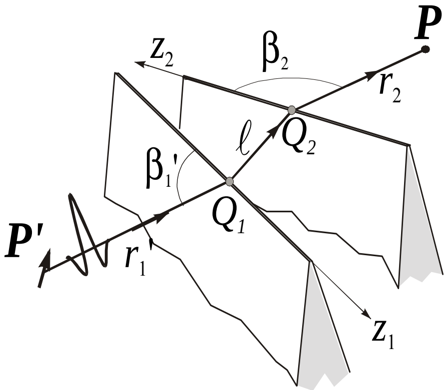
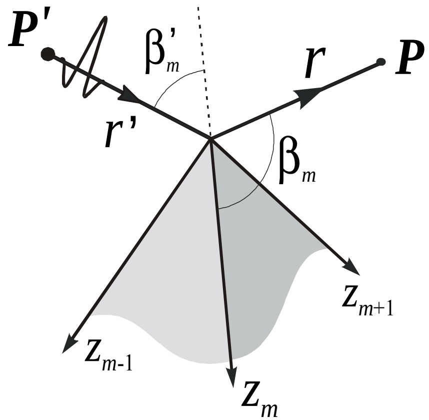
Uniform Diffraction Coefficients for Pair-of-Wedges & Vertex Diffraction Mechanisms





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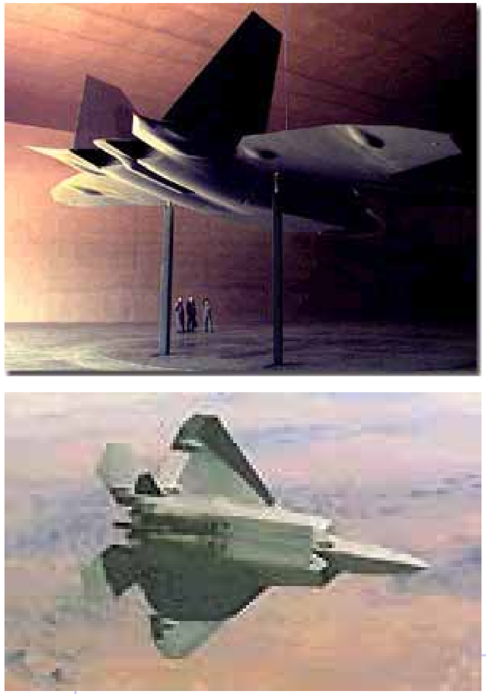
Outline

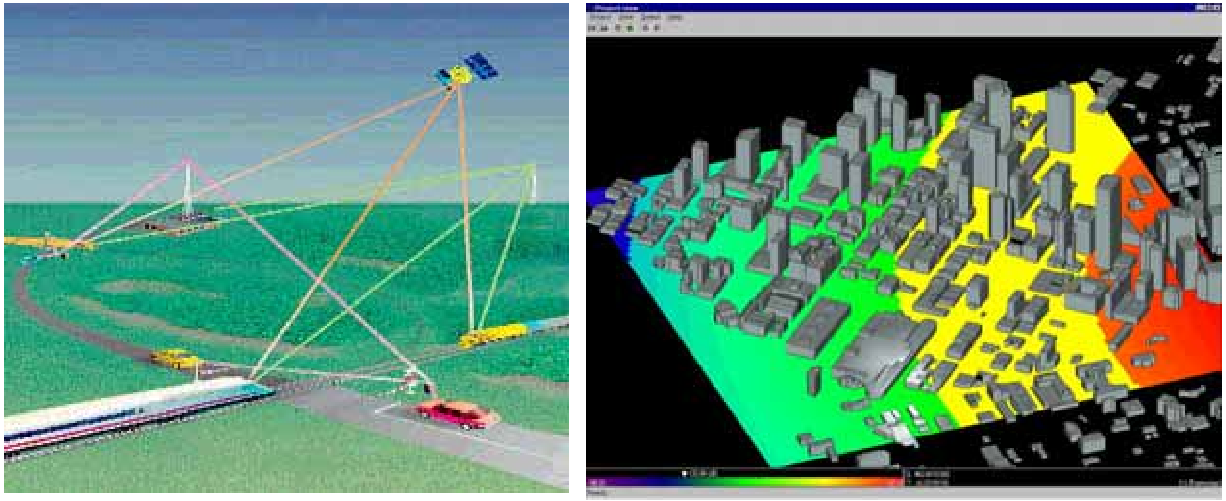
- Motivation of the work
- Double Diffraction (DD)
  - Literature/Previous Results
  - Formulation
  - Analysis of the solution (Transitional behavior)
  - Numerical examples
  - Time Domain (TD) version
- Vertex Diffraction (V)
  - Literature/Previous Results
  - Formulation
  - Analysis of the solution (Transitional behavior)
  - Numerical examples
  - Time Domain (TD) version
- Conclusions

High-frequency EM modeling

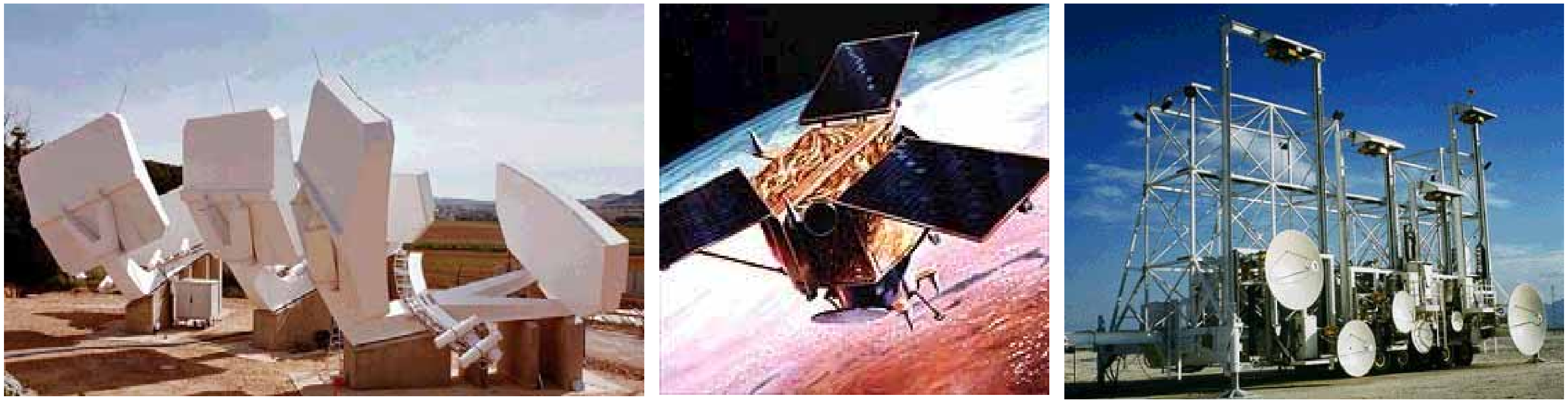
RCS



Propagation

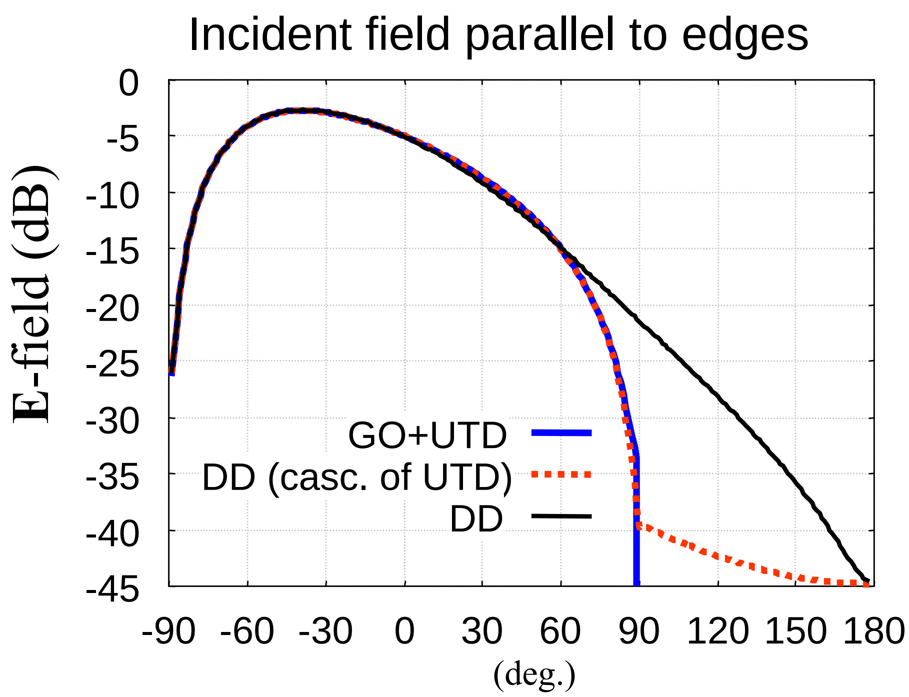


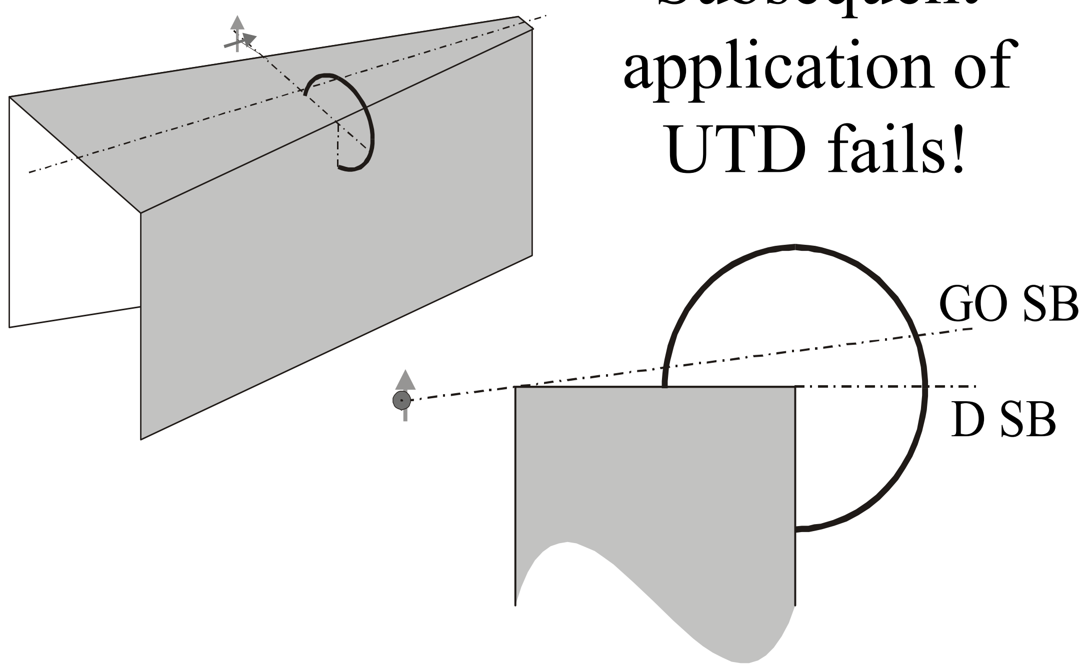
Antenna siting & coupling



Part I: Double Diffraction

- Allows field description inside the shadow regions of Geometrical Optics (GO) and of wedge single diffraction (UTD)
- Restores total field continuity at the boundaries of such shadow regions
- Augments the prediction accuracy





Subsequent application of UTD fails!

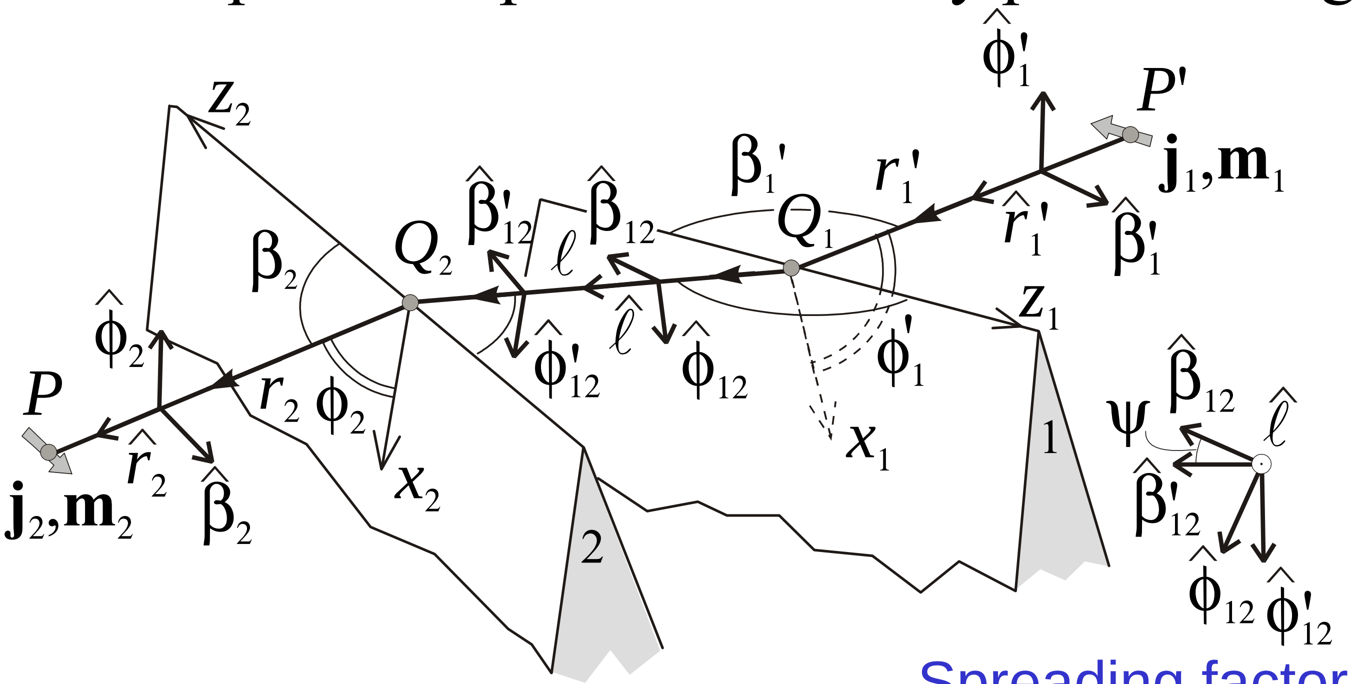
Previous results

- ✂R. Tiberio, G. Manara, G. Pelosi and R. G. Kouyoumjian, "High-frequency electromagnetic scattering of plane waves from double wedges", *Radio Sci.*, 1982.
- ✂M. Schneider and R. Luebbers, "A general UTD diffraction coefficient for two wedges", *IEEE Trans. AP*, 1991.
- ✂L. P. Ivrissimtzis and R. J. Marhefka, "Double diffraction at a coplanar skewed edge configuration", *Radio Sci.*, 1991.
- ✂F. Capolino, M. Albani, S. Maci and R. Tiberio, "Diffraction from a couple of coplanar, skew edges", *IEEE Trans. AP*, 1997.

In practical scenarios edges are not necessarily coplanar!

Formulation

Canonical problem: pair of arbitrarily placed wedges



Spreading factor

$$\mathbf{E}^{dd}\left(P\right)=\mathbf{E}^{i}\left(Q_1\right)\cdot\underline{\underline{\mathbf{D}}}^{dd}\,\frac{\sqrt{r_1'}}{\sqrt{\ell r_2}\sqrt{r_1'+\ell+r_2+\frac{r_1' r_2}{\ell}\sin^2\psi}}\,e^{-jk\left(\ell+r_2\right)}$$

Incident field Dyadic Diffraction Coefficient

$$\underline{\underline{\mathbf{D}}}^{dd}=D^{dd}_{\beta_1'\beta_2}\,\hat{\beta}_1'\hat{\beta}_2+D^{dd}_{\beta_1'\phi_2}\,\hat{\beta}_1'\hat{\phi}_2+D^{dd}_{\phi_1'\beta_2}\,\hat{\phi}_1'\hat{\beta}_2+D^{dd}_{\phi_1'\phi_2}\,\hat{\phi}_1'\hat{\phi}_2$$

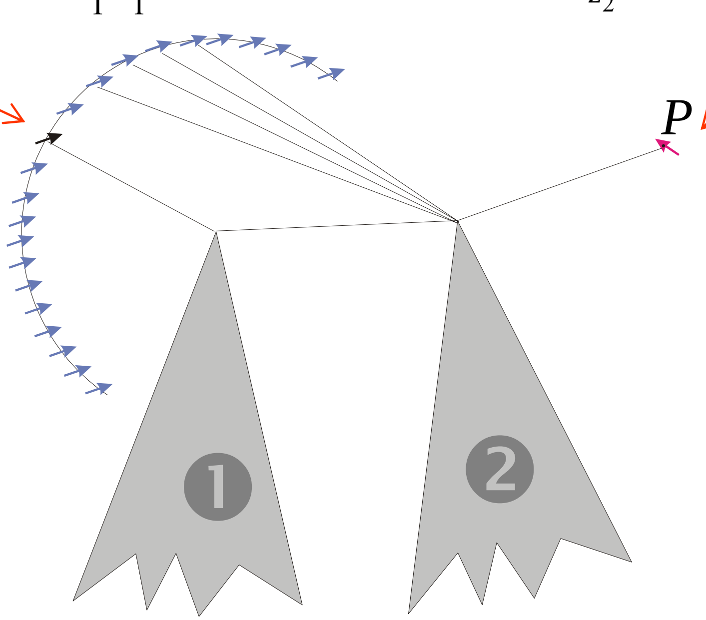
Four Dyad's entry separate analysis

TM_{z_1} illumination $\mathbf{j}_1 = I_1 \hat{z}_1$

TE_{z_1} illumination $\mathbf{m}_1 = V_1 \hat{z}_1$

TM_{z_2} observation $\mathbf{j}_2 = I_2 \hat{z}_2$

TM_{z_2} observation $\mathbf{m}_2 = V_2 \hat{z}_2$



$TM_{z_1} - TM_{z_2}$ Coupling

$$c_{j_1 j_2} = \mathbf{E}_{12}^{dd}\left(P; \mathbf{j}_1\right) \cdot \mathbf{j}_2 \rightarrow \hat{\beta}_1' \hat{\beta}_2$$

Allows to derive the $\hat{\beta}_1' \hat{\beta}_2$ term of the DD dyadic coefficient

analogously

$$c_{j_1 m_2} \rightarrow \hat{\beta}_1' \hat{\phi}_2$$

$$c_{m_1 j_2} \rightarrow \hat{\phi}_1' \hat{\beta}_2$$

$$c_{m_1 m_2} \rightarrow \hat{\phi}_1' \hat{\phi}_2$$

Singly Diffracted Field

TM_{z_1} illumination

$$\mathbf{j}_1 = I_1 \hat{z}_1$$

Incident field

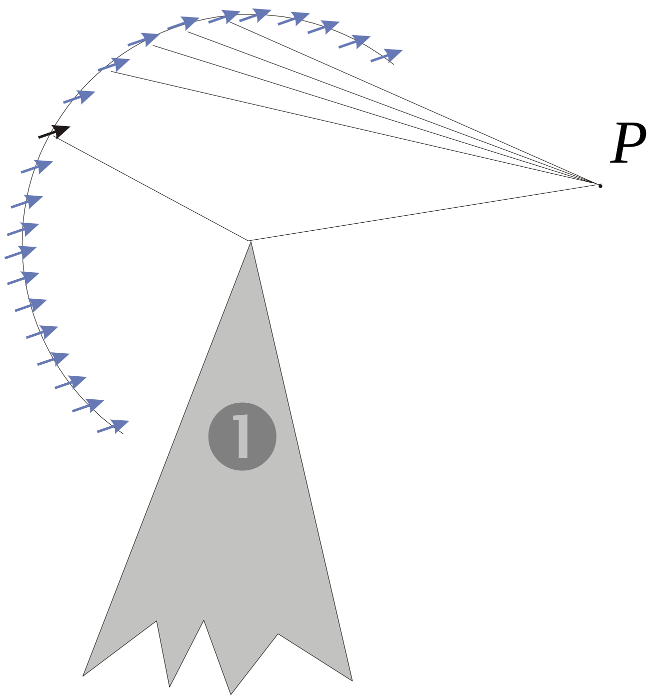
$$\mathbf{E}^i(P) = \frac{\zeta}{jk}\left(k^2 + \nabla_P \nabla_P \cdot\right) \frac{e^{-jk|P-P'|}}{4\pi |P-P'|} I_1 \hat{z}_1$$

Wedge Green's function

$$G_i^{s,h}(\phi,\phi') = -\frac{1}{2n_i} \sum_{p,q=1}^{2} \chi_i^{p+q} \cot\left(\frac{\pi + (-1)^p \phi + (-1)^q \phi'}{2n_i}\right)$$

Field Singly diffracted by Wedge 1

$$\mathbf{E}_1^d(P;\mathbf{j}_1) = \frac{\zeta}{jk}\left(k^2 + \nabla_{P'} \nabla_{P'} \cdot\right) \left[\frac{1}{2\pi j} \int_{-j\infty}^{j\infty} G_1^s(\alpha_1, \phi_1') \frac{e^{-jk|P-P'(\alpha_1)|}}{4\pi |P-P'(\alpha_1)|} d\alpha_1 I_1 \hat{z}_1 \right]$$



Integral superposition of a distribution of spectral electric dipole sources $\mathbf{j}_1(\alpha_1)$ located at

$$P'(\alpha_1) = Q_1 - r_1'(\sin\beta_1' \cos\alpha_1 \hat{x}_1 + \sin\beta_1' \sin\alpha_1 \hat{y}_1 + \cos\beta_1' \hat{z}_1)$$

oriented along $\hat{z}_1$

with a strength $\frac{1}{2\pi j} G_1^s(\alpha_1, \phi_1') I_1 d\alpha_1$

and radiating in free space.

