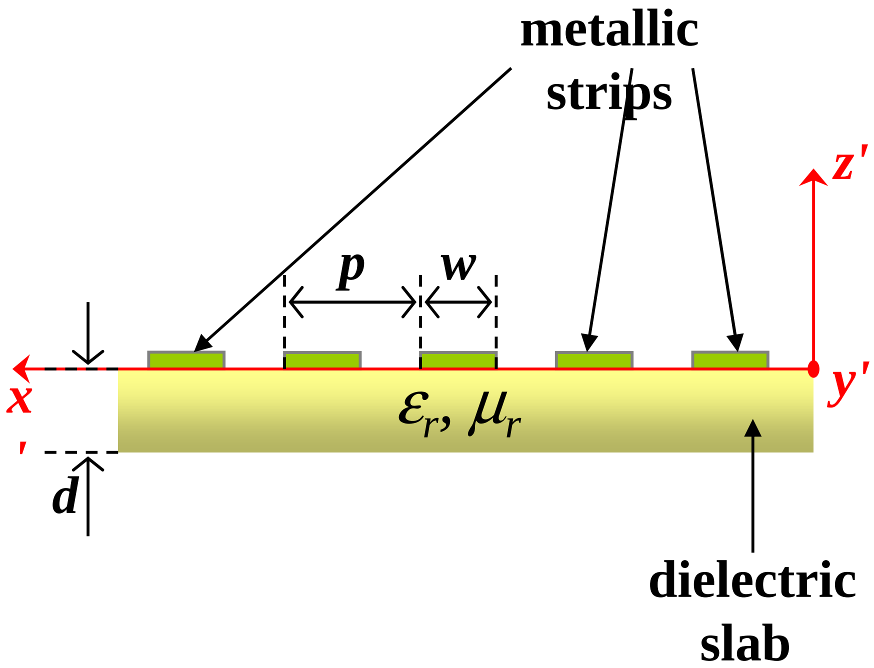


- A. Armogida, G. Manara, A. Monorchio and P. Nepa, "Validation of a Class of Heuristic Diffraction Coefficients for Edges in Penetrable Anisotropic Screens," *Proc. ICEAA 99*, Torino, Italy, Sept. 1999.

- A. Armogida, G. Manara, A. Monorchio, P. Nepa, "High-Frequency Scattering by the Edge of a Thin Dielectric Slab Loaded by a Metallic Strip Grating," *IEE Proc. – Microw., Antennas and Propag.*, 147, 2, pp. 128-133, Apr. 2000.