Doubly Diffracted Field

TM_{z_2} Observation $\mathbf{j}_2 = I_2 \hat{z}_2$

$$\mathbf{E}_2^d\left(P;\mathbf{j}_1(\alpha_1)\right)\cdot\mathbf{j}_2 \xrightarrow[\text{reciprocity}]{} \mathbf{j}_1(\alpha_1)\cdot\mathbf{E}_2^d\left(P'(\alpha_1);\mathbf{j}_2\right)$$

$$\mathbf{E}_2^d\left(P'(\alpha_1);\mathbf{j}_2\right) = \frac{\zeta}{jk}\left(k^2 + \nabla_{P'}\nabla_{P'}\cdot\right)\left[\frac{1}{2\pi j}\int_{-j\infty}^{j\infty} G_2^s(\alpha_2,\phi_2)\frac{e^{-jk\left|P(\alpha_2)-P'(\alpha_1)\right|}}{4\pi\left|P(\alpha_2)-P'(\alpha_1)\right|}d\alpha_2 I_2\hat{z}_2\right]$$

TM_{z_1} TM_{z_2}
Coupling

$$\boxed{\begin{aligned} c_{j_1 j_2} = \frac{jk\zeta I_1 I_2}{(2\pi)^2}\int_{-j\infty}^{j\infty}\int_{-j\infty}^{j\infty} G_1^s(\alpha_1,\phi_1')G_2^s(\alpha_2,\phi_2)\,\cdot \\ \cdot\left[\hat{z}_1\cdot\hat{z}_2\left(1+\frac{1}{jkR}-\frac{1}{k^2R^2}\right)-\left(\hat{z}_1\cdot\hat{R}\right)\left(\hat{z}_2\cdot\hat{R}\right)\left(1+\frac{3}{jkR}-\frac{3}{k^2R^2}\right)\right]\frac{e^{-jkR}}{4\pi R}d\alpha_1 d\alpha_2 \end{aligned}}$$

$$P(\alpha_2) - P'(\alpha_1) = R(\alpha_1,\alpha_2)\hat{R}(\alpha_1,\alpha_2)$$

Allows to derive the $\hat{\beta}_1'\hat{\beta}_2$ term of the DD dyadic coefficient

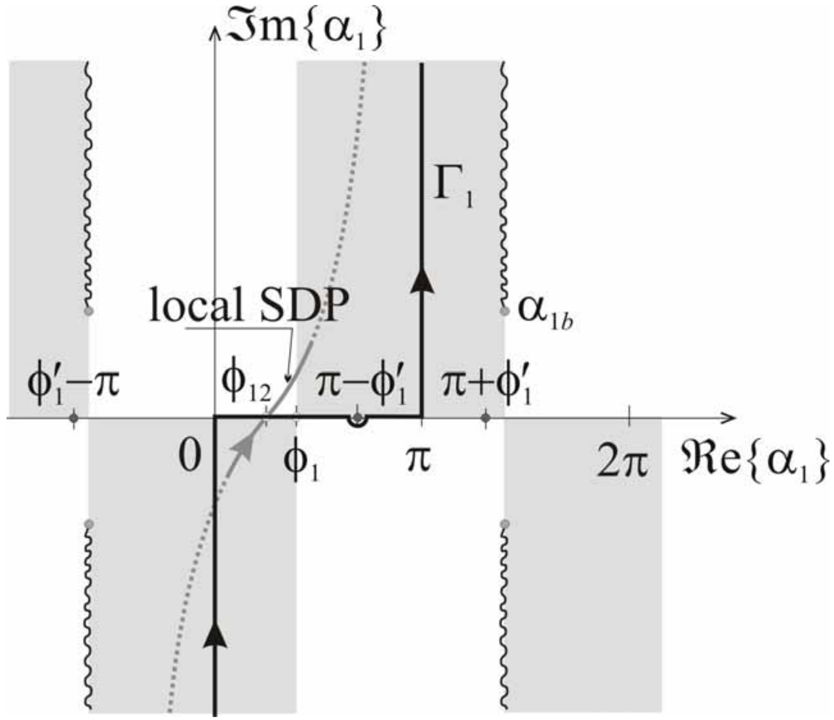
Doubly Diffracted Field

$$\begin{bmatrix} c_{j_1 j_2} & c_{m_1 j_2} \\ c_{j_1 m_2} & c_{m_1 m_2} \end{bmatrix} = \frac{jk}{(2\pi)^2} \int_{\Gamma_1}\int_{\Gamma_2} \begin{bmatrix} \zeta I_1 I_2 G_1^s G_2^s \sigma & V_1 I_2 G_1^h G_2^s \tau \\ I_1 V_2 G_1^s G_2^h \tau & \zeta^{-1} V_1 V_2 G_1^h G_2^h \sigma \end{bmatrix} \frac{e^{-jkR}}{4\pi R} d\alpha_1 d\alpha_2$$

$$P(\alpha_2) - P'(\alpha_1) = R(\alpha_1,\alpha_2)\hat{R}(\alpha_1,\alpha_2)$$

$$\sigma(\alpha_1,\alpha_2) = \hat{z}_1 \cdot \hat{z}_2 \left\{1 + \frac{1}{jkR(\alpha_1,\alpha_2)} - \frac{1}{[kR(\alpha_1,\alpha_2)]^2}\right\}$$
$$-\left[\hat{z}_1 \cdot \hat{R}(\alpha_1,\alpha_2)\right]\left[\hat{z}_2 \cdot \hat{R}(\alpha_1,\alpha_2)\right]\left\{1 + \frac{3}{jkR(\alpha_1,\alpha_2)} - \frac{3}{[kR(\alpha_1,\alpha_2)]^2}\right\}$$

$$\tau(\alpha_1,\alpha_2) = \hat{R}(\alpha_1,\alpha_2) \times \hat{z}_2 \cdot \hat{z}_1 \left[1 + \frac{1}{jkR(\alpha_1,\alpha_2)}\right]$$

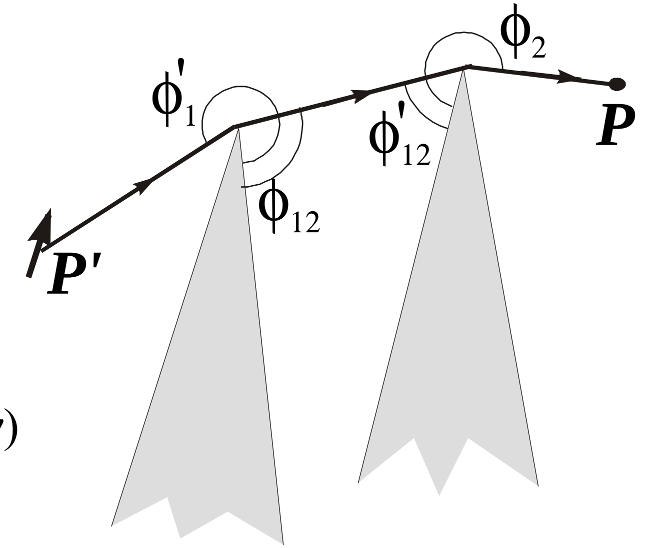


- Double Pole Singularity → GO
- Pole – Stationary Phase Point (SPP) → Singly D
- Double SPP → DD

Asymptotic Evaluation

2D stationary phase point (double diffraction) $(\alpha_1,\alpha_2)=(\phi_{12},\phi'_{12})$

defined by $\nabla R=0$



$$\underline{\underline{\mathbf{D}}}^{dd}=\underline{\underline{\tilde{\mathbf{D}}}}^{dd}+\underline{\underline{\tilde{\tilde{\mathbf{D}}}}}^{dd}$$

$$\tilde{D}^{dd}=\frac{\gamma}{8j\pi k n_1 n_2\sin\beta'_1\sin\beta_2}\sum_{p,q,r,s=0}^{1}\chi_1^{p+q}\chi_2^{r+s}\cot\left(\frac{\Phi_1^{pq}}{2n_1}\right)\cot\left(\frac{\Phi_2^{rs}}{2n_2}\right)\tilde{T}(a_{pq},b_{rs},w)$$

$$\tilde{\tilde{D}}^{dd}=\frac{\gamma\cos\psi}{32\pi k^2\left(\ell+\frac{r'_1 r_2}{r'_1+\ell+r_2}\sin^2\psi\right)\left(n_1 n_2\sin\beta'_1\sin\beta_2\right)^2}\sum_{p,q,r,s=0}^{1}\chi_1^{p+q}\chi_2^{r+s}(-1)^{q+r}\csc^2\left(\frac{\Phi_1^{pq}}{2n_1}\right)\csc^2\left(\frac{\Phi_2^{rs}}{2n_2}\right)\tilde{\tilde{T}}(a_{pq},b_{rs},w)$$

Component	χ_1	χ_2	γ
$\hat{\beta}'_1\hat{\beta}_2$	-1	-1	$\cos\psi$
$\hat{\beta}'_1\hat{\phi}_2$	-1	1	$\sin\psi$
$\hat{\phi}'_1\hat{\beta}_2$	1	-1	$-\sin\psi$
$\hat{\phi}'_1\hat{\phi}_2$	1	1	$\cos\psi$

with $\Phi_1^{pq}=\pi+(-1)^p\phi'_1+(-1)^q\phi_{12}$

$\Phi_2^{rs}=\pi+(-1)^r\phi_2+(-1)^s\phi'_{12}$

Transition Functions

$$\widetilde{T}(a,b,w) = \frac{ja^2b^2}{\pi\left(1-w^2\right)^{3/2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{e^{-jk\left(\xi^2+2w\xi\eta+\eta^2\right)}}{\left(\xi^2+\frac{a^2}{1-w^2}\right)\left(\eta^2+\frac{b^2}{1-w^2}\right)} d\xi d\eta$$

They can be expressed in terms of the Generalized Fresnel Integral ***G***

$$\widetilde{\widetilde{T}}(a,b,w) = \frac{2a^2b^2}{\pi w\sqrt{1-w^2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\xi\eta\, e^{-jk\left(\xi^2+2w\xi\eta+\eta^2\right)}}{\left(\xi^2+\frac{a^2}{1-w^2}\right)\left(\eta^2+\frac{b^2}{1-w^2}\right)} d\xi d\eta$$

$$\widetilde{T}(a,b,w) = \frac{2\pi jab}{\sqrt{1-w^2}}\left\{\left[\mathcal{G}\left(a,\frac{b+wa}{\sqrt{1-w^2}}\right)+\mathcal{G}\left(b,\frac{a+wb}{\sqrt{1-w^2}}\right)\right]+\left[\mathcal{G}\left(a,\frac{b-wa}{\sqrt{1-w^2}}\right)+\mathcal{G}\left(b,\frac{a-wb}{\sqrt{1-w^2}}\right)\right]\right\}$$

$$\widetilde{\widetilde{T}}(a,b,w) = \frac{-4\pi a^2b^2}{w\sqrt{1-w^2}}\left\{\left[\mathcal{G}\left(a,\frac{b+wa}{\sqrt{1-w^2}}\right)+\mathcal{G}\left(b,\frac{a+wb}{\sqrt{1-w^2}}\right)\right]-\left[\mathcal{G}\left(a,\frac{b-wa}{\sqrt{1-w^2}}\right)+\mathcal{G}\left(b,\frac{a-wb}{\sqrt{1-w^2}}\right)\right]\right\}$$

Arguments

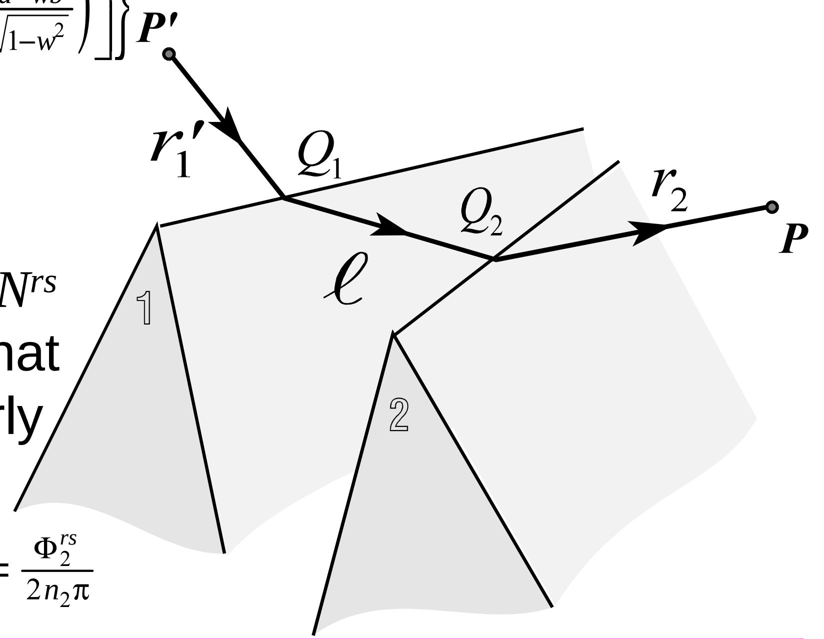
$$a_{pq} = \sin\beta_1' \sqrt{\frac{2kr_1'\left(\ell+\frac{r_1'r_2}{r_1'+\ell+r_2}\sin^2\psi\right)}{r_1'+\ell}} \sin\left(\frac{\Phi_1^{pq}-2n_1\pi N^{pq}}{2}\right)$$

$$b_{rs} = \sin\beta_2 \sqrt{\frac{2kr_2\left(\ell+\frac{r_1'r_2}{r_1'+\ell+r_2}\sin^2\psi\right)}{r_2+\ell}} \sin\left(\frac{\Phi_2^{rs}-2n_2\pi N^{rs}}{2}\right)$$

$$w = \cos\psi \frac{\sqrt{r_1'r_2}}{\sqrt{r_1'+\ell}\sqrt{r_2+\ell}}$$

with N^{pq} e N^{rs} integers that most nearly satisfy

$$N^{pq} = \frac{\Phi_1^{pq}}{2n_1\pi} \quad N^{rs} = \frac{\Phi_2^{rs}}{2n_2\pi}$$



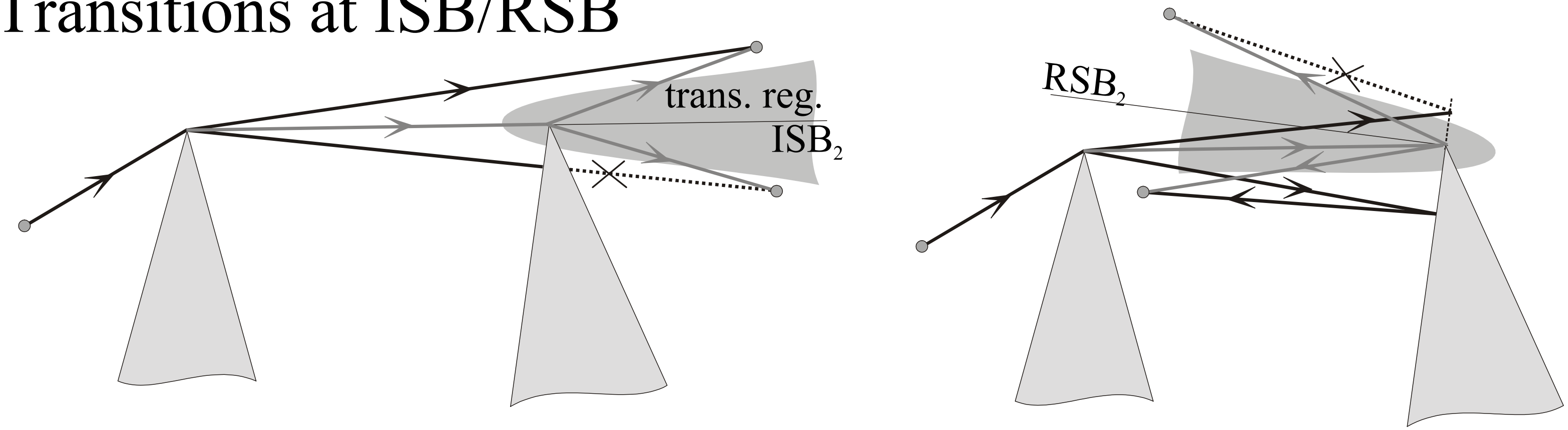
Analysis of the solution

- Out of transition $\quad a,b \to \infty \quad \tilde{T},\tilde{\tilde{T}} \to 1$

$$\tilde{\underline{\underline{\mathbf{D}}}}^{dd} = \underline{\underline{\mathbf{D}}}_1 \cdot \underline{\underline{\mathbf{D}}}_2 = O\left(k^{-1}\right) \quad \text{GTD}$$

$$\tilde{\tilde{\underline{\underline{\mathbf{D}}}}}^{dd} = \frac{\partial}{\partial \phi_1}\underline{\underline{\mathbf{D}}}_1 \cdot \frac{\partial}{\partial \phi_2}\underline{\underline{\mathbf{D}}}_2 = O\left(k^{-2}\right) \quad \text{GTD slope}$$

- Transitions at ISB/RSB



$$b_{rs} \to 0 \quad \tilde{T}(a,b \to 0,w) \approx \sqrt{j\pi b^2}\, F\left(\tfrac{a^2}{1-w^2}\right)$$

DD compensates for the discontinuity of 1st order UTD

$$\tilde{\underline{\underline{\mathbf{D}}}}^{dd} = O\left(k^{-\frac{1}{2}}\right)$$ Continuity of the total field

$$\tilde{\tilde{\underline{\underline{\mathbf{D}}}}}^{dd} = O\left(k^{-1}\right)$$ Continuity of the derivative of total field

Analogously source

$$a_{pq} \to 0$$

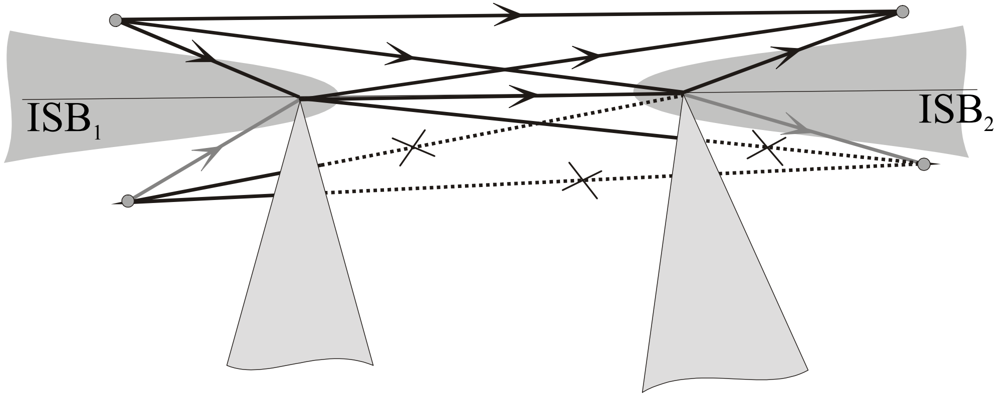
Analysis of the solution (cont'd)

- Double transition

$$a_{pq}, b_{rs} \to 0$$

$$\tilde{T}(a \to 0, b \to 0, w) \approx j\pi\sqrt{\frac{a^2b^2}{1-w^2}}$$

$$\tilde{\tilde{T}}(a \to 0, b \to 0, w) \approx -4\sin^{-1} w \frac{a^2b^2}{w\sqrt{1-w^2}}$$



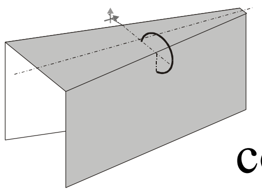
$\underline{\underline{\tilde{\mathbf{D}}}}^{dd} = O\left(k^0\right)$ Continuity of the total field

$\underline{\underline{\tilde{\tilde{\mathbf{D}}}}}^{dd} = O\left(k^0\right)$ Continuity of the derivative of total field

- Coplanar edges $\psi \to 0, \pi$

$$\underline{\underline{\mathbf{D}}}^{dd} = D^{dd}_{\beta_1'\beta_2}\hat{\beta}_1'\hat{\beta}_2 + \cancelto{0}{D^{dd}_{\beta_1'\phi_2}\hat{\beta}_1'\hat{\phi}_2} + \cancelto{0}{D^{dd}_{\phi_1'\beta_2}\hat{\phi}_1'\hat{\beta}_2} + D^{dd}_{\phi_1'\phi_2}\hat{\phi}_1'\hat{\phi}_2$$

Previous result



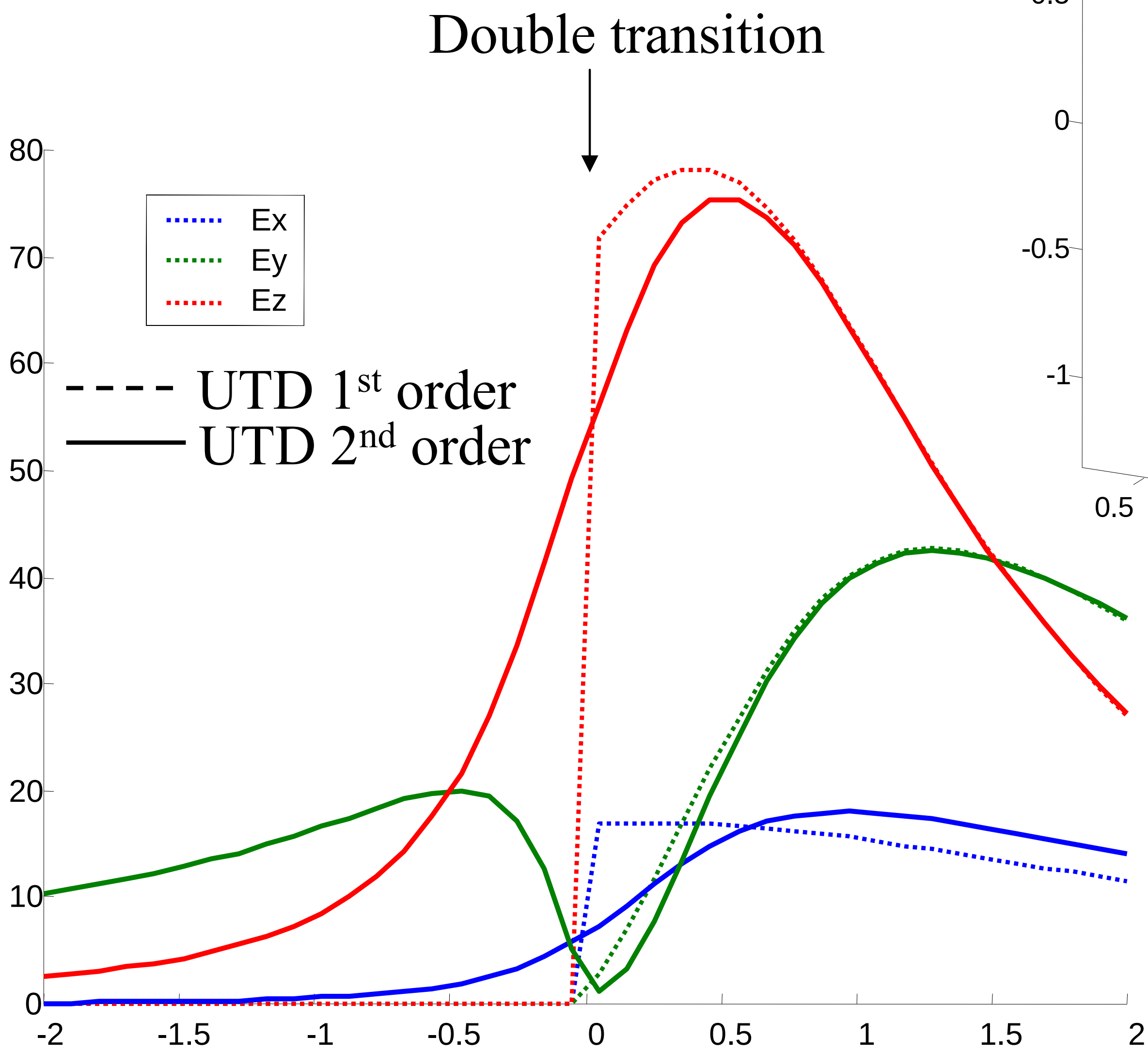
Wedges sharing a common face

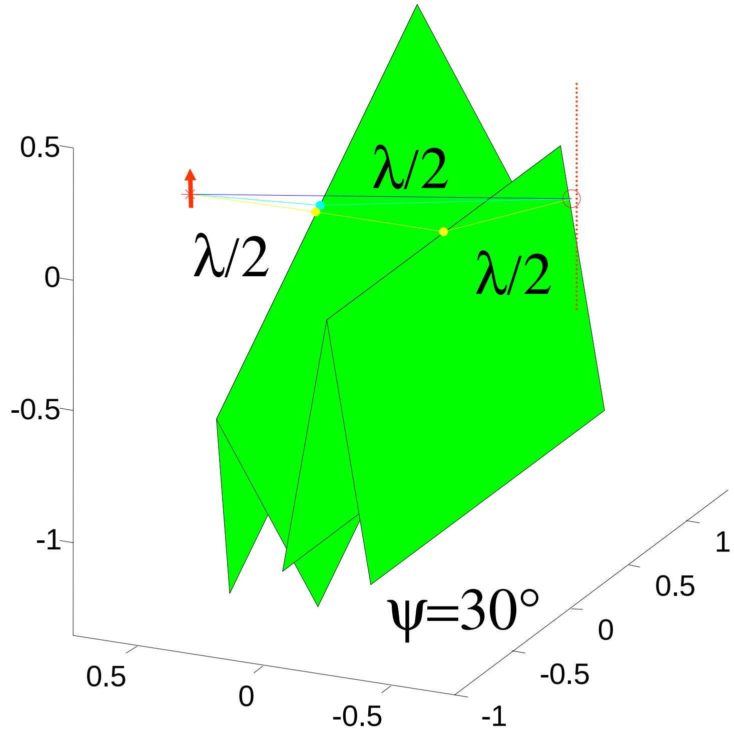
$$D^{dd}_{\beta_1'\beta_2} = \cancelto{0}{\tilde{D}^{dd}_{\beta_1'\beta_2}} + \tilde{\tilde{D}}^{dd}_{\beta_1'\beta_2}$$

Slope interaction

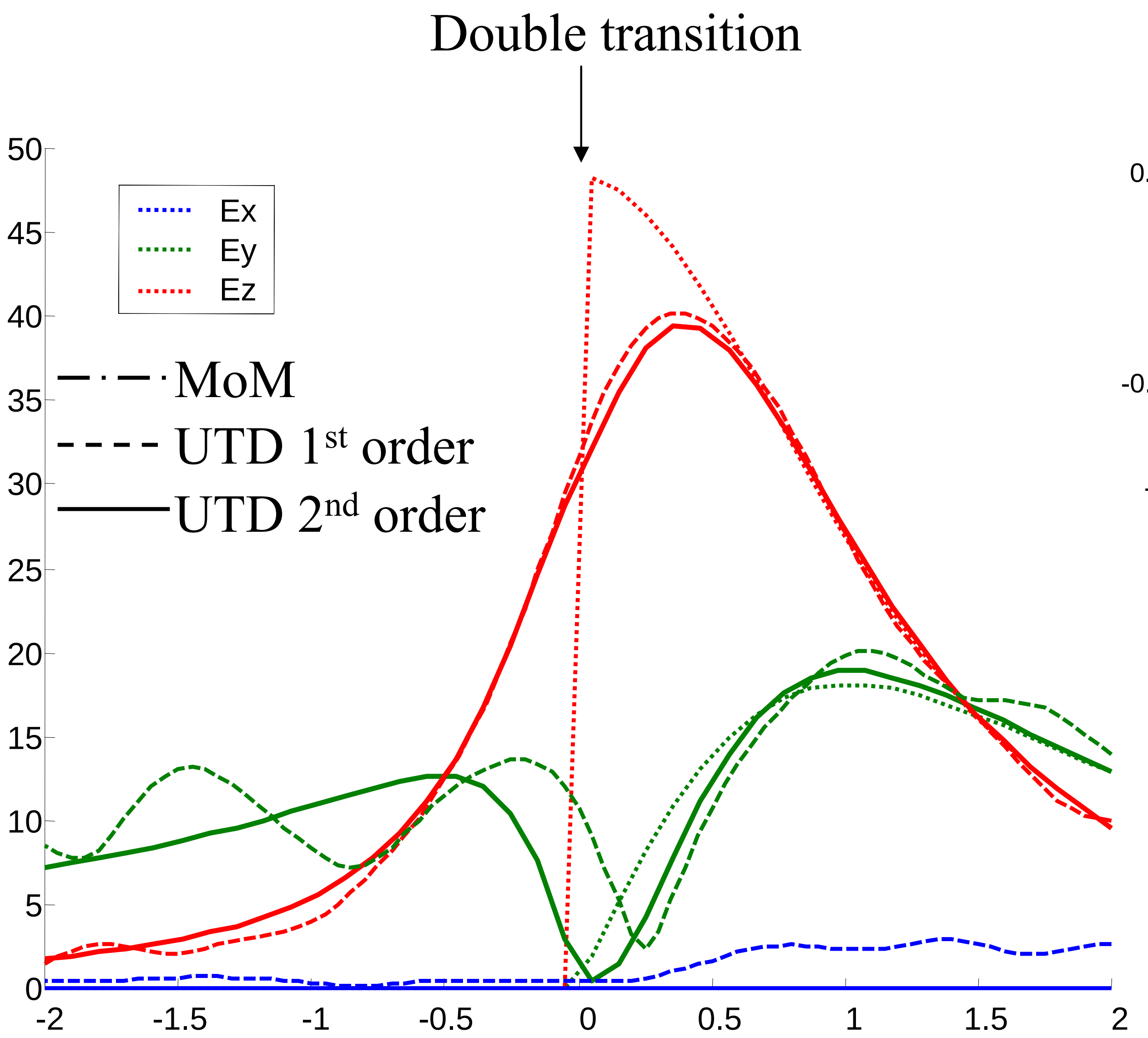
$$D^{dd}_{\phi_1'\phi_2} = \tilde{D}^{dd}_{\phi_1'\phi_2} + \cancelto{0}{\tilde{\tilde{D}}^{dd}_{\phi_1'\phi_2}}$$

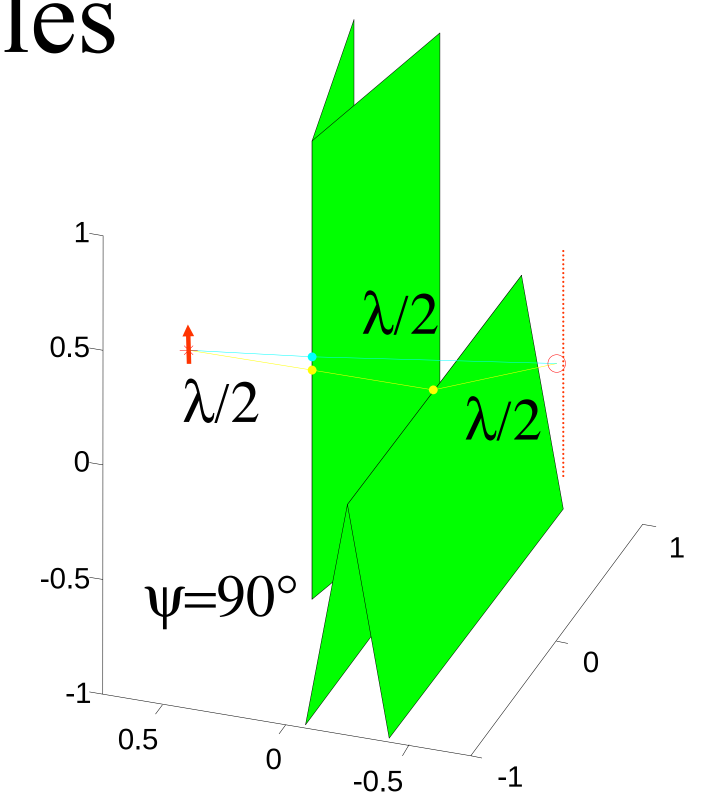
Numerical examples



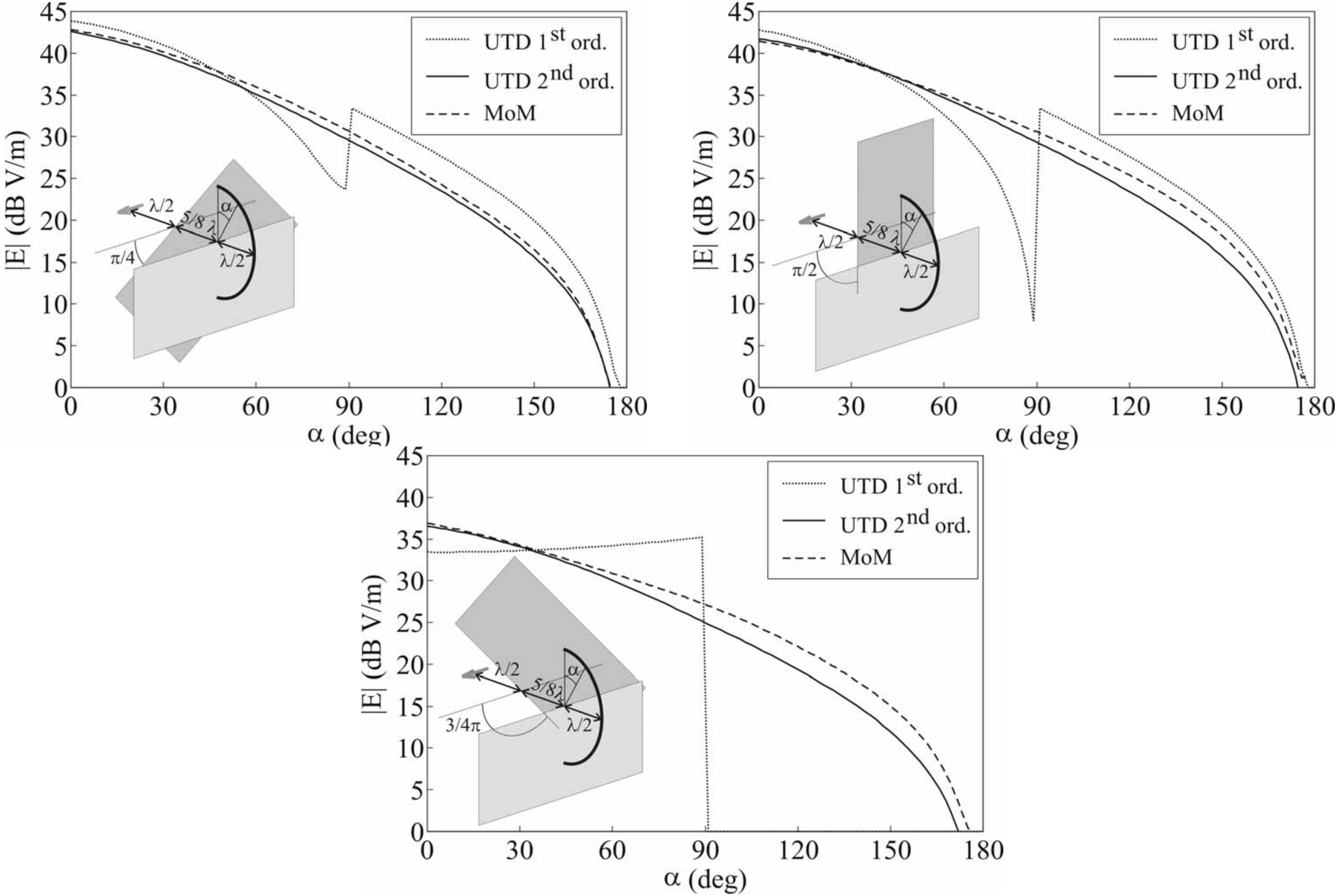


Numerical examples





Numerical examples



Time Domain (TD) Version

Transient field response to pulsed excitation via Ray techniques.

- Wide Band analysis obtained from **short-pulse excitation**.
- **TD**-diffraction coefficients are **simpler** and **easier to evaluate** than **FD**-diffraction coefficients.
- Multipath **delay** information

Efficient Field Evaluation

- In the TD, wavefront approximations for diffracted fields are in closed form (no Fresnel integrals and no generalized Fresnel integrals).