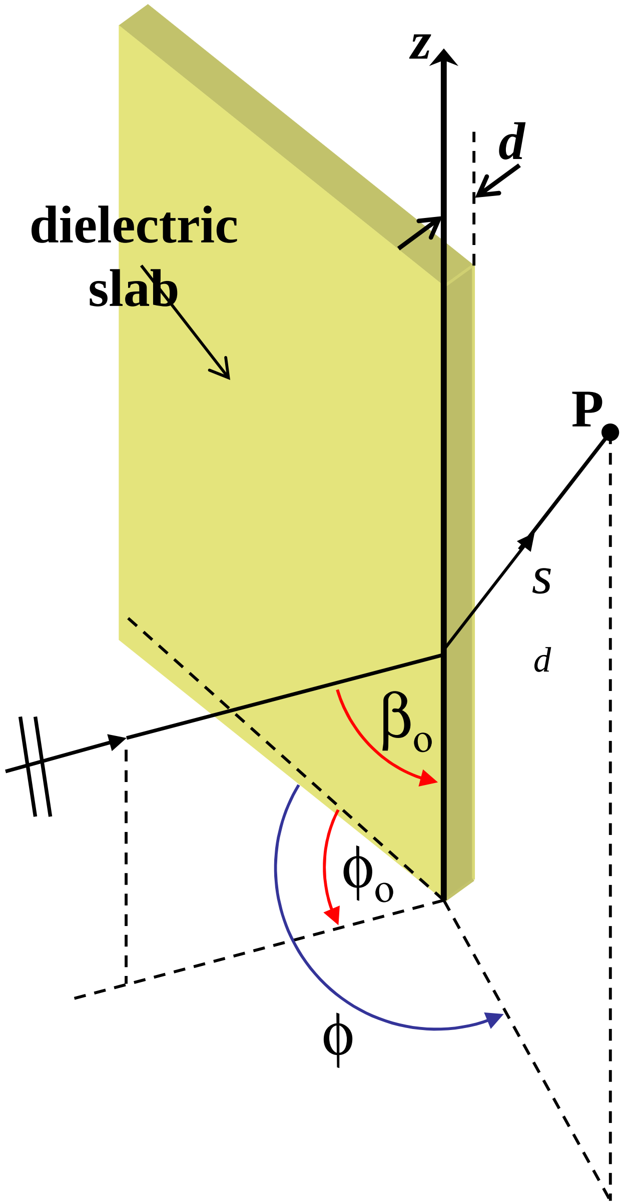
Formulation





$$d \ll \lambda_o$$

Heuristic Solution for a Thin Dielectric Slab



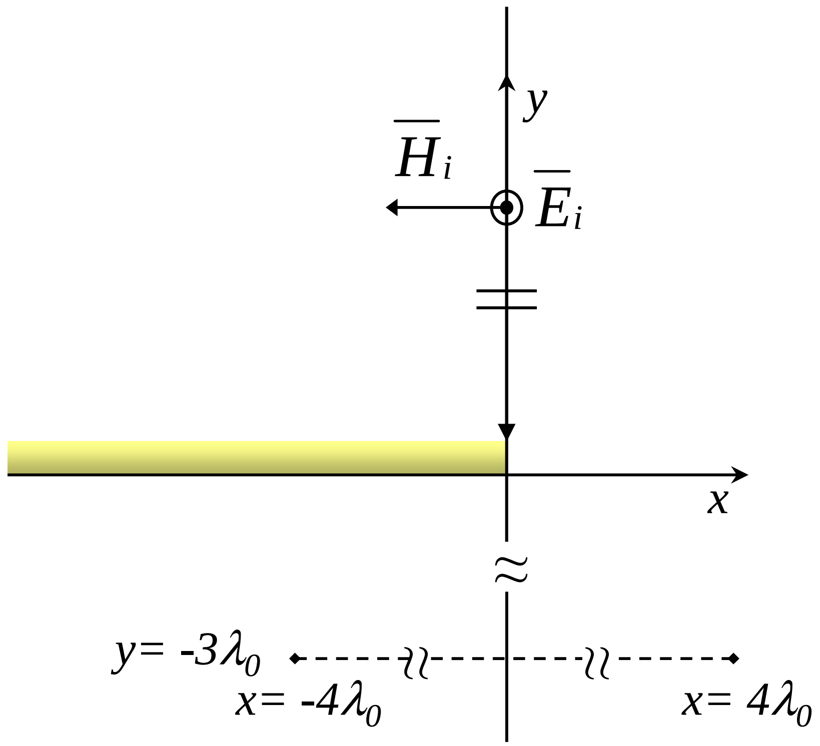
$d << \lambda_o$

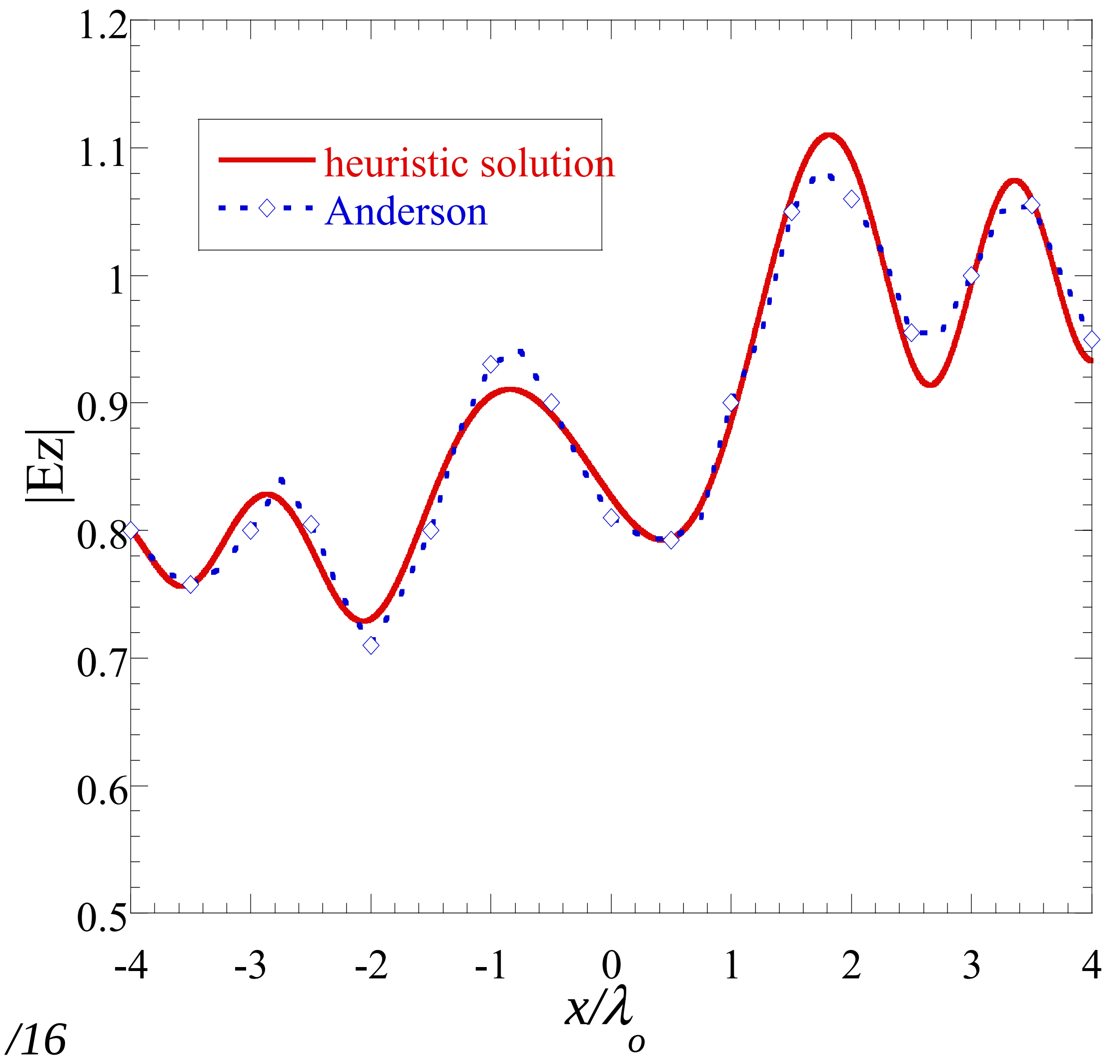
$$\overline{\overline{D}} = \frac{-e^{-j\frac{\pi}{4}}}{4\sqrt{2\pi k}\sin\beta_o}\left\{\left(\overline{\overline{I}} - \overline{\overline{T}}\right)\cdot\frac{F[kLa(\phi-\phi_o)]}{\cos\left(\frac{\phi-\phi_o}{2}\right)} + \overline{\overline{R}}\cdot\frac{F[kLa(\phi+\phi_o)]}{\cos\left(\frac{\phi+\phi_o}{2}\right)}\right\}$$

$$L = s_d \sin^2\beta_o \qquad a(x) = 2\cos^2(x/2) \qquad F(X) = 2j\sqrt{X}e^{jX}\int_{\sqrt{X}}^{\infty} e^{-j\tau^2}d\tau$$

W.D. Burnside and K.W. Burgener, “High Frequency Scattering by a Thin Lossless Dielectric Slab”, *IEEE Trans. on Antennas and Prop.*, vol. AP-31, No. 1, pp. 104-110, January 1983.