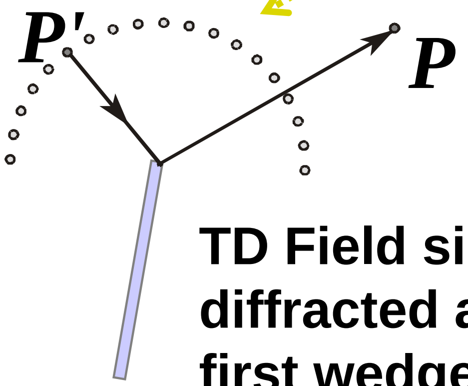
- Felsen and Marcuvitz (1972)
- Verruttipong (1990)
- Ianconescu and Heyman (1994)
- Rousseau and Pathak (1995)

TD impulsive excitation

$$\hat{\psi}\{P, P'; t\} = \frac{\delta\left(t - \frac{|P-P'|}{c}\right)}{4\pi|P-P'|}$$

Impulsive spherical source

Time Domain Spectral Synthesis (Double Diffraction)



The diffracted field can be interpreted as superposition of spectral spherical sources radiating in free space

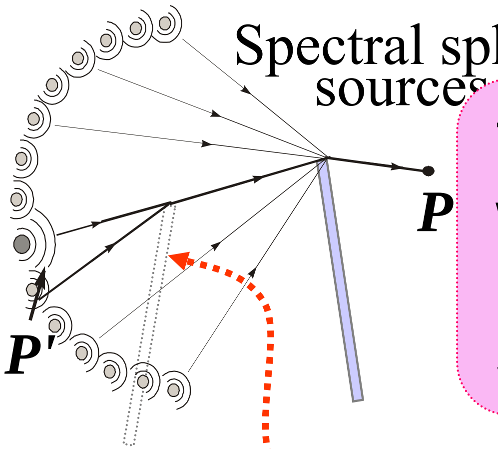
TD Field singly diffracted at the first wedge

$$\hat{\psi}_1^d\{P,P';t\} = \frac{1}{2\pi j}\int_{C_{\alpha_1}} \hat{\psi}\{P,P'(\alpha_1 \pm \pi);t\}\, G(\phi_1,\phi_1';\alpha_1,n_1)\, d\alpha_1$$

Wedge Green's function
soft **or** ***hard*** **b.c.**

$$G(\phi',\phi;\alpha,n) = \sum_{i,j}^{2} (\mp 1)^j\, B\left[(-1)^i\left(\phi-(-1)^j\phi'\right);\alpha,n\right]$$

$$B[\phi;\alpha,n] = \frac{1}{2n}\,\frac{\sin\left(\frac{\pi-\phi}{n}\right)}{\cos\left(\frac{\pi-\phi}{n}\right)-\cos\left(\frac{\alpha}{n}\right)}$$



The presence of the second wedge is taken into account by evaluating its response to each spectral spherical source

$$\hat{\psi}_2^d\{P,P'(\alpha_1 \pm \pi);t\} = \frac{1}{2\pi j}\int_{-j\infty}^{+j\infty} \hat{\psi}\{P'(\alpha_1+\pi),P(\alpha_2+\pi);t\}\, G(\phi_2,\phi'_2;\alpha_2+\pi,n_2)\, d\alpha_2$$

TD DD Field

Superposition of spectral impulsive spherical sources:

$$R(\alpha_1,\alpha_2)=\left|P(\alpha_1+\phi_{12}\pm\pi)-P'(\alpha_2+\phi'_{12}\pm\pi)\right|$$

$$\hat{\psi}_{12}^{dd}\{P,P';t\}=-\frac{1}{4\pi^2}\int_{-j\infty}^{j\infty}\int_{-j\infty}^{j\infty}\frac{\delta\left(t-\frac{R(\alpha_1,\alpha_2)}{c}\right)}{4\pi R(\alpha_1,\alpha_2)}G(\phi'_1,\phi_{12};\alpha_1,n_1)G(\phi'_{12},\phi_2;\alpha_2,n_2)\,d\alpha_2\,d\alpha_1$$

TD spectral representation for doubly diffracted field (12 mechanism)

2D stationary phase point

$$(\alpha_1,\alpha_2)=(0,0)$$

$$\nabla_{\alpha_1,\alpha_2}R(0,0)=0$$

Pole singularities in each variable

$$\alpha_1=\Phi_1^{p,q}=(-1)^p\phi'_1+(-1)^q\phi_{12}+\pi$$

$$\alpha_2=\Phi_2^{r,s}=(-1)^r\phi_2+(-1)^s\phi'_{12}+\pi$$

Change of variables

$$\begin{cases} u = \sin\frac{\alpha_1}{2} \\ v = \sin\frac{\alpha_2}{2} \end{cases}$$

Taylor expansion of the phase:

$$r(u,v)\approx t^{dd}+\tfrac{1}{2c}[r_{uu}u^2+2r_{uv}uv+r_{vv}v^2]$$

Mapping of the poles

$$\begin{cases} u_{p,q} = \sin\frac{\Phi_1^{p,q}}{2} \\ v_{r,s} = \sin\frac{\Phi_2^{r,s}}{2} \end{cases}$$

Arrival time of the doubly diffracted wavefront:

$$t^{dd}=\frac{r(0,0)}{c}=\frac{r'_1+\ell+r_2}{c}$$

Wavefront Approximations (Double Diffraction)

Canonical Integrals (TD Transition Functions)

$$\hat{T}^{I}(\overline{a},\overline{b},w;t)=\frac{\overline{a}\,\overline{b}}{\pi\left(1-w^{2}\right)^{3/2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\delta\left[t-\left(\xi^{2}+2w\xi\eta+\eta^{2}\right)\right]}{\left(\xi^{2}+\frac{\overline{a}}{1-w^{2}}\right)\left(\eta^{2}+\frac{\overline{b}}{1-w^{2}}\right)}d\xi d\eta$$

$$\hat{T}^{II}(\overline{a},\overline{b},w;t)=\frac{2\,\overline{a}\,\overline{b}}{\pi w\sqrt{1-w^{2}}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\xi\eta\,\frac{\delta\left[t-\left(\xi^{2}+2w\xi\eta+\eta^{2}\right)\right]}{\left(\xi^{2}+\frac{\overline{a}}{1-w^{2}}\right)\left(\eta^{2}+\frac{\overline{b}}{1-w^{2}}\right)}d\xi d\eta$$

Nondimensional parameters

$$\hat{a}(t)=\overline{a}/t$$

$$\hat{b}(t)=\overline{b}/t$$

$$\hat{c}=1-w^{2}+\hat{a}+\hat{b}$$

Evaluated in Closed Form

$$\hat{T}^{I}(\hat{a}(t),\hat{b}(t),w)=\frac{\sqrt{\hat{a}\hat{b}}}{\hat{c}^{2}-4w\hat{a}\hat{b}}\left[\sqrt{\frac{\hat{b}}{1+\hat{a}}}(\hat{c}-2w^{2}\hat{a})+\sqrt{\frac{\hat{a}}{1+\hat{b}}}(\hat{c}-2w^{2}\hat{b})\right]$$

$$\hat{T}^{II}(\hat{a}(t),\hat{b}(t),w)=\frac{\hat{a}\hat{b}}{2[\hat{c}^{2}-4w\hat{a}\hat{b}]}\left[\sqrt{\frac{\hat{a}}{1+\hat{a}}}(\hat{c}-2\hat{b})+\sqrt{\frac{\hat{b}}{1+\hat{b}}}(\hat{c}-2\hat{a})\right]$$

Far from transitions $T^{I,II}\left(\frac{\overline{a}_{pq}}{t}>>1,\frac{\overline{b}_{rs}}{t}>>1,w\right)\to 1$

Uniform Wavefront approximation for TD-DD field

Scalar field

$$\hat{\psi}_{12}^{dd}\{P, P'; t\} \sim A^{inc}(r_1')\, A(r_1', \ell, r_2)\, \hat{D}_{12}^{s,h}(t - t^{dd})$$

Spreading of the Incident field at Q_1 $\dfrac{1}{4\pi r_1'}$

DD spreading factor $\dfrac{\sqrt{r_1'}}{\sqrt{\ell r_2}\sqrt{r_1' + \ell + r_2}}$

Double diffraction coefficient (***soft, hard***)

$$\hat{D}_{12}^{s,h} = \hat{D}_{12}^{I,s,h} + \hat{D}_{12}^{II,s,h}$$

Electromagnetic TD-DD field

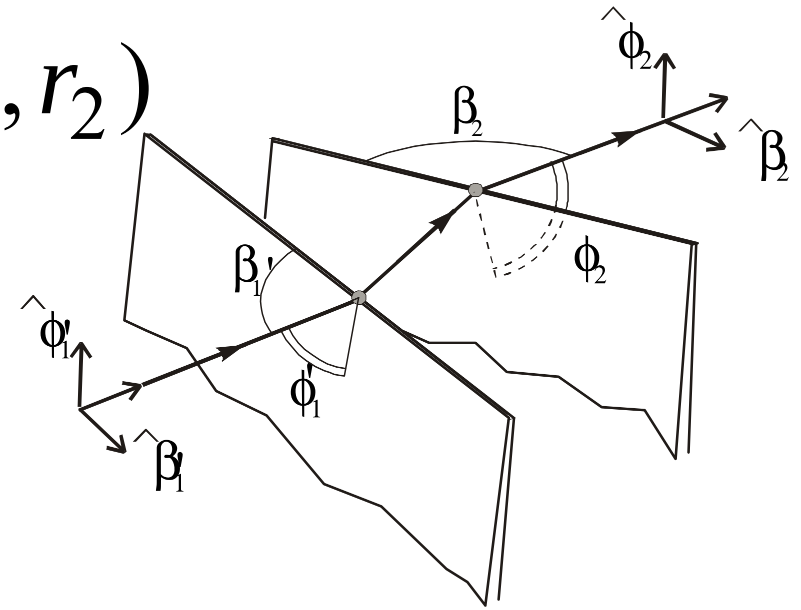
$$\mathbf{E}_{12}^{dd}(P) \sim \mathbf{E}^{inc}(Q_1)\, \underline{\underline{\mathbf{D}}}_{12}\, A(r_1', \ell, r_2)$$

Spreading of the Incident field at Q_1

Dyadic Diffraction Coefficient

$$\underline{\underline{\mathbf{D}}}_{12} = \varepsilon_{12}(\hat{\beta}_1' \hat{\beta}_2 D_{12}^{s} + \hat{\phi}_1' \hat{\phi}_2 D_{12}^{h})$$

soft and ***hard*** scalar diffraction coefficients



Unit vectors of the Ray Fixed coordinate systems

Uniform Early-time Double Diffraction coefficients

Step wavefront

$$\hat{D}_{12}^{I,s,h} = \frac{cU(t)}{8\pi n_1 n_2 \sin\beta_1' \sin\beta_2} \sum_{p,q,r,s=1}^{2} (\mp 1)^{p+q+r+s} \cot\frac{\Phi_1^{pq}}{2n_1} \cot\frac{\Phi_2^{rs}}{2n_2} \hat{T}^{I}\left(\frac{\bar{a}_{pq}}{t}, \frac{\bar{b}_{rs}}{t}, w\right)$$

$$\hat{D}_{12}^{II,s,h} = \frac{-\varepsilon_{12} c^2 t U(t)}{32\pi\ell (n_1 n_2 \sin\beta_1' \sin\beta_2)^2} \sum_{p,q,r,s=1}^{2} (\mp 1)^{p+s} (\pm 1)^{q+r} \csc^2\frac{\Phi_1^{pq}}{2n_1} \csc^2\frac{\Phi_2^{rs}}{2n_2} \hat{T}^{II}\left(\frac{\bar{a}_{pq}}{t}, \frac{\bar{b}_{rs}}{t}, w\right)$$

Ramp wavefront

Non-uniform evaluation

Transition Functions

Delay parameters

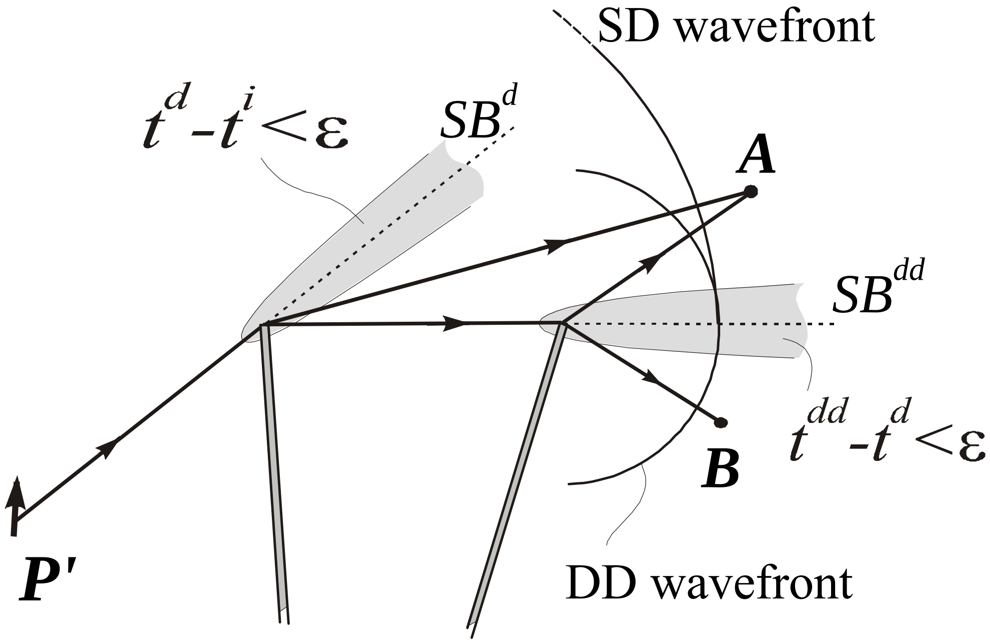
Delay between incident and diffracted arrival times for the leading and trailing diffraction

$$\bar{a}_{pq} = \frac{2}{c} \frac{r_1'\ell}{r_1' + \ell} \sin^2\beta_1' \sin^2\left(\frac{\Phi_1^{pq} - 2n_1\pi N^{pq}}{2}\right)$$

$$\bar{b}_{rs} = \frac{2}{c} \frac{r_2\ell}{r_2 + \ell} \sin\beta_2 \sin^2\left(\frac{\Phi_2^{rs} - 2n_2\pi N^{rs}}{2}\right)$$

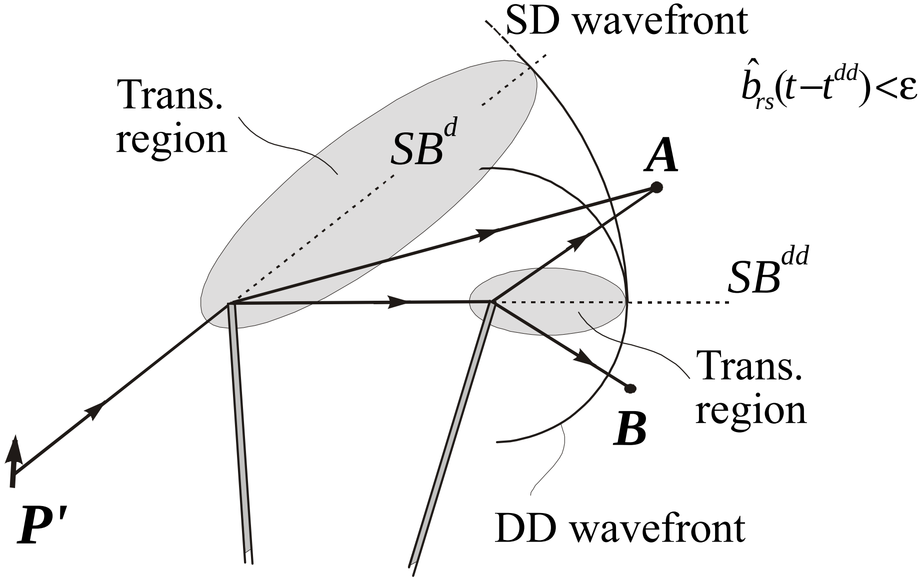
$$N^{pq} = \frac{\Phi_1^{pq}}{2n_1\pi} \qquad N^{rs} = \frac{\Phi_2^{rs}}{2n_2\pi}$$

DD Transition Regions

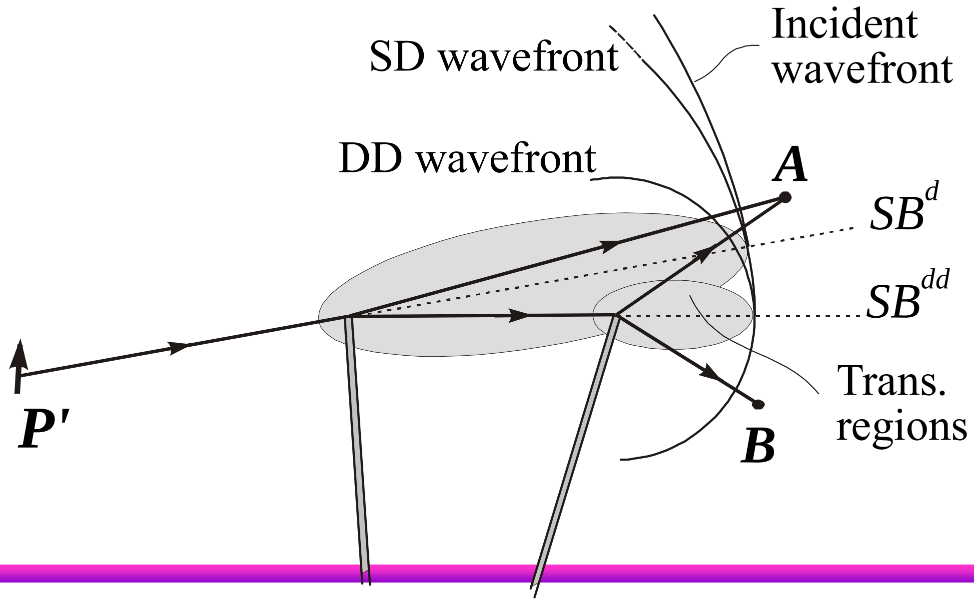


SD and DD ray wavefonts, shadow boundaries (SB). The SB^{dd} plane bounds the domain of existence of SD field. Conditions $t^{dd} - t^d < \varepsilon$ and $t^d - t^i < \varepsilon$ define parabolas (if $r'_1 >> \ell >> r_2$) in which wavefronts arrive "almost" simultaneously (delay<ε)

TD transition regions (ellipses if $\ell >> r_2$)



Overlapping TD transition regions:
SD and DD arrive at the observer A at instants t^i , t^d and t^{dd} , respectively ($t^i < t^d < t^{dd}$).



Numerical Examples - Excitation

(you turn me on!)

Band-Limited Short-Pulse Excitation

Spherical decaying pulse

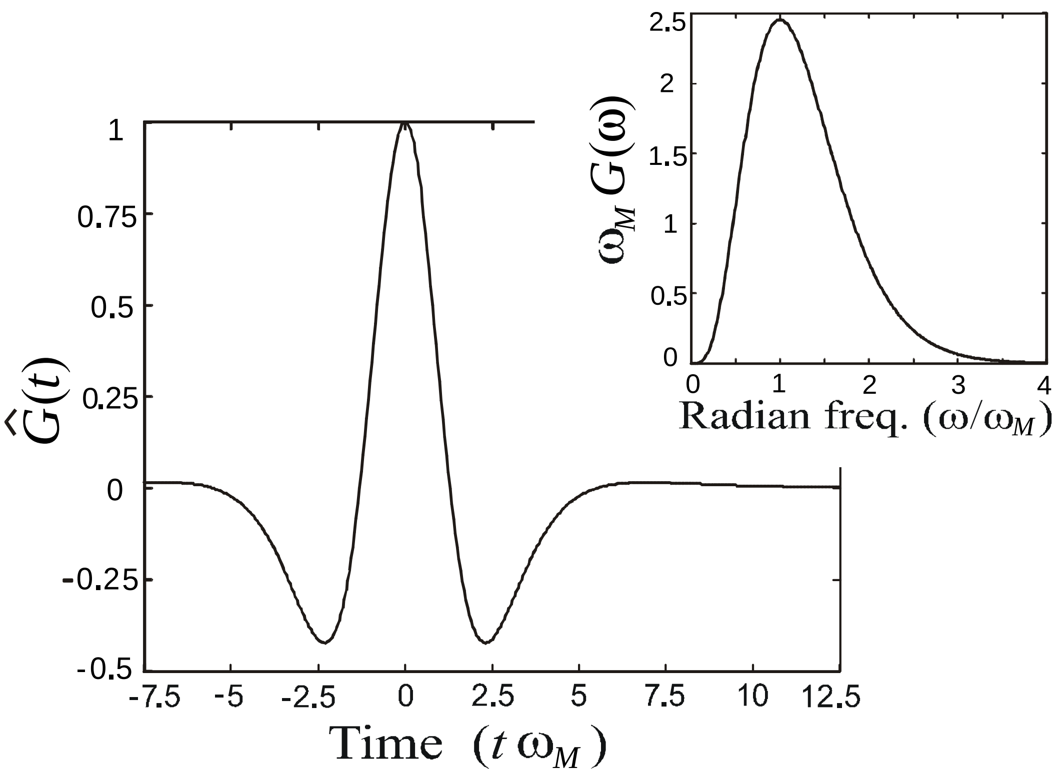
$$\hat{\psi}^{inc}(P,P',t)=\frac{\hat{G}\left(t-\frac{|P-P'|}{c}\right)}{4\pi|P-P'|}$$

Normalized Rayleigh pulse

$$\hat{G}(t)=\Re e\left[\frac{j}{(j+\omega_M t/4)^5}\right]$$

Frequency spectrum

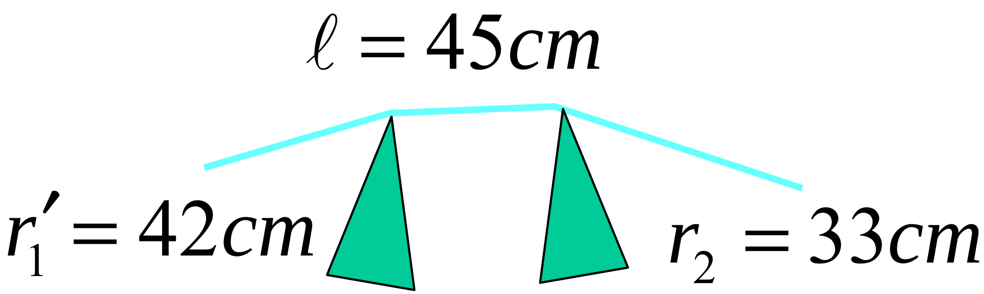
$$G(\omega)=\frac{\pi}{6\omega_M}\left(\frac{j4\omega}{\omega_M}\right)^4 e^{-4|\omega|/\omega_M}$$

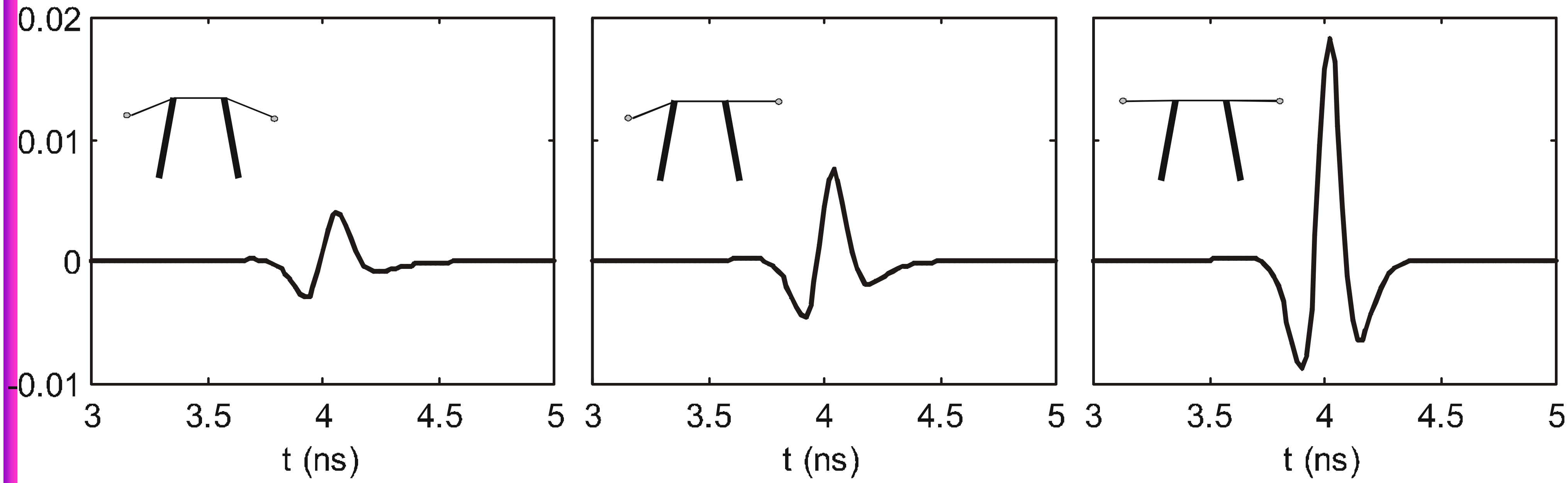


No zero-frequency components!

Different regimes of DD signals

Rayleigh pulse $f_M = 3GHz \; (\lambda_M = 10cm)$



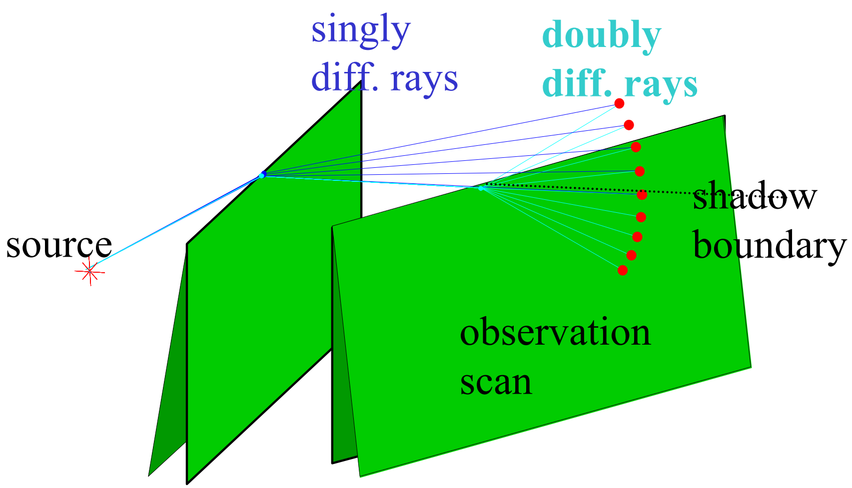


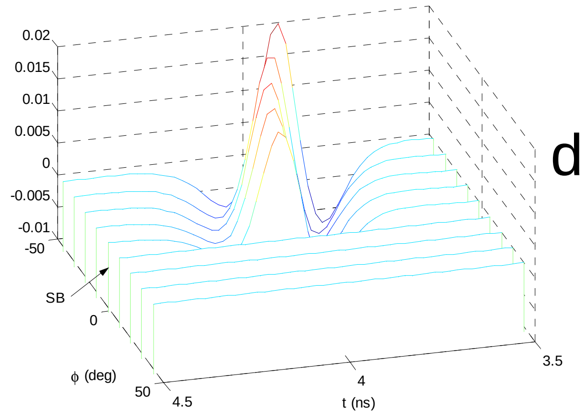
Both source and observer 30° out of transition

Source 30° out of transition, observer in transition

Both source and observer in transition

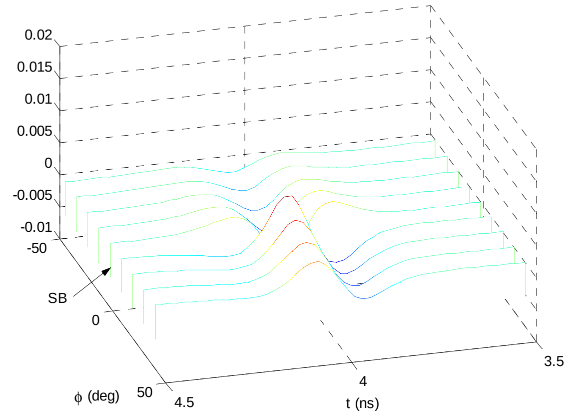
Crossing a shadow boundary plane of diffracted field

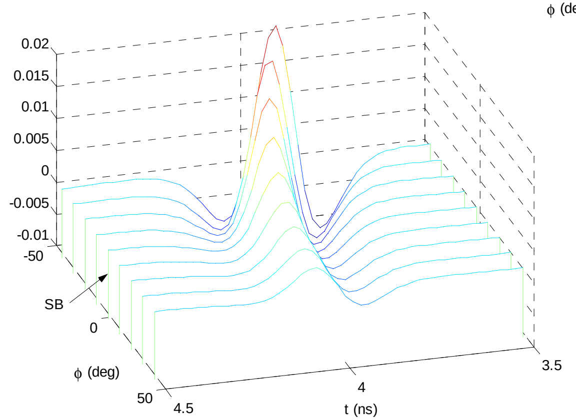




Singly diffracted field

Doubly diffracted field





Total field

At the **Shadow Boundary Plane**

- Singly diffracted signal experiences a discontinuity
- Doubly diffracted contribution exhibits a proper transition
- Total field is continuous at and behind the wavefront.

Note the change of shape of the signal (total field) from the lit to the shadowed region.

Part II: Vertex Diffraction

- Allows field description inside the shadow regions of Geometrical Optics (GO) and of wedge single diffraction (UTD)
- Restores total field continuity at the boundaries of such shadow regions
- Augments the prediction accuracy

Previous results

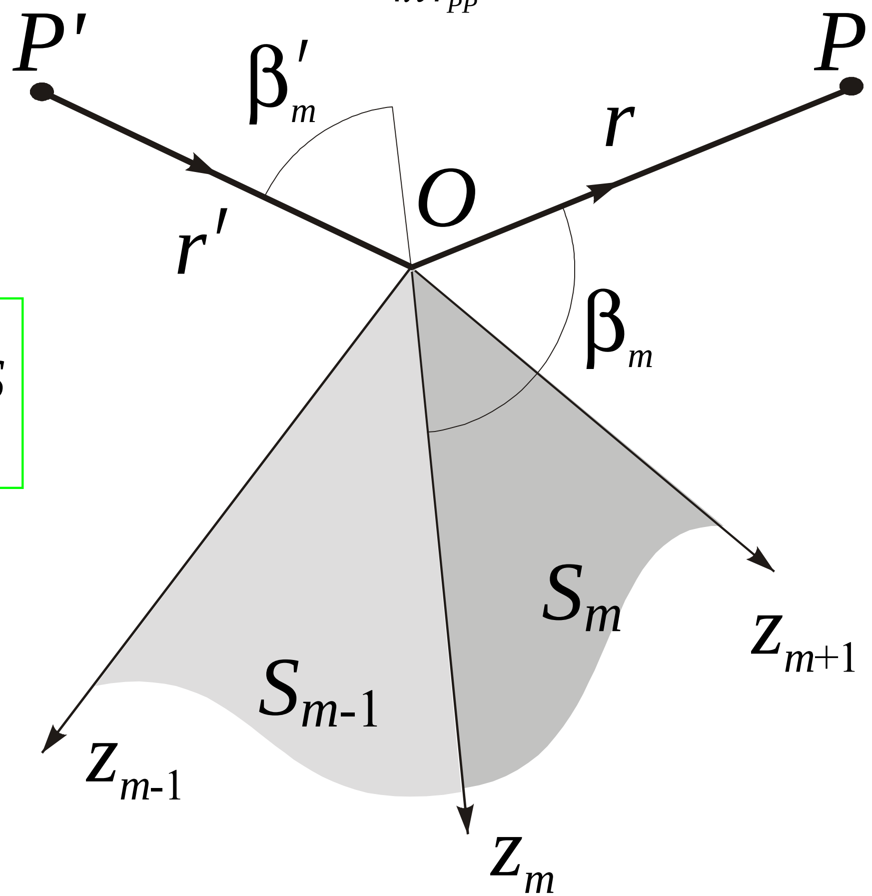
- F. A. Sikta, W. D. Burnside, T. T. Chu and L. Peters, Jr., "First-order equivalent current and corner diffraction from flat plate structures," *IEEE Trans. Antennas Propagat.*, vol. 31, no.4, pp. 584-589, July 1983.
- A. Michaeli, "Comments on 'first-order equivalent current and corner diffraction from flat plate structures," *IEEE Trans. Antennas Propagat.*, vol. 32, no. 9, Sept. 1984, p. 1011.
- S. Maci, R. Tiberio and A. Toccafondi, "Diffraction at a plane angular sector," *J. Electromagn. Wave Applicat.*, vol. 8, no. 9/10, pp. 1247-1276, Sept. 1994.
- F. Capolino, S. Maci "Uniform high-frequency description of singly, doubly, and vertex diffracted rays for a plane angular sector," *J. Electromagn. Wave Applicat.*, vol. 10, no 9, pp. 1175-1197, Oct. 1996.
- V. P. Smyshlyaev, "The high-frequency diffraction of electromagnetic waves by cones of arbitrary cross-section," *Soc. Indust. Appl. Math.*, vol. 53, no. 3, pp. 670-688, 1993.
- R. S. Satterwhite, "Diffraction by a quarter plane, the exact solution and some numerical results", IEEE Trans. Antennas Propagat., vol. AP-22, no. 3, pp. 500-503, Mar. 1974.
- L. P. Ivrissimtzis and R. J. Marhefka, "Edge wave vertex and edge diffraction", *Radio Sci.*, vol. 24, no. 6, pp. 771-784, 1989.

Formulation

- Acoustic scalar case

Incident field

$$\Psi_o(P,P') = \frac{e^{-jkr_{PP'}}}{4\pi r_{PP'}}$$



Helmholtz-Huygens principle

Total field

$$\boxed{\Psi^{tot}(P,P') = \Psi_o(P,P') + \sum_{m=1}^{M} \iint_{S_m} \vec{V}(P,Q,P') \cdot \hat{n}\, dS}$$

$$\vec{V}(P,Q,P') = \frac{e^{-jkr_{PQ}}}{4\pi r_{PQ}} \nabla_Q \Psi^{tot}(Q,P) - \Psi^{tot}(Q,P) \nabla_Q \frac{e^{-jkr_{PQ}}}{4\pi r_{PQ}}$$

Note that $\nabla_Q \cdot \vec{V}(P,Q,P') = 0$

Formulation (2)

Using Miyamoto-Wolf vector potential

$$\vec{V}(P,Q,P') = \nabla_Q \times \vec{W}(P,Q,P')$$

A. Rubinowicz, "The Miyamoto-Wolf Diffraction Wave," *Progress in Optics*, vol. 4, pp. 199-240, 1965

Using Stokes theorem

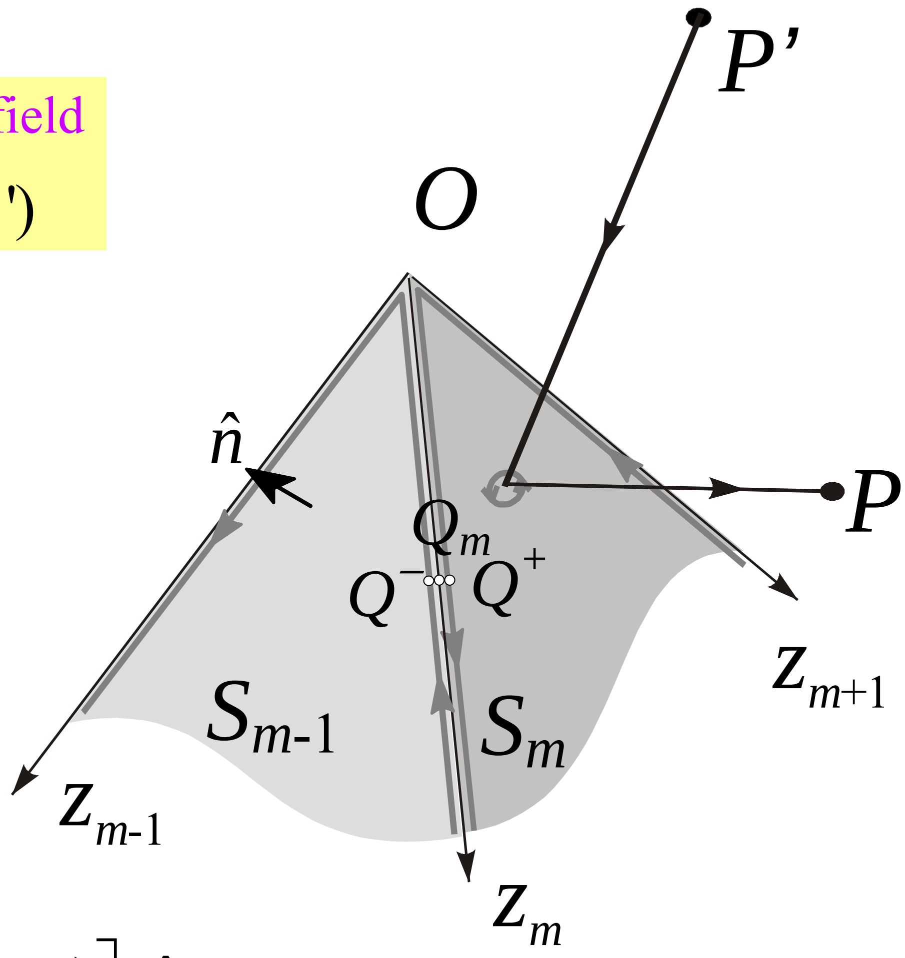
Total field Geometrical Optics Diffracted field

$$\Psi^{tot}(P,P') = U^{GO}\Psi^{GO}(P,P') + \Psi^{d}(P,P')$$

Incremental distribution along edges

$$\boxed{\Psi^{d}(P,P') = \sum_{m=1}^{M}\int_{0}^{\infty}\psi^{d}(P,Q_m,P')dz_m}$$

S. Maci, R. Tiberio, A. Toccafondi "Incremental diffraction coefficients for source and observation at finite distance from an edge," IEEE Trans. Antennas Propagat., vol. 44, no. 5, May 1996.



Exact incremental field

$$\psi^{d}(P,Q_m,P') = \lim_{Q^{\pm}\to Q_m}\left[\vec{W}(P,Q^{+},P') - \vec{W}(P,Q^{-},P')\right]\cdot\hat{z}_m$$

Formulation (3)

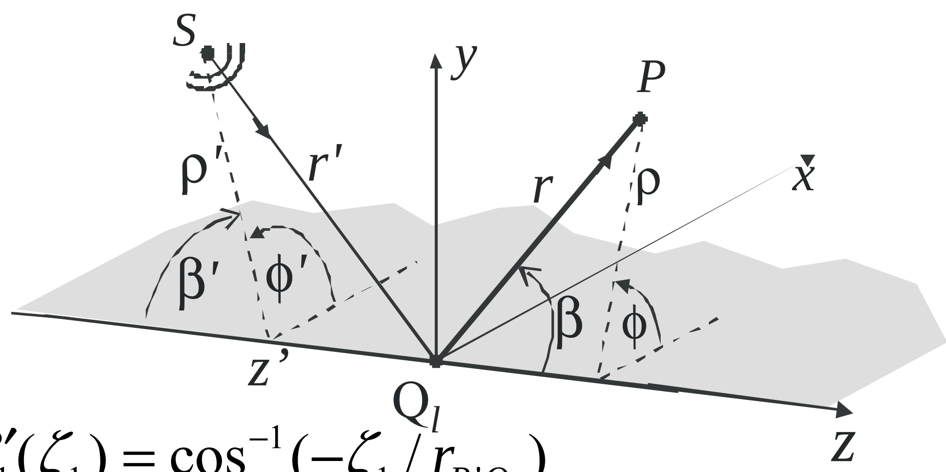
Approx. incremental field (Wedge)

$$\psi^d(P,Q,P') \approx 2\cdot G\left(\phi,\phi', ju(z_Q)\right)\frac{e^{-jkr_{PQ}}}{4\pi r_{PQ}}\frac{e^{-jkr_{P'Q}}}{4\pi r_{P'Q}}$$

Soft/hard

$$G(\phi,\phi',\alpha)=\left[B(\pi+\phi-\phi',\alpha)+B(\pi-\phi+\phi',\alpha)\right]\mp \left[B(\pi+\phi+\phi',\alpha)+B(\pi-\phi-\phi',\alpha)\right]$$

$$B(\Phi,\alpha)=-\tfrac{1}{2n}\frac{\operatorname{sen}\left(\frac{\Phi}{n}\right)}{\cos\left(\frac{\Phi}{n}\right)-\cos\left(\frac{\alpha}{n}\right)}$$