Thin Dielectric Slab: Validation

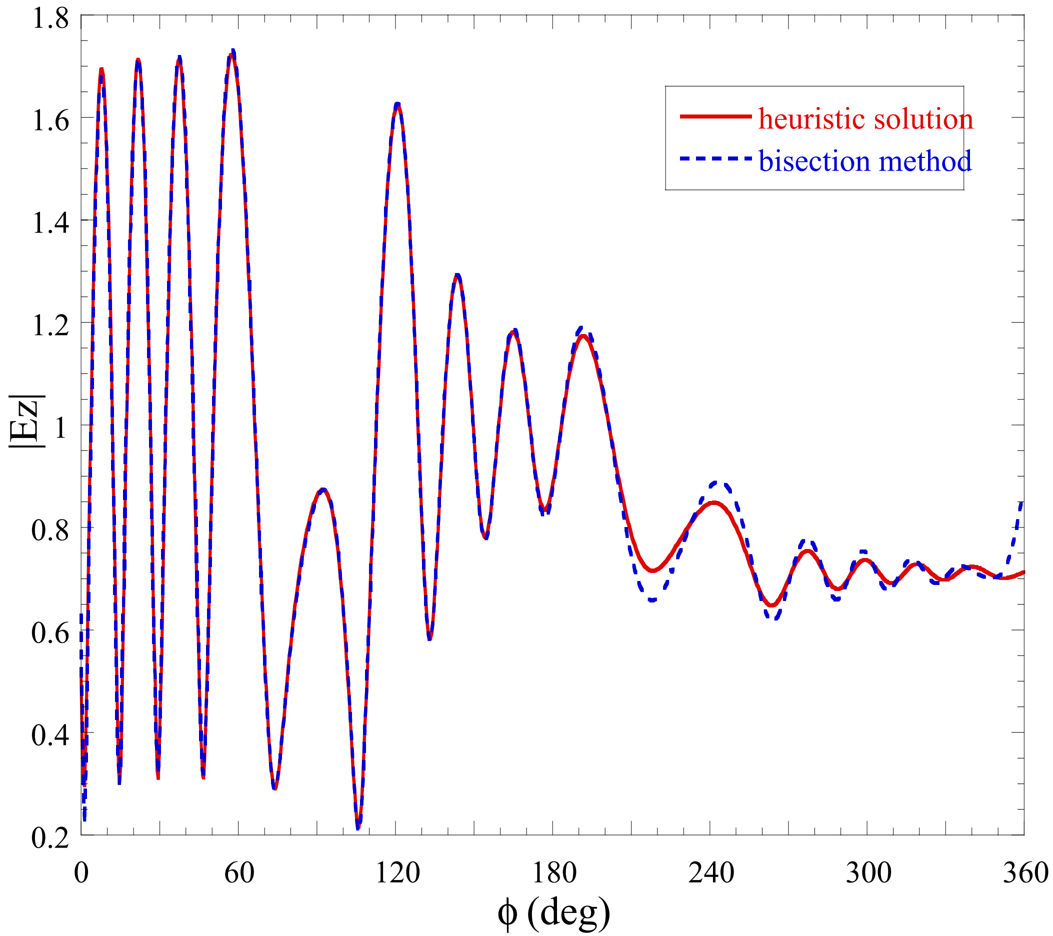


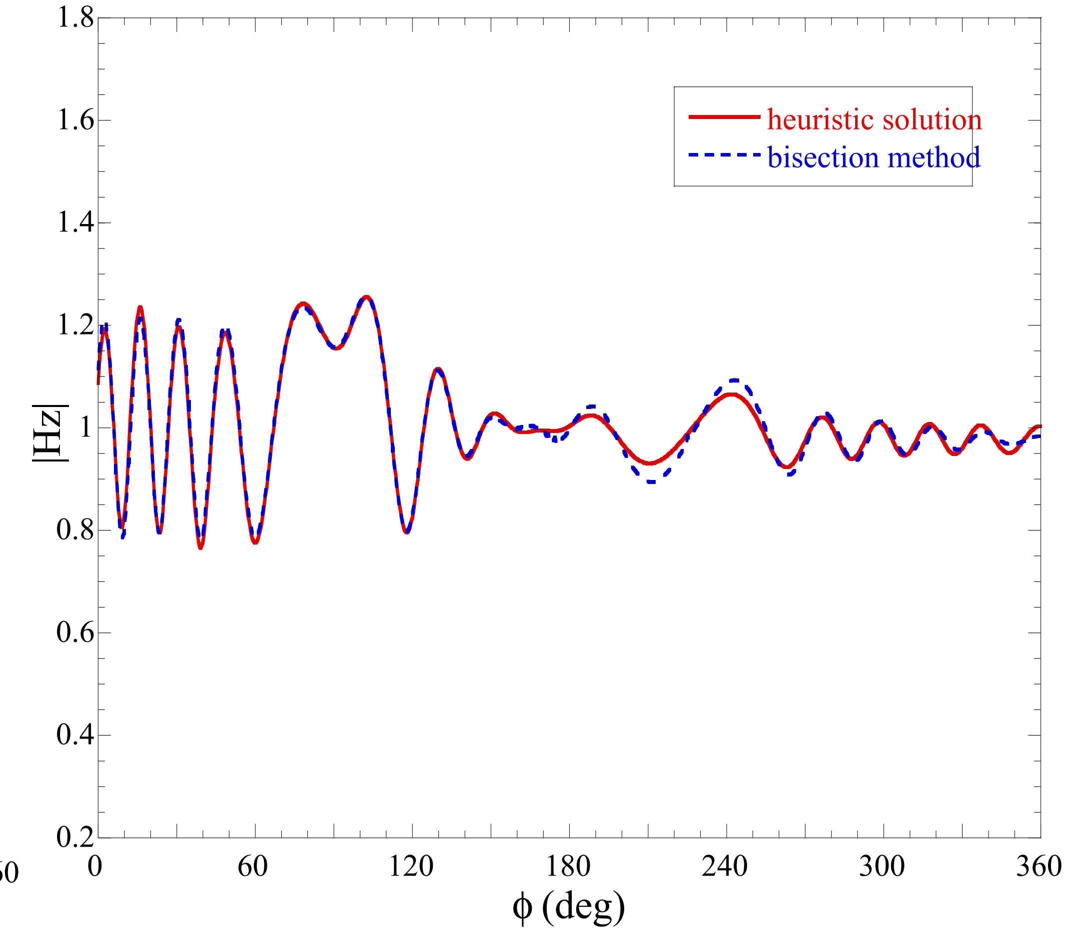


physical parameters: $\varepsilon_r = 5$, $\mu_r = 1$, $d = \lambda_0 / 16$

I. Anderson, "Plane Wave Diffraction by a Thin Dielectric Half-Plane", *IEEE Trans. on Antennas and Prop.*, vol. AP-27, No. 5, pp. 584-589, September 1979.

Thin Dielectric Slab: Validation





$E_\beta = 1$, $E_\phi = 0$, $\beta_0 = 90^o$, $\phi_0 = 90^o$, $s_d = 3\lambda_0$

physical parameters: $\varepsilon_r=5$, $d = \lambda_0/16$

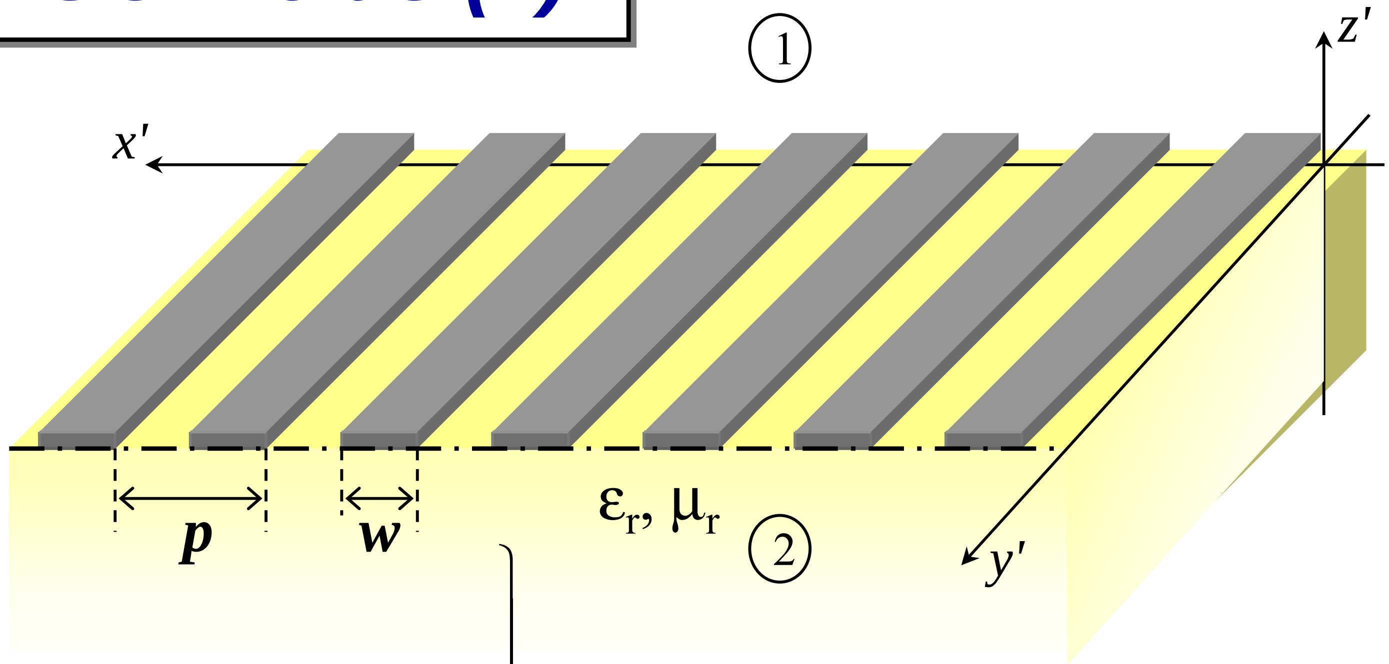
$E_\beta = 0$, $E_\phi = 1$, $\beta_0 = 90^o$, $\phi_0 = 45^o$, $s_d = 3\lambda_0$

physical parameters: $\varepsilon_r=5$, $d = \lambda_0/32$

H.C. Ly, R.G. Rojas and P.H. Pathak, "EM Plane Wave Diffraction by a Planar Junction of Two Thin Material Half-Planes - Oblique Incidence", *IEEE Trans. on Antennas and Prop.*, vol. 41, No. 4, pp. 429-441, April 1993.

Approximate GO Fields (1)

$$p < 0.1\,\lambda_o$$



$$E_{x'1} = E_{x'2}\,, \qquad E_{y'1} = E_{y'2}$$

$$E_{z'1} = \frac{l_2}{2}\left[j\omega\mu_a\left(H_{y'2} - H_{y'1}\right) + \frac{1}{\varepsilon_a}\frac{\partial}{\partial z'}\left(\varepsilon_2 E_{x'2} - \varepsilon_1 E_{x'1}\right)\right]$$

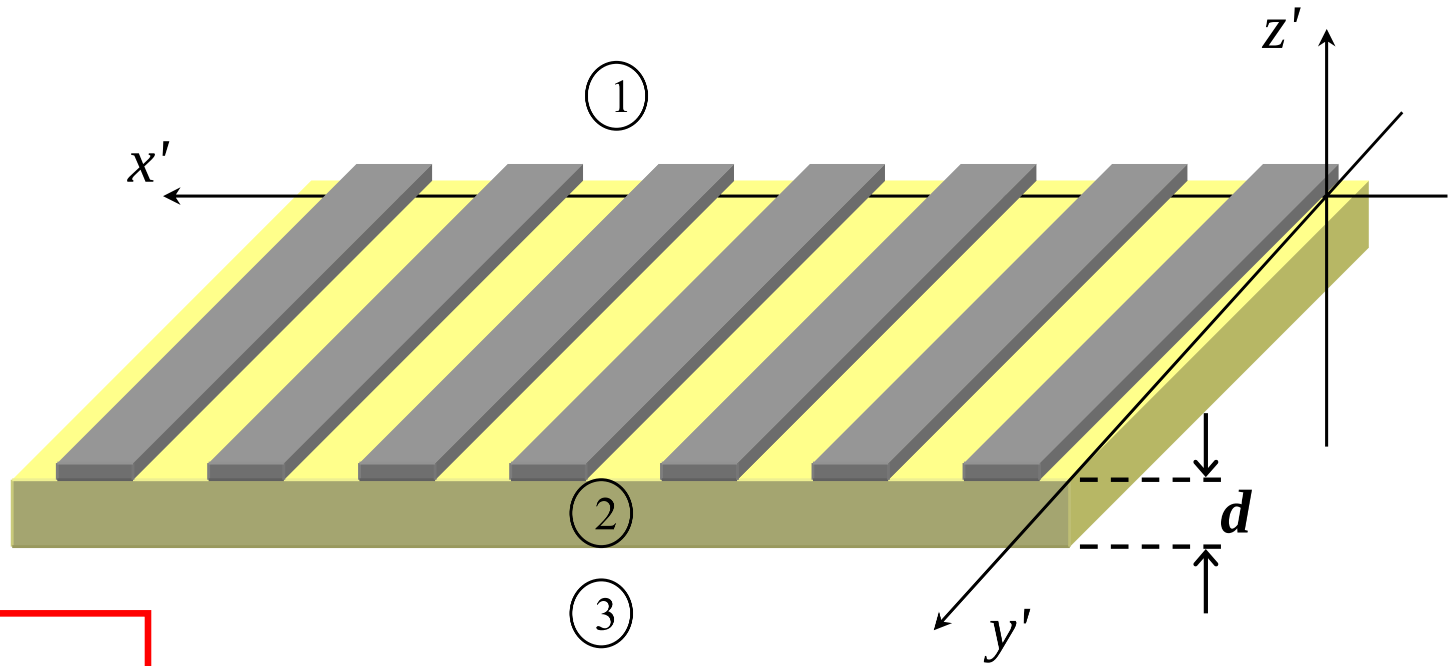
$$H_{z'2} - H_{z'1} = 2l_1\left(-j\omega\varepsilon_a E_{y'1} + \frac{1}{\mu_a}\frac{\partial B_{x'1}}{\partial z'}\right)$$

$$\forall\, x', y' \quad \text{at } z' = 0$$

with: $$l_1 = \frac{p}{\pi}\ln\left(sec\frac{\pi w}{2p}\right), \quad l_2 = \frac{p}{\pi}\ln\left(csc\frac{\pi w}{2p}\right), \quad \varepsilon_a = \frac{\varepsilon_1 + \varepsilon_2}{2}, \quad \mu_a = \frac{2\mu_1\mu_2}{\mu_1 + \mu_2}$$

R.R. DeLyser, "Use of Equivalent Boundary Conditions for the Solution of a Class of Strip Grating Structures", *IEEE Trans. on Antennas and Prop.*, vol. 41, No. 1, pp. 103-105, January 1993.