Rubinowicz's parameter

$$u_1(\zeta_1)=\ln\left\{\tan\left[\frac{\beta_1(\zeta_1)}{2}\right]\cot\left[\frac{\beta_1'(\zeta_1)}{2}\right]\right\}$$

$$\beta_1'(\zeta_1)=\cos^{-1}(-\zeta_1/r_{P'Q_1})$$

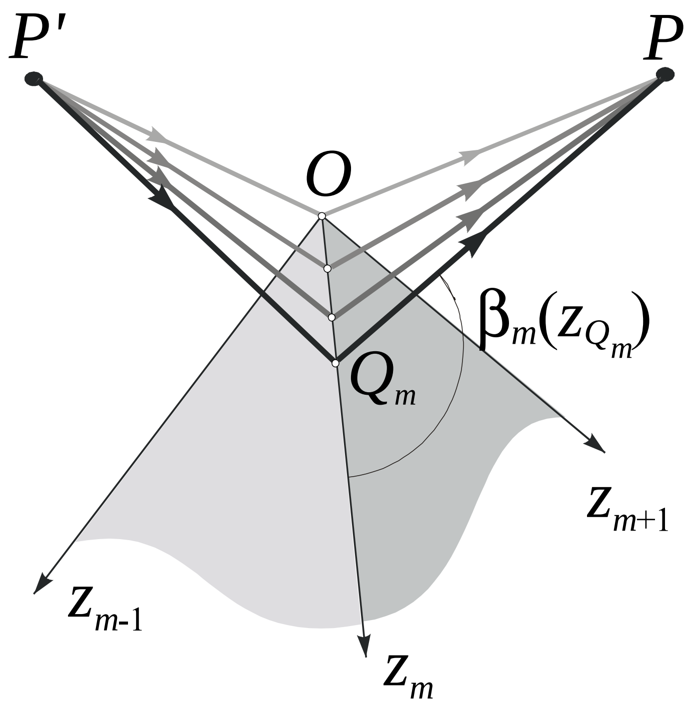
$$\beta_1(\zeta_1)=\cos^{-1}(\zeta_1/r_{PQ_1})$$

Approx. diffracted field

$$\Psi^d(P,P')\approx\sum_{m=1}^{M}\int_0^{\infty}2G\left(\phi_m,\phi_m', ju_m(z_{Q_m})\right)\frac{e^{-jkr_{PQ_m}}}{4\pi r_{PQ_m}}\frac{e^{-jkr_{P'Q_m}}}{4\pi r_{P'Q_m}}dz_{Q_m}$$

Asymptotic Evaluation

$$\Psi_m^d(P,P') = \Psi_m^{UTD}(P,P')\,U(\beta_m' - \beta_m) + \Psi_m^{tip}(P,P')$$



Saddle point Wedge UTD contribution

$$\boxed{\Psi_m^{UTD} = \int_{-\infty}^{\infty} 2G\left(\phi_m, \phi_m', ju_m(z_{Q_m})\right) \frac{e^{-jkr_{PQ}}}{4\pi r_{PQ}} \frac{e^{-jkr_{P'Q}}}{4\pi r_{P'Q}} dz_Q}$$

End-point Pyramid Tip contribution

$$\boxed{\Psi_m^{tip} = \int_0^{-\mathrm{sgn}(z_s)\infty} 2G\left(\phi_m, \phi_m', ju_m(z_{Q_m})\right) \frac{e^{-jkr_{PQ}}}{4\pi r_{PQ}} \frac{e^{-jkr_{P'Q}}}{4\pi r_{P'Q}} dz_Q}$$

Critical points

End point (vertex) $\zeta_1 = 0$

Saddle point (edge diff.) $\zeta_1 = z_1^d = \dfrac{\rho_1 z_1' + \rho_1' z_1}{\rho_1 + \rho_1'}$

Pole singularities (GO) $\zeta_1 = z_1^p \in \mathbb{C}$

Incident field at O

Spreading and propagation factor from O to P

$$\boxed{\Psi^{tip}(P,P') \sim \frac{e^{-jkr'}}{4\pi r'} \cdot D^{tip} \cdot \frac{e^{-jkr}}{r}}$$

Tip diffraction coefficient

UTD Tip diffraction coefficient

$$\boxed{D^{tip} = \sum_{m=1}^{M} D_m^{tip}}$$

$$u_m = u(O) = \log\left[\tan\tfrac{\beta_m}{2}\right] - \log\left[\tan\tfrac{\beta'_m}{2}\right]$$

$$D_m^{tip} = \frac{1}{2\,jk\pi(\cos\beta_m{}' - \cos\beta_m)} \cdot \Big\{\Big[B\big(\pi+(\phi_m-\phi'_m),u_m\big)T_{GFI}\big(b_m, a_m^{+}(\phi_m-\phi'_m)\big) + B\big(\pi-(\phi_m-\phi'_m),u_m\big)T_{GFI}\big(b_m, a_m^{-}(\phi_m-\phi'_m)\big)\Big]$$
$$\mp\Big[B\big(\pi+(\phi_m+\phi'_m),u_m\big)T_{GFI}\big(b_m, a_m^{+}(\phi_m+\phi'_m)\big) + B\big(\pi-(\phi_m+\phi'_m),u_m\big)T_{GFI}\big(b_m, a_m^{-}(\phi_m+\phi'_m)\big)\Big]\Big\}$$

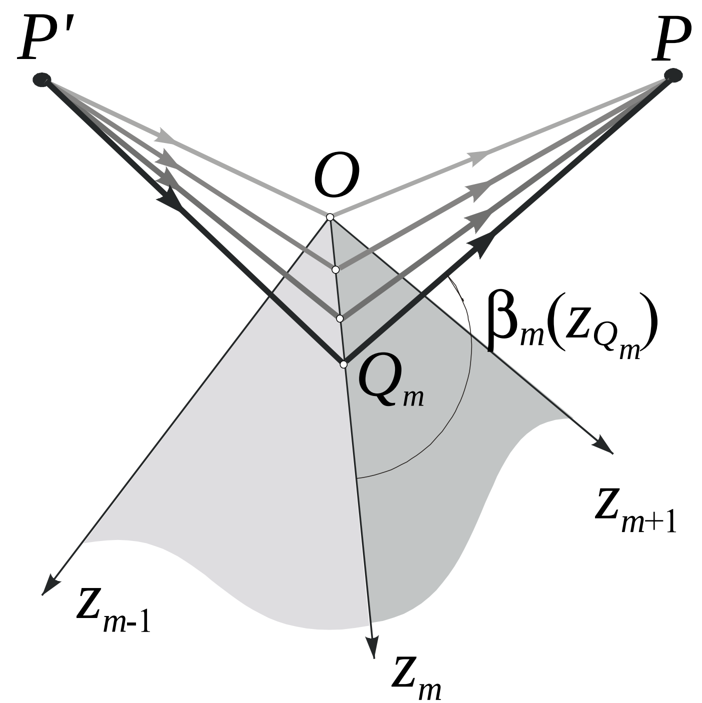
Transition function (end-point close to sadlle point + poles)

$$T_{GFI}(a,b) = 2j\sqrt{b}\,(b+a)\,e^{jb}\int_{\sqrt{b}}^{\infty}\frac{e^{-j\tau^2}}{\tau^2+a}\,d\tau$$

Distance parameters

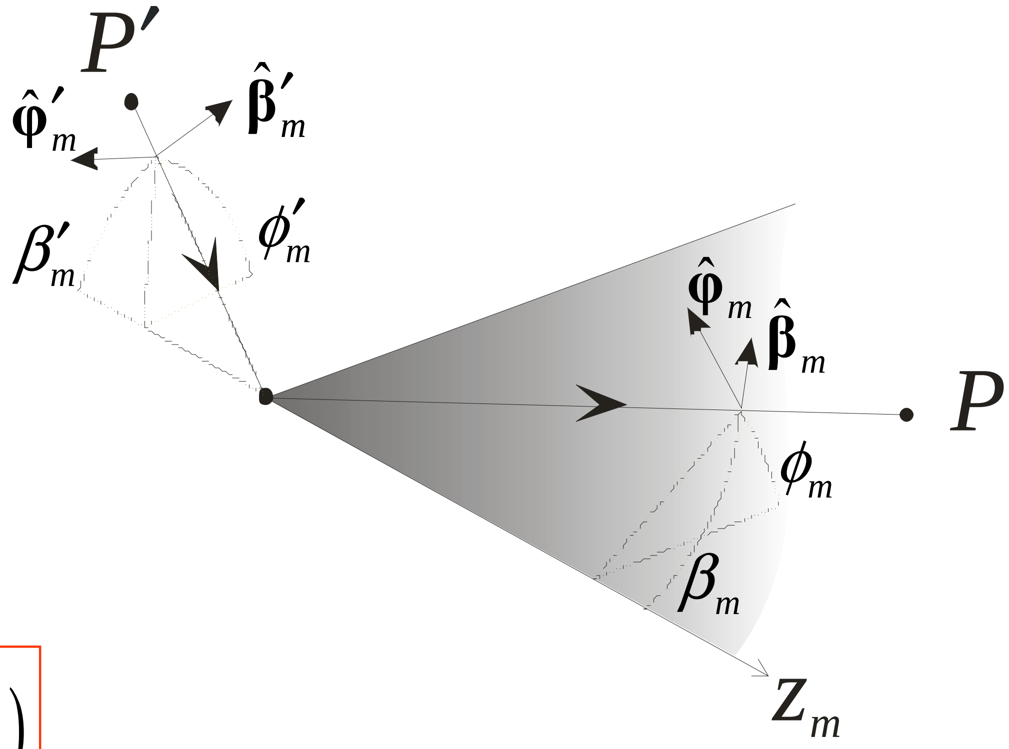
$$b_m = k\frac{rr'}{r+r'}\left[1-\cos\left(\beta_m-\beta'_m\right)\right]$$

$$a_m^{\pm}(\Phi_m) = k\frac{rr'}{r+r'}\sin\beta_m\sin\beta'_m\left[1+\cos\left(\Phi_m - 2N_m^{\pm}n_m\pi\right)\right]$$



Electromagnetic Case

$$\mathbf{E}^{tip}(P,P') \sim \underline{\mathbf{D}}^{tip} \cdot \mathbf{E}^{inc}(O,P')\frac{e^{-jkr}}{r}$$



UTD tip Dyadic coefficient

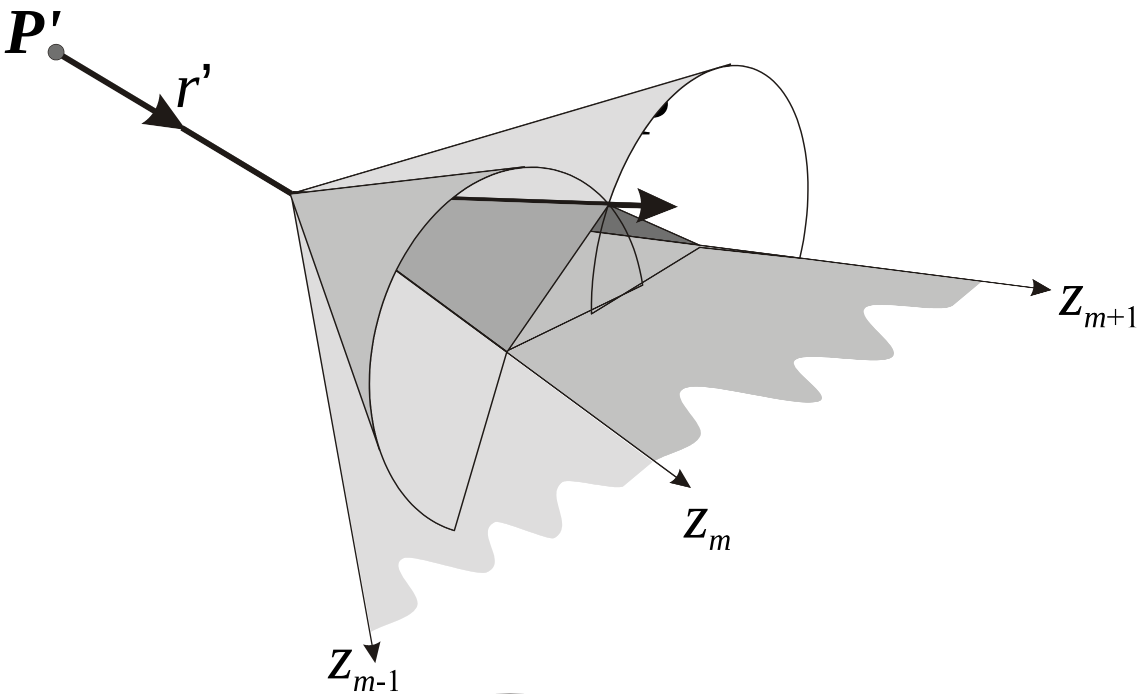
$$\underline{\mathbf{D}}^{tip} = \sum_{m=1}^{M}\left(-\hat{\beta}'_m\hat{\beta}_m D_m^{tip,s} - \hat{\phi}'_m\hat{\phi}_m D_m^{tip,h}\right)$$

soft and ***hard*** scalar diffraction coefficients

Ray fixed coordinate systems

$$(\hat{r},\hat{\beta}_m,\hat{\phi}_m) \quad (\hat{r}',\hat{\beta}'_m,\hat{\phi}'_m)$$

Analysis



Far from transition

$$T_{GFI} \to 1 \qquad \Psi^{tip} = O(k^{-1})$$

In Transition (Approaching a SBC)

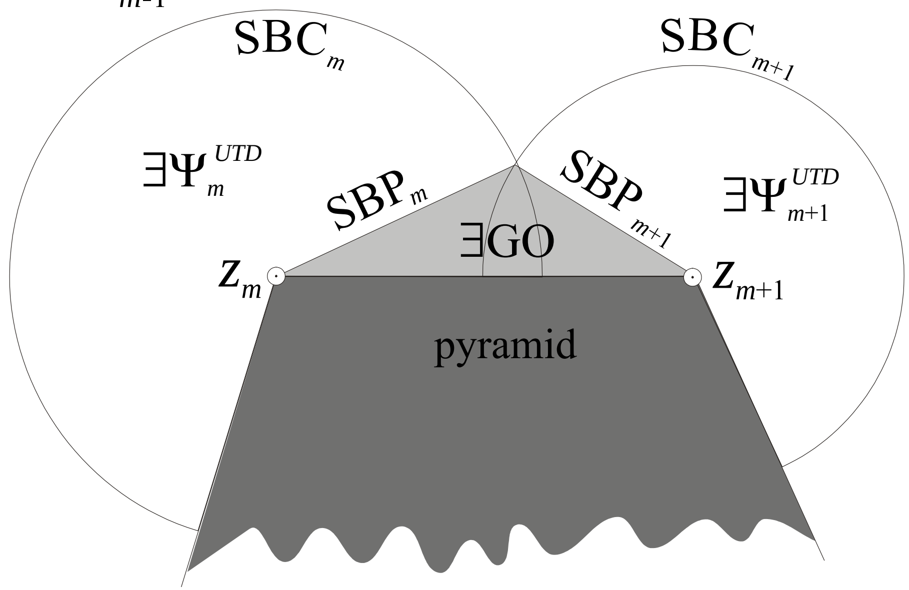
$$\beta_m \approx \beta'_m$$

$$b_m \to 0 \qquad \lim_{b\to 0} T_{GFI}(b,a) = \sqrt{j\pi b}\,F(a)$$

$$\Psi_m^{tip} = O(k^{-\frac{1}{2}})$$

UTD discontinuity compensation

$$\boxed{\Psi_m^{tip} \approx \tfrac{1}{2}\Psi_m^{UTD}\,\mathrm{sgn}(\beta_m - \beta'_m)}$$



Analysis

Double transition

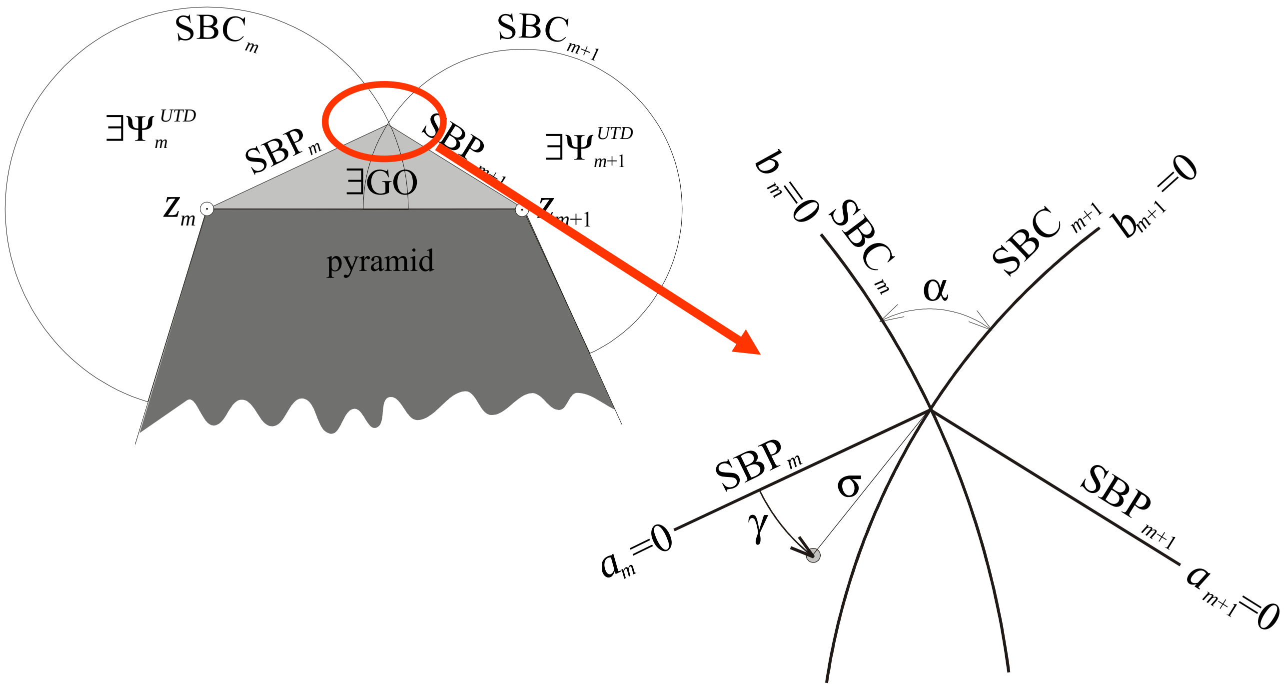
$$\beta_m \approx \beta'_m \quad \beta_{m+1} \approx \beta'_{m+1}$$

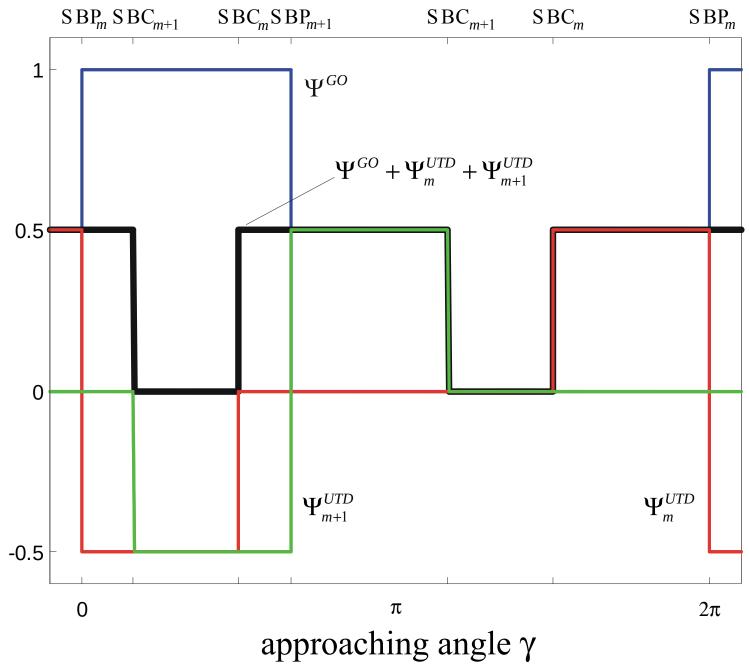
$$b_m \to 0 \quad b_{m+1} \to 0$$

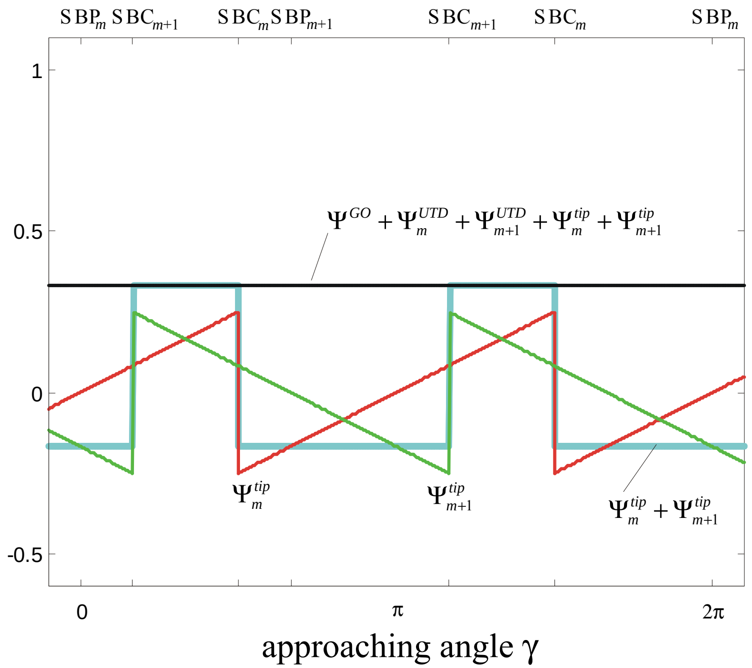
$$\lim_{b,a\to 0} T_{GFI}(b,a) = 2j\sqrt{\frac{b}{a}}\,(a+b)\tan^{-1}\left(\sqrt{\frac{a}{b}}\right)$$

$$\boxed{\Psi^{tot} = \Psi^{GO} + \Psi^{UTD}_{m} + \Psi^{UTD}_{m+1} + \Psi^{tip}_{m} + \Psi^{tip}_{m+1} \to \frac{\pi-\alpha}{2\pi}\Psi^{GO}}$$