Approximate GO Fields (2)



$p < 0.1\,\lambda_o \qquad d > 0.3\,p$

$$\left.\begin{array}{ccc} E_{x'2} = E_{x'3} & & H_{x'2} = H_{x'3} \\ & \text{and} & \\ E_{y'2} = E_{y'3} & & H_{y'2} = H_{y'3} \end{array}\right\} \quad \forall\, x', y' \quad \text{at } z' = -d$$

Reflection and Transmission Matrices

$$\begin{bmatrix} E^r_{(//,\perp)} \\ E^r_{(//,\perp)} \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \cdot \begin{bmatrix} E^i_{(//,\perp)} \\ E^i_{(//,\perp)} \end{bmatrix}$$

$$R_{11} = \frac{C_i B_r - D_r A_i}{C_r B_r - D_r A_r} \qquad R_{12} = \frac{D_i B_r - D_r A_i}{C_r B_r - D_r A_r} \qquad R_{21} = \frac{A_i C_r - C_i A_r}{C_r B_r - D_r A_r} \qquad R_{12} = \frac{C_r B_i - D_i A_r}{C_r B_r - D_r A_r}$$

$$A_{i,r} = -\left(\frac{cos\psi cos\theta_1}{\eta_2 cos\theta_2} \frac{1+\Gamma_a}{1-\Gamma_a} \mp \frac{cos\psi}{\eta_1} \right) - j\left(2\omega\varepsilon_a l_1 cos\theta_1 cos\psi\right)$$

$$B_{i,r} = \left(\mp \frac{sin\psi cos\theta_2}{\eta_2} \frac{1-\Gamma_b}{1+\Gamma_b} - \frac{sin\psi cos\theta_1}{\eta_1} \right) \mp j\left(2\omega\varepsilon_a l_1 sin\psi - \frac{2k_1\mu_1 l_1 sin^2\theta_1 sin\psi}{\mu_a\eta_1} \right)$$

$$C_{i,r} = -cos\theta_1 sin\psi + j\frac{l_2}{2} sin\psi \left[-\frac{1}{\eta_2} M \pm \frac{1}{\eta_1} + \frac{1}{\varepsilon_a}\left(\varepsilon_2 k_2 sin^2\theta_2 M \mp \varepsilon_1 k_1 sin^2\theta_1\right) \right], \qquad M = \frac{cos\theta_1}{cos\theta_2} \frac{1+\Gamma_a}{1-\Gamma_a}$$

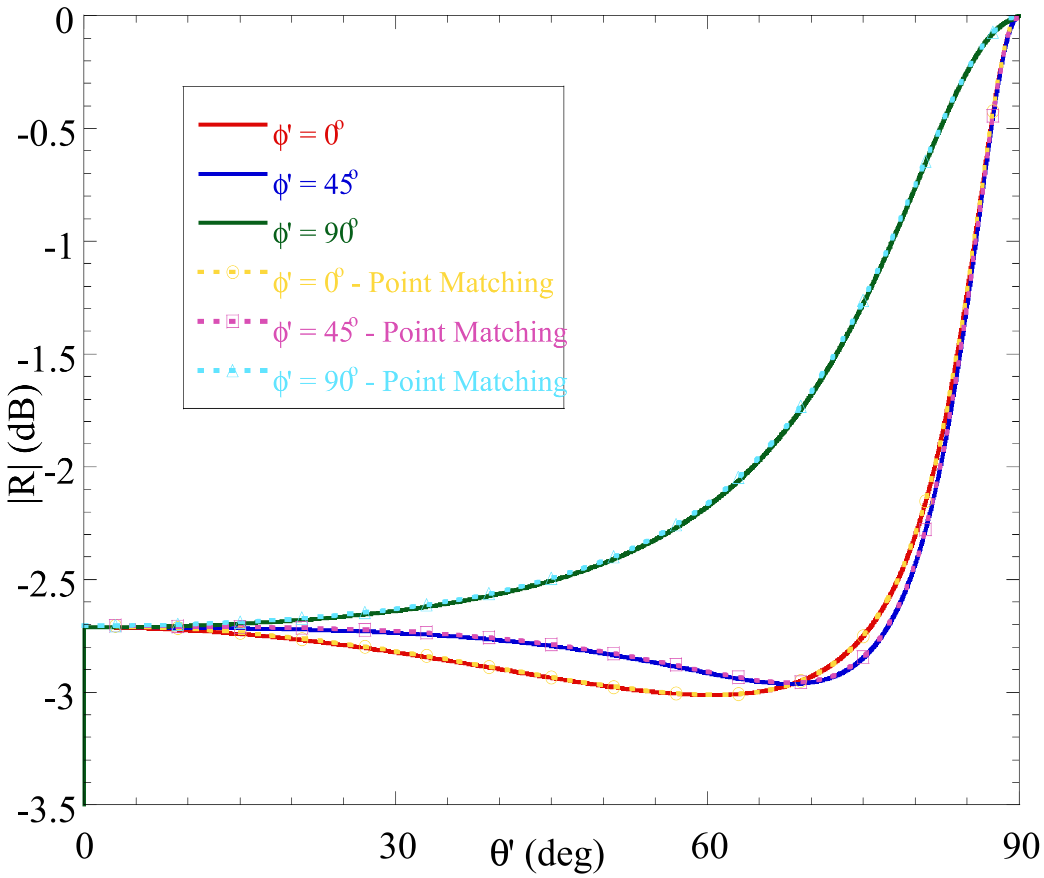
$$D_{i,r} = \pm cos\psi \pm j\omega\mu_a \frac{l_2}{2} \left(\frac{cos\psi cos\theta_2}{\eta_2} \frac{1-\Gamma_b}{1+\Gamma_b} \mp \frac{cos\psi cos\theta_1}{\eta_1} \right) \qquad \varepsilon_a = \frac{\varepsilon_1 + \varepsilon_2}{2}$$

$$\Gamma_a = -\frac{A^-}{A^+} e^{-j2\nu} \qquad \Gamma_b = -\frac{B^-}{B^+} e^{-j2\nu} \qquad A^{\mp} = \frac{1}{\eta_2} \mp \frac{1}{\eta_1} \frac{cos\theta_2}{cos\theta_1}, \qquad B^{\mp} = \frac{1}{\eta_2} cos\theta_2 \mp \frac{1}{\eta_1} cos\theta_1$$