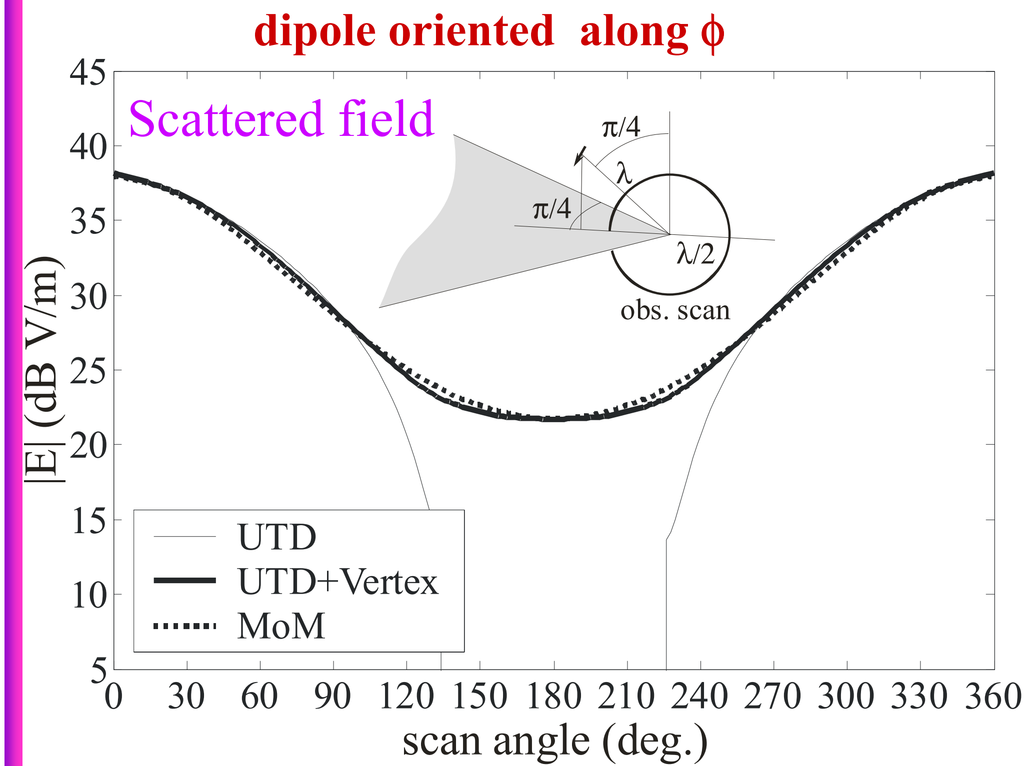
Smooth continuous behavior of total field

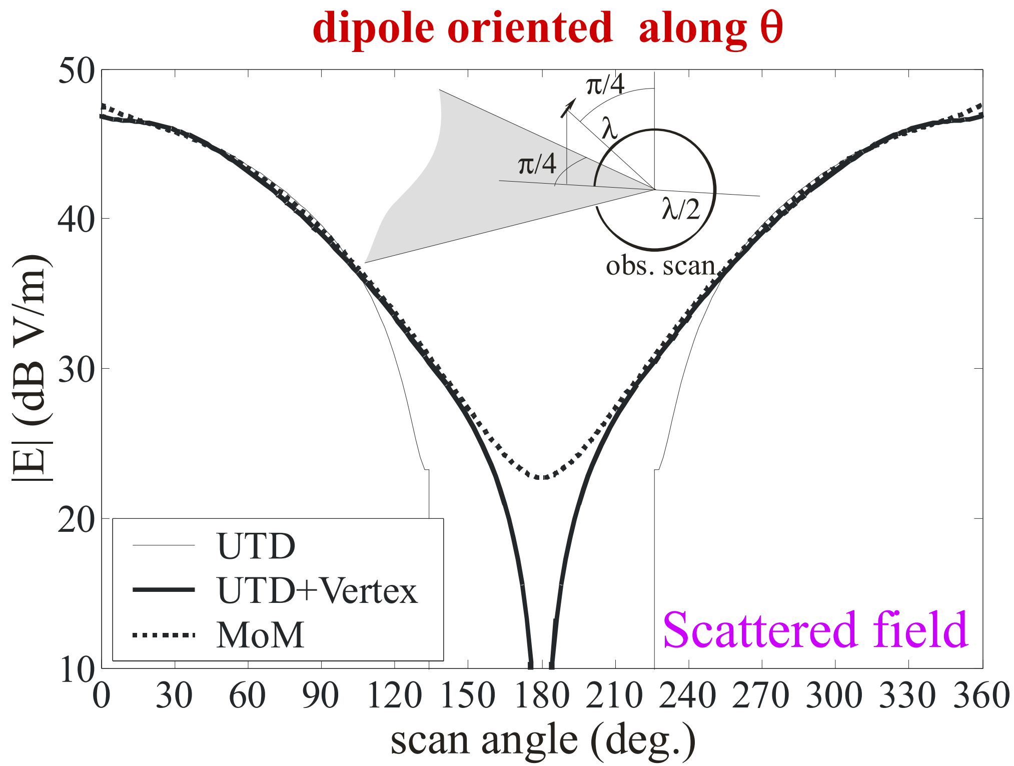






Numerical example 1



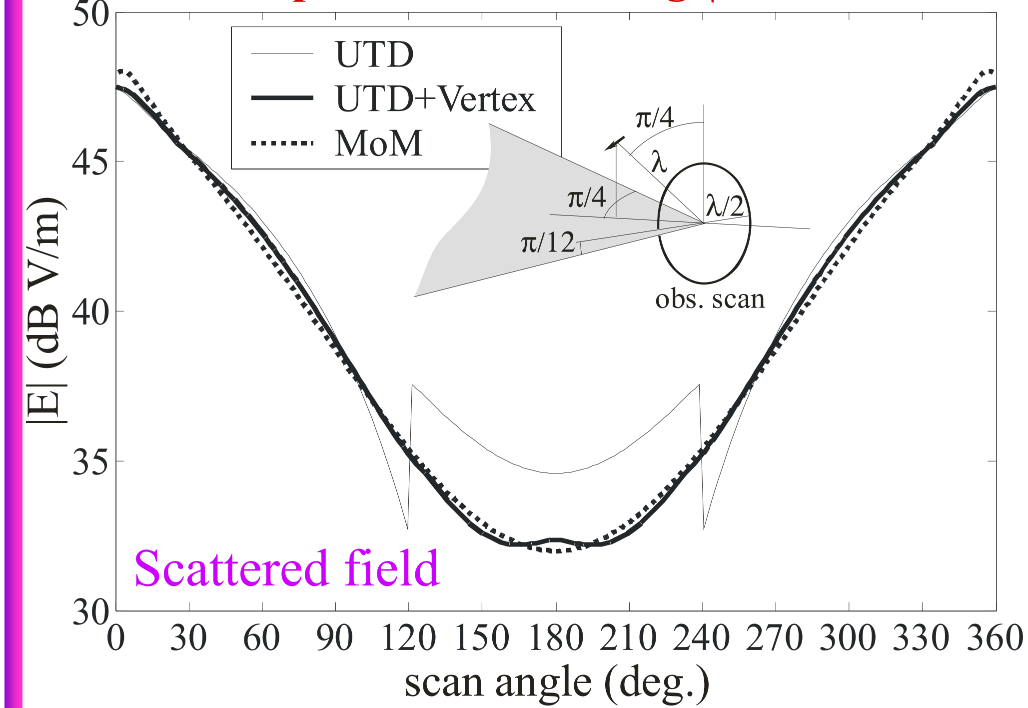


MoM reference solution 4λ×4λ square plate

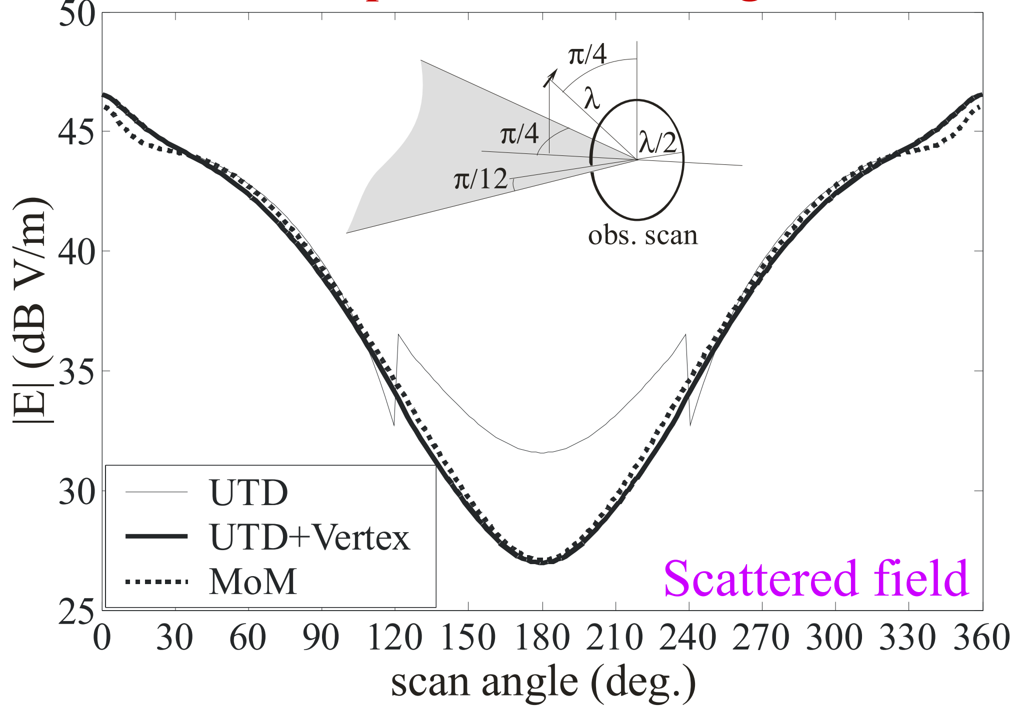
The two SBCs are crossed simultaneously at θ=135° and θ=225° where the UTD edge diffracted fields exhibit a jump discontinuity that is compensated by the vertex diffracted field.

Numerical example 2

dipole oriented along φ



dipole oriented along θ

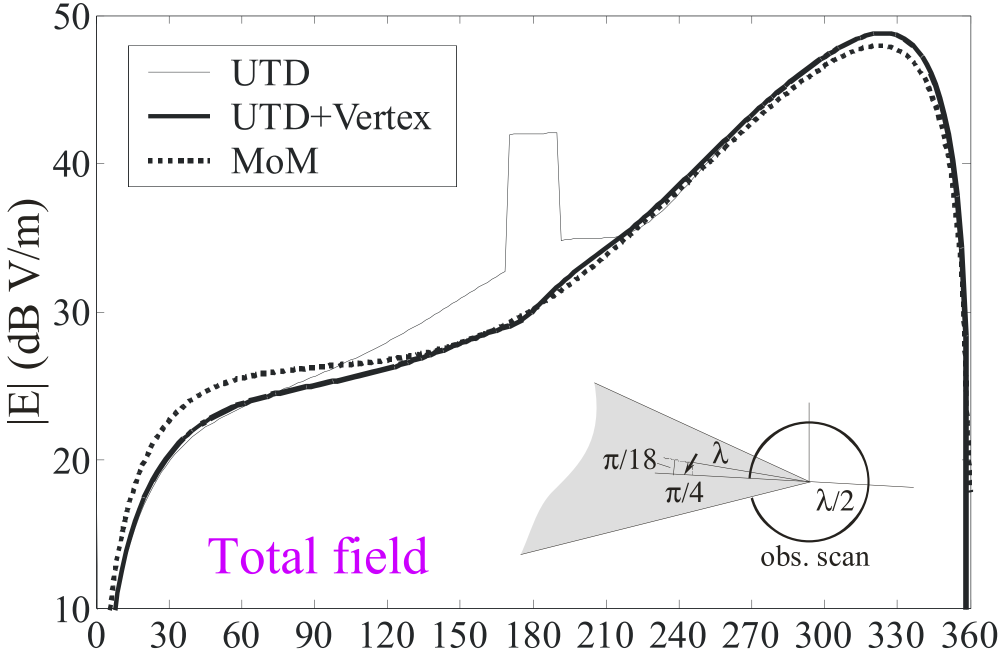


MoM reference solution 4λ×4λ square plate

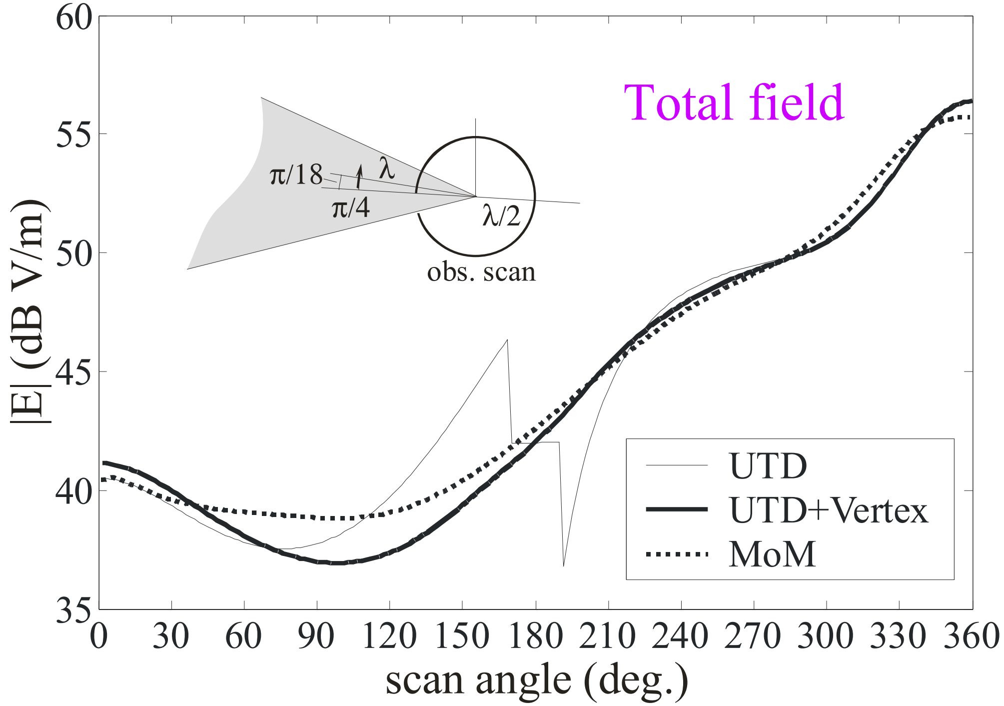
Two SBCs are crossed at θ=120° and θ=240° where the UTD field exhibits a jump discontinuity that is compensated by the vertex diffracted field.

Numerical example 3

dipole oriented along ϕ



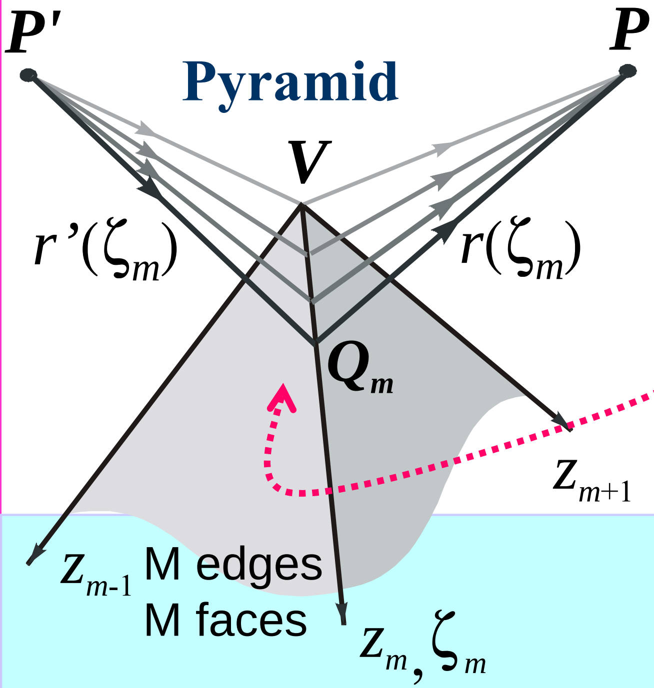
dipole oriented along θ



MoM reference solution $4\lambda \times 4\lambda$ square plate

The two SBCs are crossed simultaneously at θ=165° and 195° where the UTD edge diffracted fields exhibit a jump discontinuity that is compensated by the vertex diffracted field.