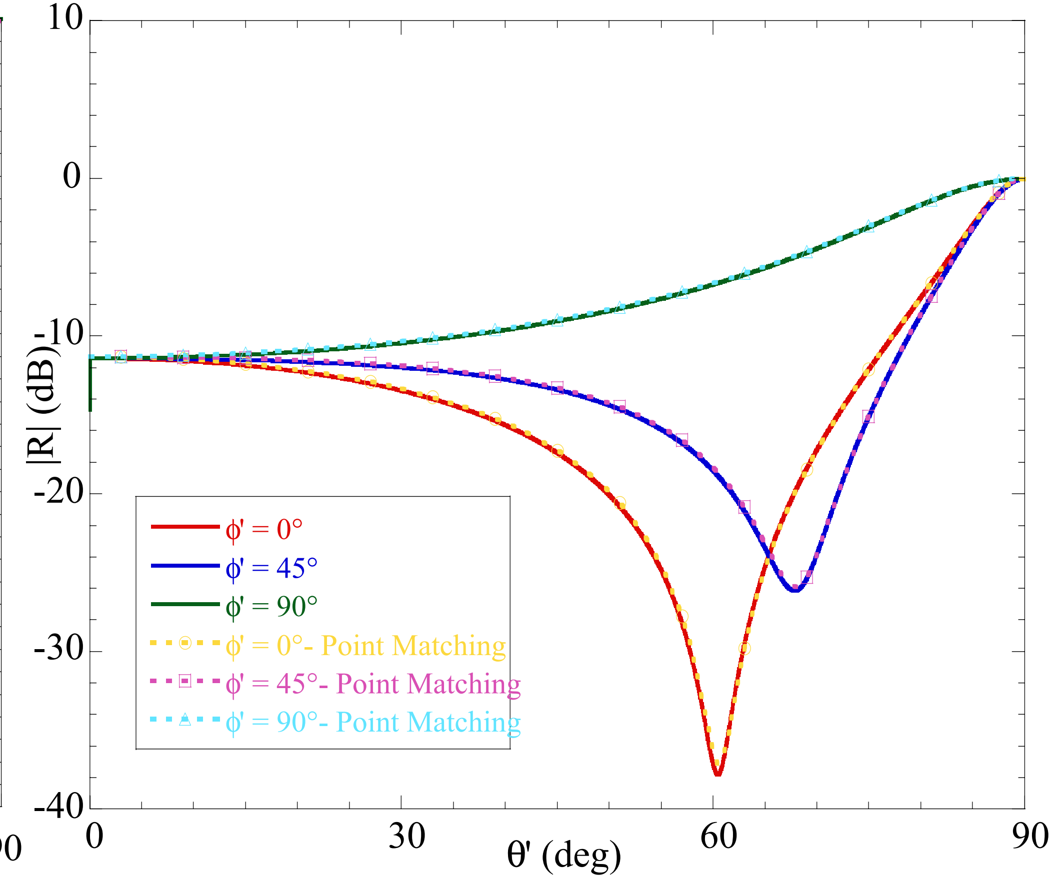
Validation: Mode Matching Method

physical parameters: $\varepsilon_r = 2.5$, $\mu_r = 1$, $d = 0.05\lambda_0$, $p = 0.05\lambda_0$, $w = 0.025\,\lambda_0$

$\alpha = 45^\circ$



$\alpha = 90^\circ$

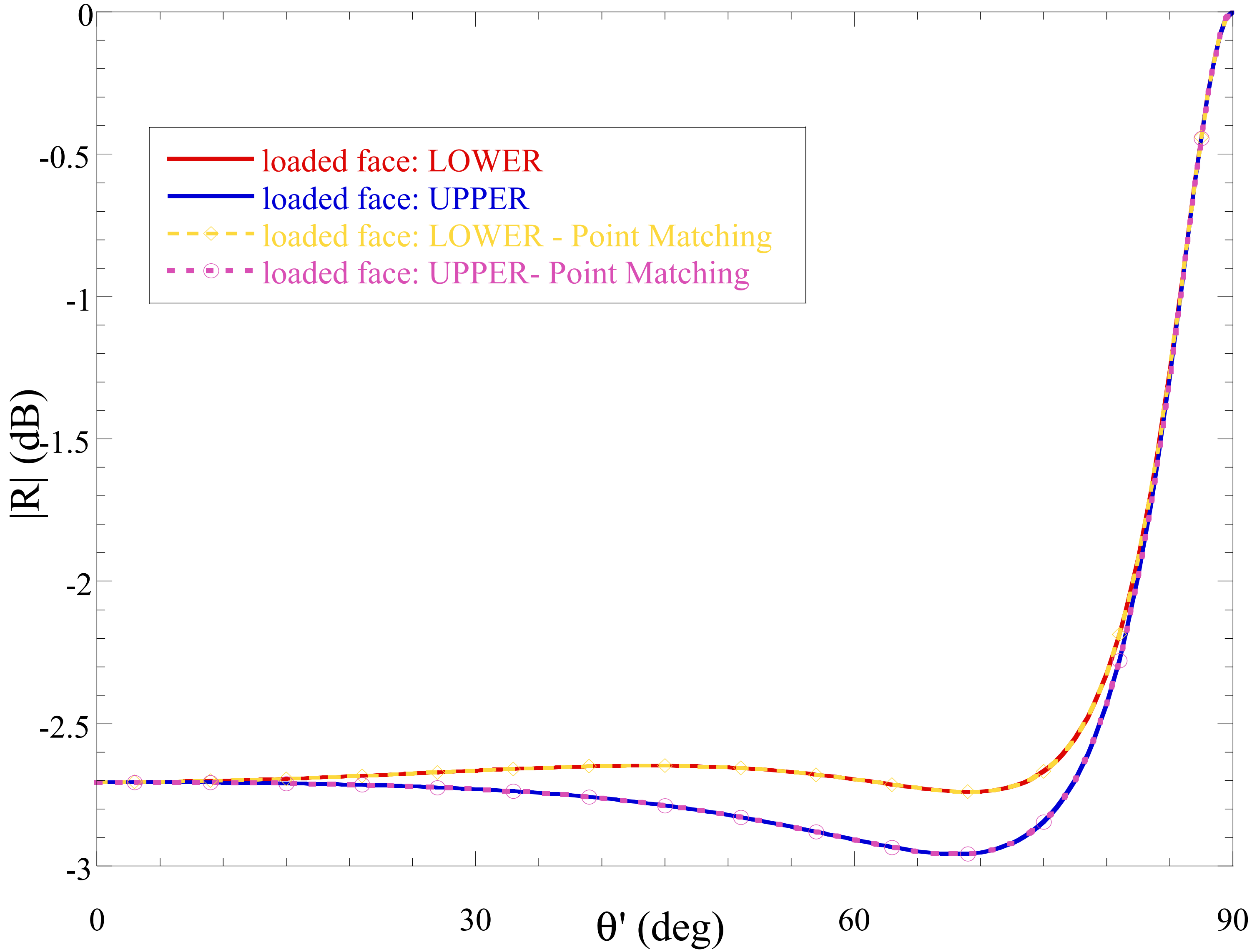


Validation: Mode Matching Method

physical parameters: $\varepsilon_r = 2.5$, $\mu_r = 1$, $d = 0.05\lambda_0$, $p = 0.05\lambda_0$, $w = 0.025\lambda_0$

$\alpha = 45^\circ$

$\phi' = 45^\circ$



Heuristic Diffraction Coefficient

$$\overline{\overline{\mathrm{D}}} = \frac{-e^{-j\frac{\pi}{4}}}{4\sqrt{2\pi k}\sin\beta_o}\left\{\left[\overline{\overline{\mathrm{I}}} - \overline{\overline{\mathrm{T}}}_0(\phi_o, \beta_o)\right]\cdot cot\left[\frac{\pi-(\phi-\phi_o)}{4}\right]\cdot F\left[kLa^-(\phi-\phi_o)\right]+\right.$$

$$+\overline{\overline{\mathrm{R}}}_0(\phi_o, \beta_o)\cdot cot\left[\frac{\pi-(\phi+\phi_o)}{4}\right]\cdot F\left[kLa^-(\phi+\phi_o)\right]+$$

$$+\left[\overline{\overline{\mathrm{I}}} - \overline{\overline{\mathrm{T}}}_{2\pi}(2\pi-\phi_o, \beta_o)\right]\cdot cot\left[\frac{\pi+(\phi-\phi_o)}{4}\right]\cdot F\left[kLa^+(\phi-\phi_o)\right]+$$

$$\left.+\overline{\overline{\mathrm{R}}}_{2\pi}(2\pi-\phi_o, \beta_o)\cdot cot\left[\frac{\pi+(\phi+\phi_o)}{4}\right]\cdot F\left[kLa^+(\phi+\phi_o)\right]\right\}$$

R.G. Kouyoumjian and P.H. Pathak, “A Uniform Geometrical Theory of Diffraction for an Edge in a Perfectly Conducting Surface”, *Proceedings of the IEEE*, vol. 62, No. 11, pp. 1448-1461, November 1974.

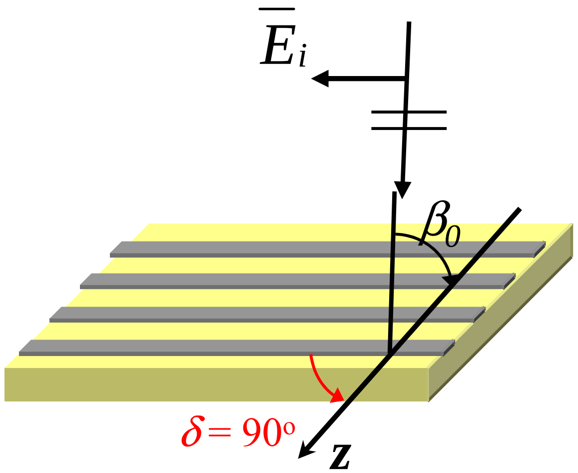
Numerical Results (1)

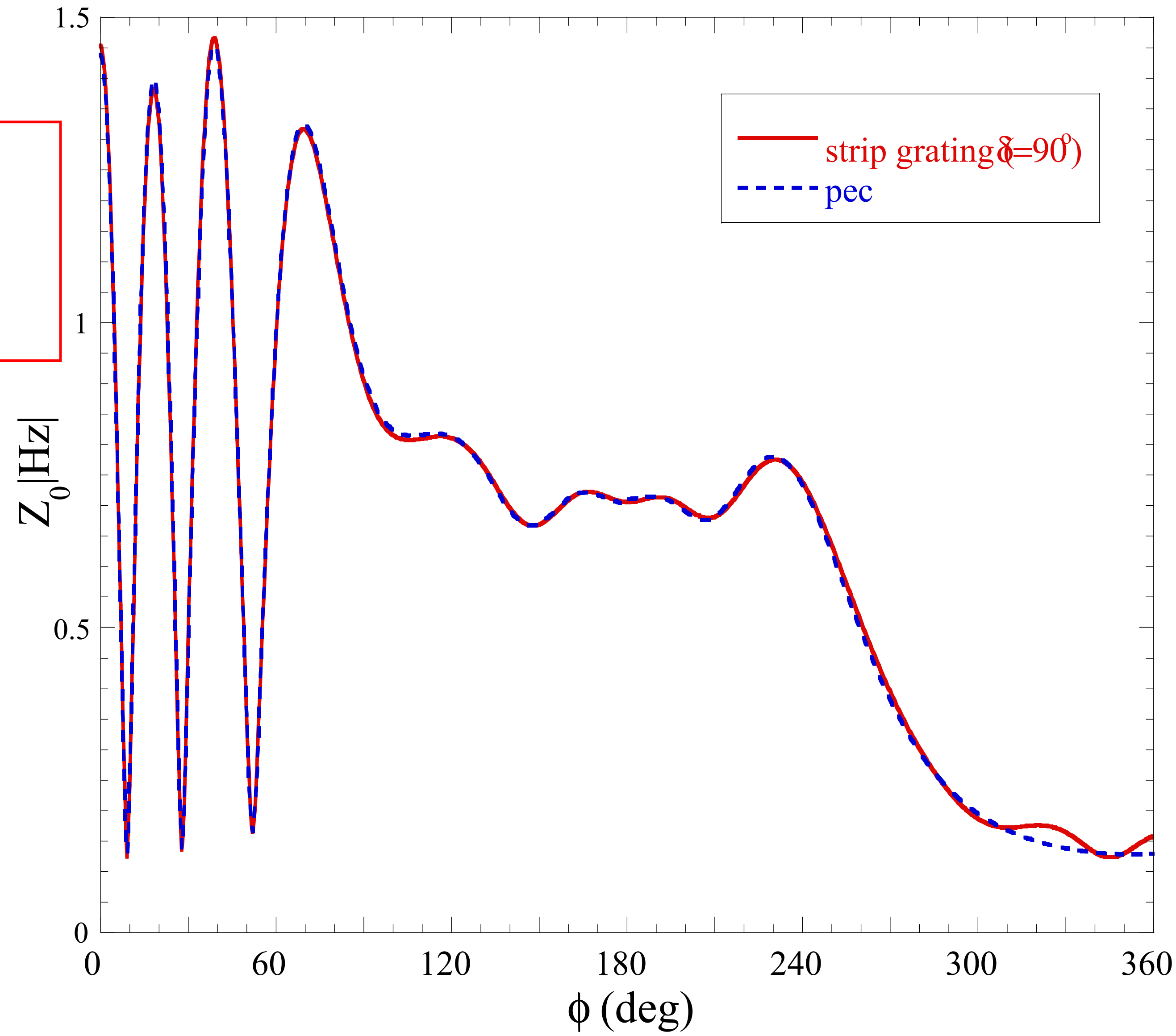
physical parameters: $\varepsilon_r = 2.5$, $\mu_r = 1$, $d = 0.05\lambda_0$, $p = 0.05\lambda_0$, $w = 0.025\,\lambda_0$