Time Domain Spatial Synthesis (Vertex Diffraction)



TD Diffracted field at each truncated illuminated edge is represented as integral superposition of TD incremental field contributions

$$u_m = \ln\left[\tan\frac{\beta_m(\zeta)}{2}\right] - \ln\left[\tan\frac{\beta'_m(\zeta)}{2}\right]$$

Spatial Representation for Total TD Diffracted Field

$$\hat{\psi}\{P, P'; t\} = \frac{j}{8\pi}\sum_{m=1}^{M}\int_0^{\infty} \underline{G[\phi_m, \phi'_m; ju_m(\zeta), n_m]}\,\frac{\delta\left[t - \left(r'_m(\zeta) + r_m(\zeta)\right)/c\right]}{r'_m(\zeta)\,r_m(\zeta)}\,d\zeta$$

ζ–Saddle point

TD Edge diffracted field

End Point
TD Vertex diffracted field

Poles
TD GO field

Approximationof the phase at the End Point

$$[r_m(\zeta) + r'_m(\zeta)]/c \approx t^v + A\zeta + B\zeta^2$$

Arrival time of Vertex Diffracted Wavefront

$$t^v = [r_m(0) + r'_m(0)]/c = [r + r']/c$$

Wavefront Approximations (Vertex Diffraction)

Canonical Integral (TD Transition Function)

$$\hat{T}_G(\bar{x},\bar{y};t) = 2\sqrt{\bar{x}}(\bar{x}+\bar{y})\int_{\sqrt{\bar{x}}}^{\infty}\frac{\delta\left[t-\left(\xi^2-\bar{x}\right)\right]}{\xi^2+\bar{y}}\,d\xi$$

Evaluated in Closed Form

$$\hat{T}_G(\hat{x}(t),\hat{y}(t)) = \frac{1}{\left(1+\frac{1}{\hat{x}+\hat{y}}\right)\sqrt{1+\frac{1}{\hat{x}}}}$$

Nondimensional parameters

$$\hat{x}(t) = \frac{\bar{x}}{t}$$

$$\hat{y}(t) = \frac{\bar{y}}{t}$$

Far from transitions

$$T_G\left(\tfrac{x}{t} >> 1, \tfrac{y}{t} >> 1\right) \rightarrow 1$$

Uniform Wavefront approximation for TD-VD field

Total TD Vertex Diffracted Field

$$\hat{\psi}\{P,P';t\} = \sum_{m=1}^{M} \hat{\psi}^{d}_{m}(t)\, U(\beta'_m - \beta_m) + \hat{\psi}^{v}(t)$$

Vertex diffraction coefficient

Wavefront approximation for TD VD Field

$$\hat{\psi}^{v}(t) = A^{inc}\,\frac{1}{r}\left[\sum_{m=1}^{M} \hat{D}_{m}^{v,(s,h)}(t - t^{v})\right]$$

Spreading of the Incident field at *V* $\dfrac{1}{4\pi r'}$

vertex spreading factor

Dyadic Electromagnetic TD-VD field

$$\mathbf{E}^{v}(P) \sim \mathbf{E}^{i}(V)\,\underline{\underline{\mathbf{D}}}^{v}\,\frac{1}{r}$$

Spreading of the Incident field at *V*

Dyadic Diffraction Coefficient

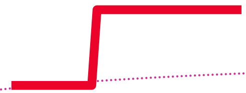
$$\underline{\underline{\mathbf{D}}}^{v}_{m} = \hat{\beta}'_m \hat{\beta}_m D_m^{v(s)} + \hat{\phi}'_m \hat{\phi}_m D_m^{v(h)}$$

soft and ***hard*** scalar diffraction coefficients

Uniform Early-time Vertex Diffraction coefficients

$$u_m = \ln\left(\frac{\tan\frac{\beta_m}{2}}{\tan\frac{\beta'_m}{2}}\right)$$

Step wavefront



$$\hat{D}_m^{v,(s,h)}(t) = \frac{c \quad U(t)}{2\pi(\cos\beta'_m - \cos\beta_m)} \sum_{i,j}^{2} (\mp 1)^j B\left[(-1)^i\left(\phi_m - (-1)^i \phi'_m\right); ju_m, n_m\right] \cdot \hat{T}_G\left(\frac{t_\delta^v}{t}, \frac{\hat{a}^i(\phi_m - \phi'_m)}{t}\right)$$

Non-uniform evaluation **Transition Function**

Delay parameters

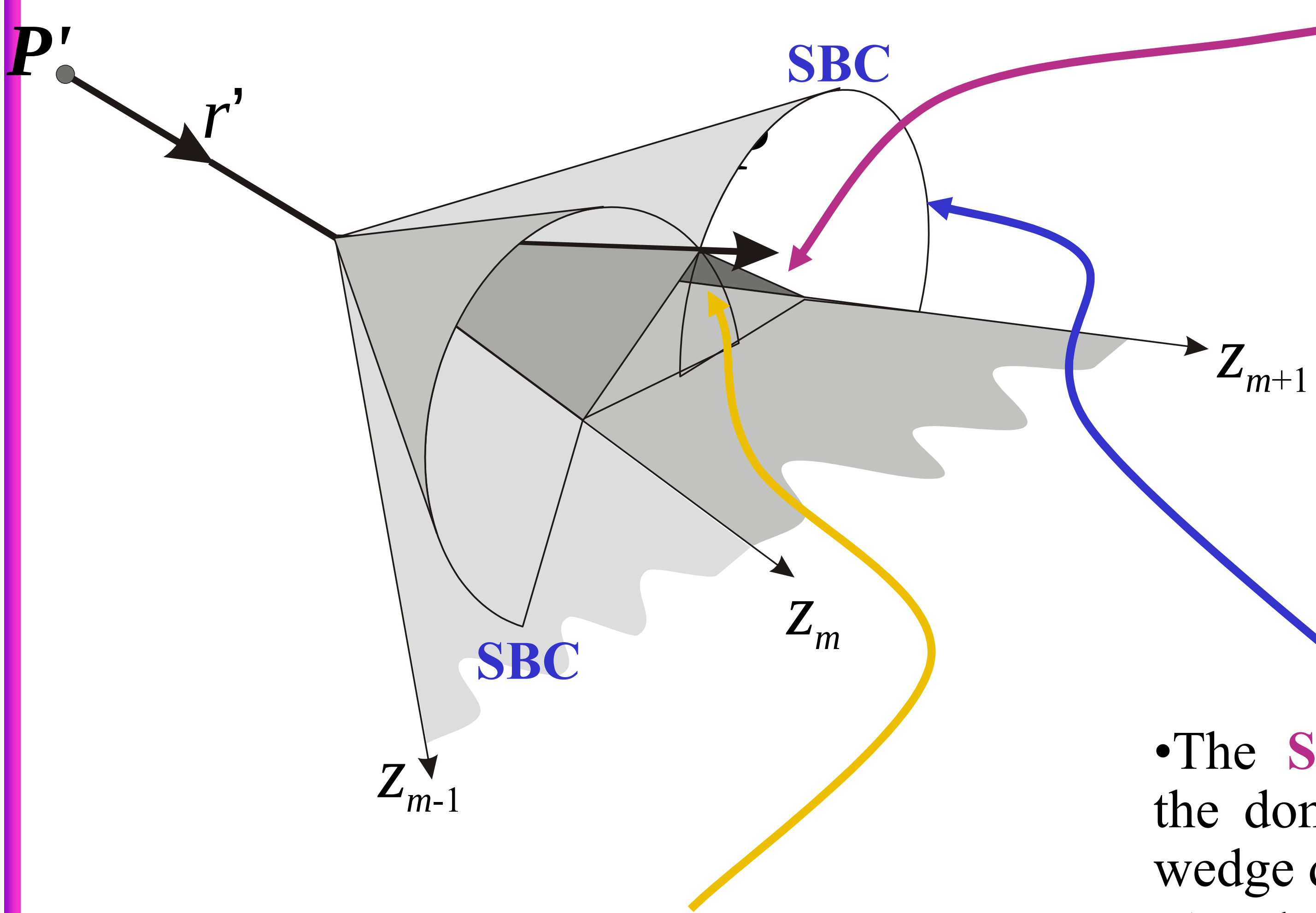
Delay between incident, edge-diffracted and vertex-diffracted wave arrival times.

$$t_\delta^v = \frac{2}{c}\frac{rr'}{r+r'}\sin^2\left(\frac{\beta_m - \beta'_m}{2}\right)$$

$$a^i(\Phi_m) = \frac{1}{c}\frac{rr'}{r+r'}\sin\beta'_m \sin\beta'_m\left[1 + \cos(\Phi_m - 2N_m^i n_m \pi)\right]$$

$$\Phi_m - 2N_m^i n_m \pi = (\pm 1)^i \pi, \quad i = 1,2$$

VD Shadow Boundaries and Transition Regions



•The **SB Plane** bounds the domain of existence of the reflected field.

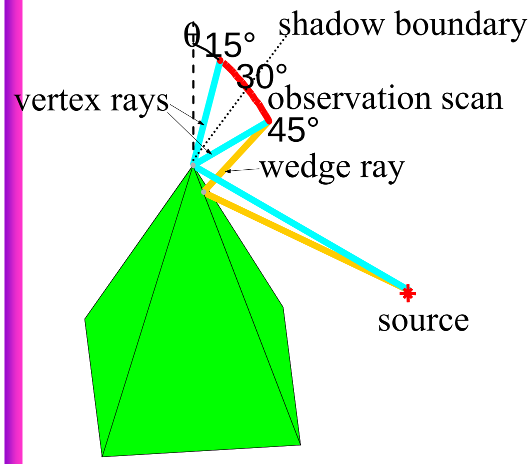
•At the SB Plane the wedge diffracted field experience a transition compensating the discontinuity of the reflected field.

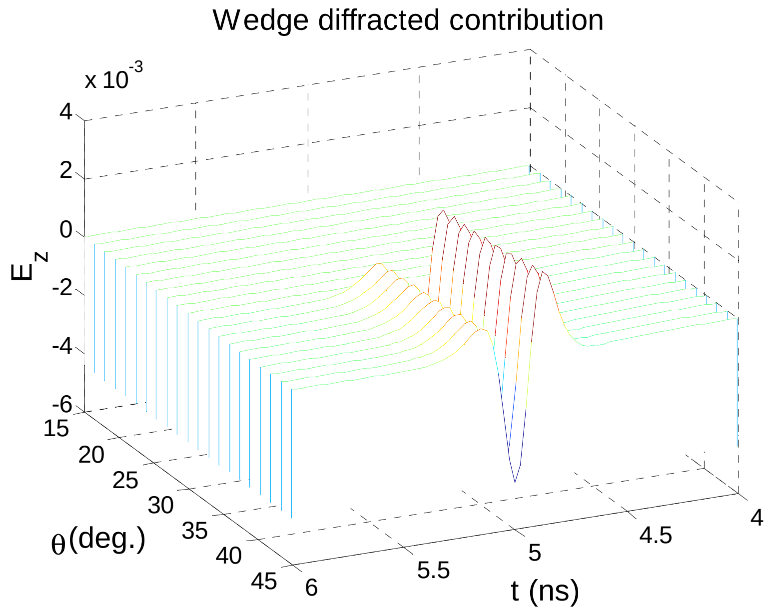
•The **SB Cone (SBC)** bounds the domain of existence of the wedge diffracted fields.

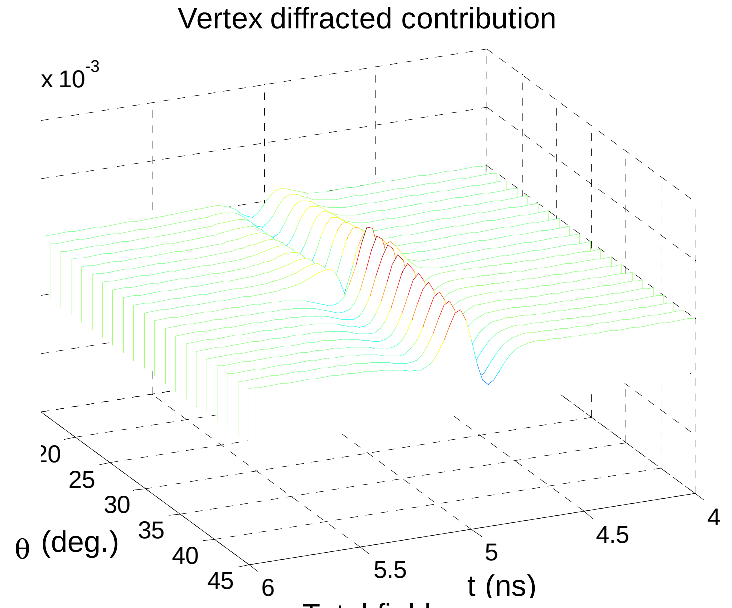
•At the SB Cone the Vertex diffracted field experience a transition restoring the total continuity for the total field

The two **SB Planes** and the two **SB Cones** all intersect at the same line. Here, the Vertex Ray experience a **double transition** able to restore the continuity of the total field

Crossing a shadow boundary cone

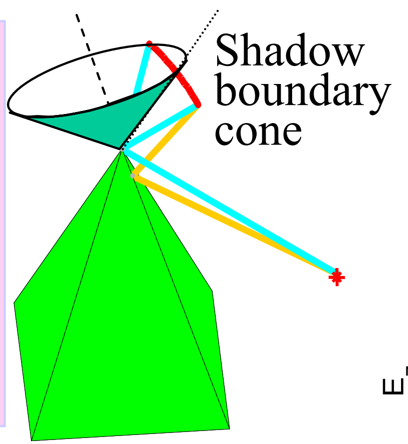


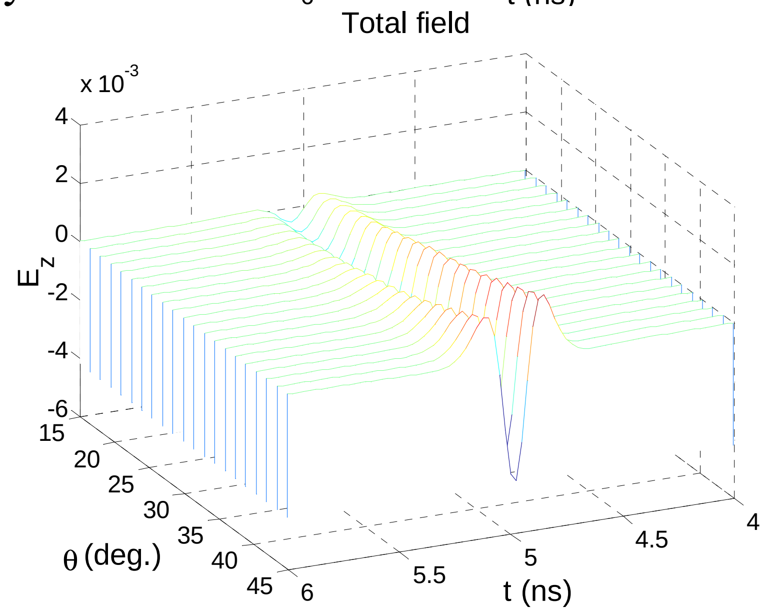




At the **Shadow Boundary Cone**

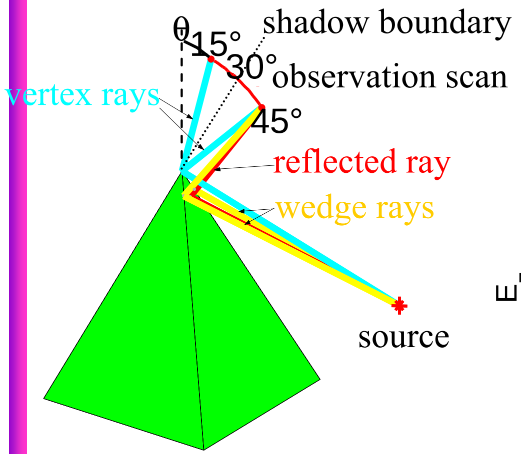
- Wedge diffracted signal experiences a discontinuity;
- Vertex contribution exhibits a transition
- Total field is continuous at and behind the wavefront.

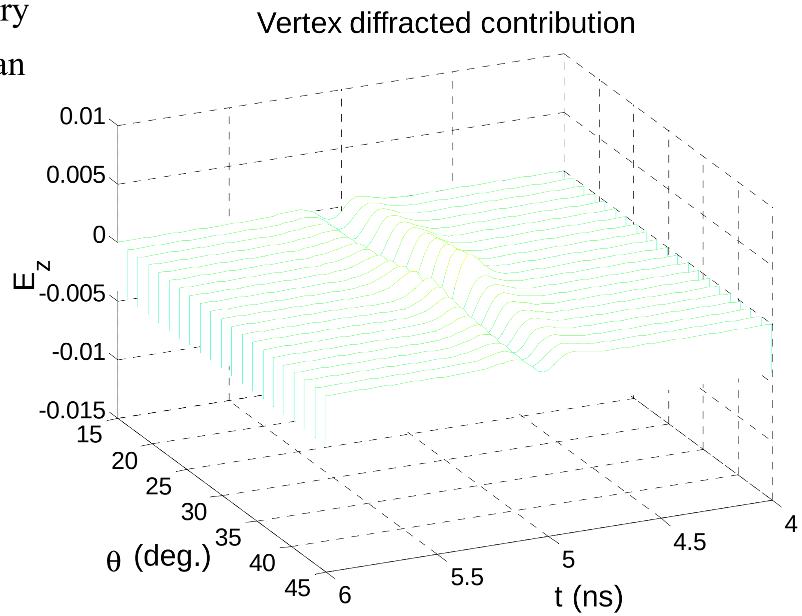


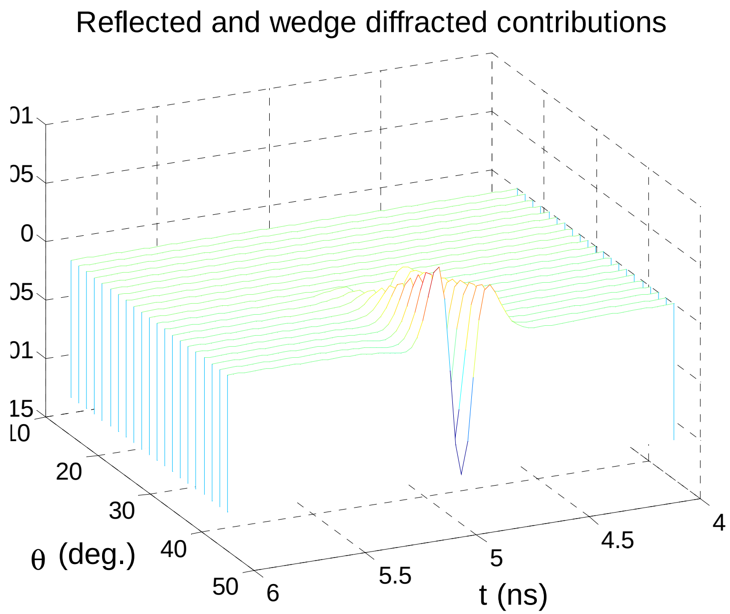


Note the change of shape of the signal (total field) from deep inside to far outside the SBC.

Crossing intersection of two shadow boundary cones



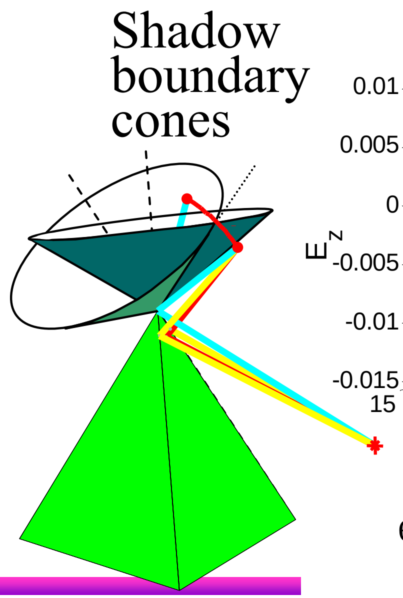


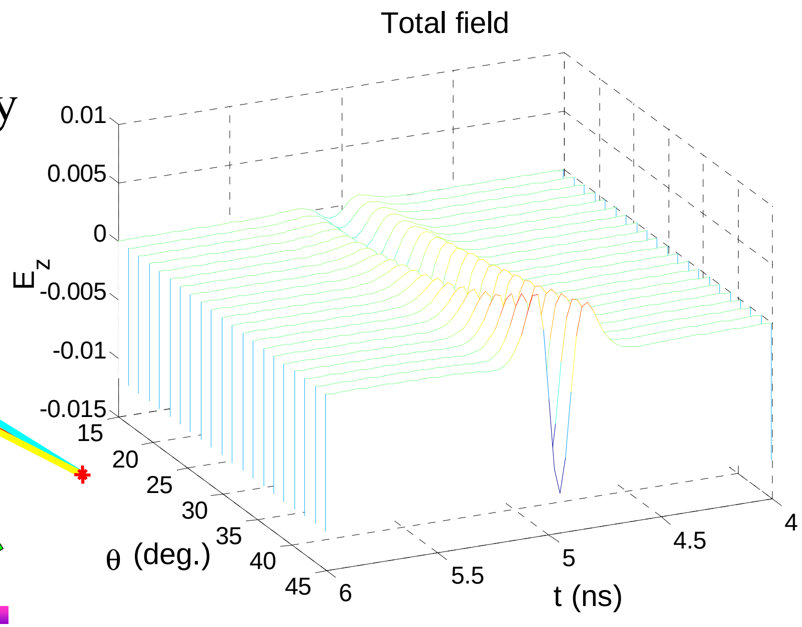


At the **intersection of two SBC's**

- Two wedge diffracted ray and a reflected ray simultaneously disappear;
- Vertex contribution restores the desired continuity of the total field

Note that disappearing wedge contributions are in their transitional regime.





Conclusions

- Uniform diffraction coefficients was presented for DD & Vertex
- Transitional behaviors are described by proper transition functions
- These contributions augment standard UTD ray field description to order k^{-1} and restore continuity across SBs