$$\beta_0 = 45^{o},\ \phi_0 = 90^{o}$$

$$E_{\beta_0} = 0,\ E_{\phi_0} = 1 \ (\alpha = 0^{o})$$

$$2\pi(\rho/\lambda_0)\sin\beta_0 = 10$$





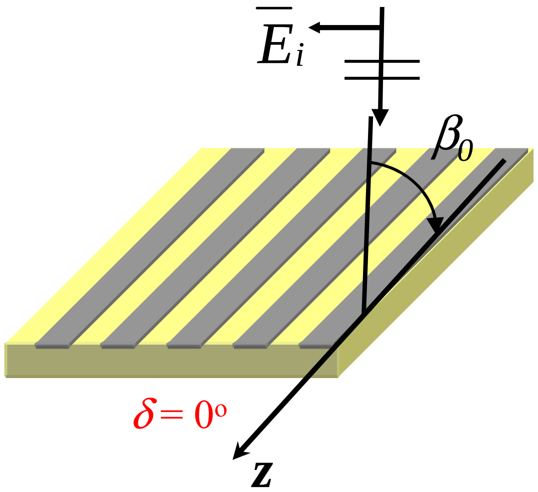
Numerical Results (2)

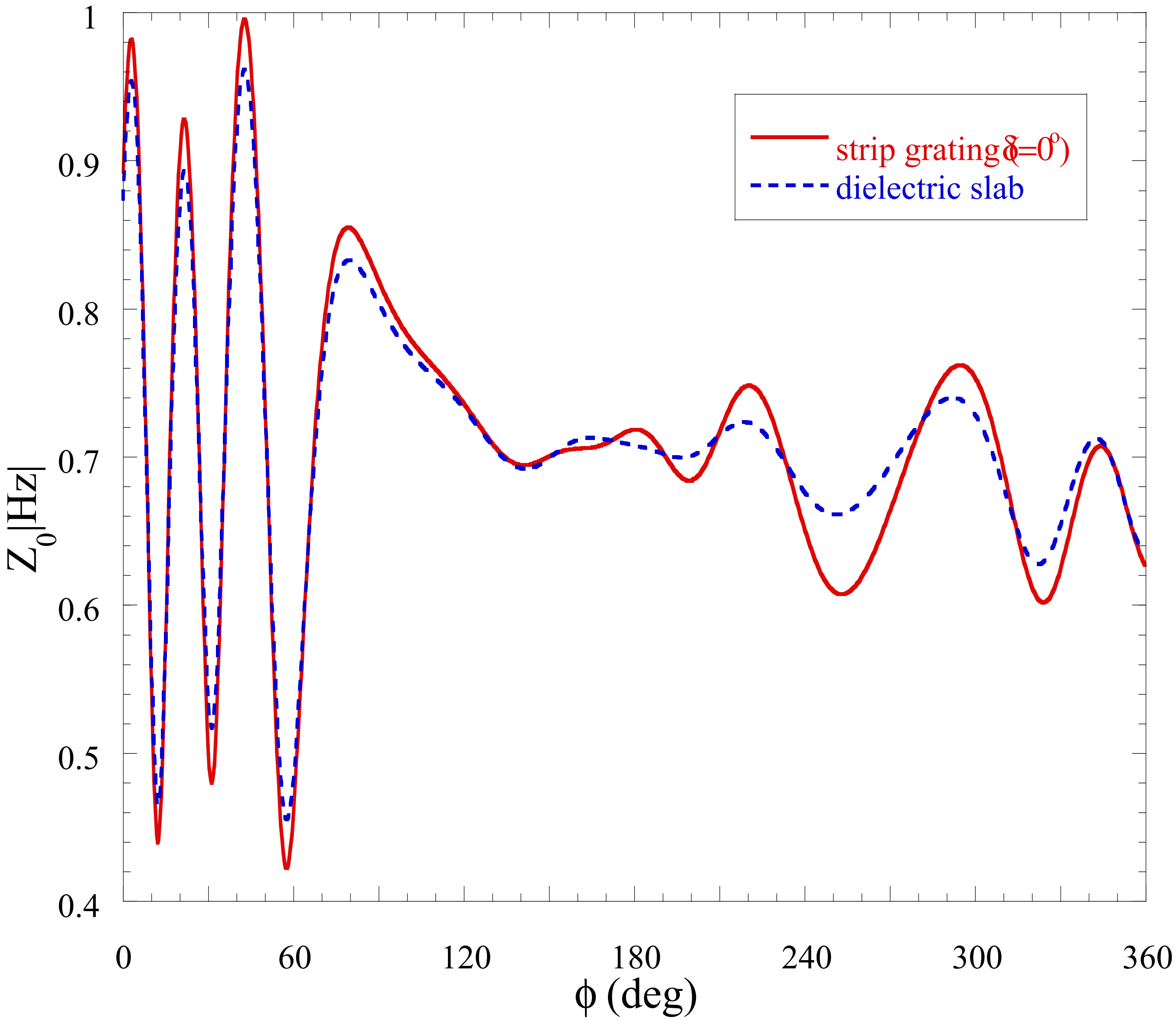
physical parameters: $\varepsilon_r = 2.5$, $\mu_r = 1$, $d = 0.05\lambda_0$, $p = 0.05\lambda_0$, $w = 0.025\,\lambda_0$

$\beta_0 = 45^\circ$, $\phi_0 = 90^\circ$

$E_{\beta_0} = 0, \; E_{\phi_0} = 1$ ($\alpha = 90^\circ$)

$$2\pi(\rho/\lambda_0)\sin\beta_0 = 10$$





Conclusions

High-frequency analytical techniques can be extended to describe the scattering from finite electrically large non-metallic surfaces.

Exact solutions to specific canonical problems …..

- ... represent a starting point for the derivation of perturbative and heuristic solutions
- ... form a steadily expanding set of benchmark solutions which can be used to check the accuracy of approximate diffraction coefficients valid for more general configurations (UAPO, heuristic solutions, PE method, hybrid PO-MoM methods)
- …are useful to induce extensions of the analytical method toward the solution of more general configurations