# GO Field Representation

- Astigmatic GO Ray Tube

$$\boxed{\overline{E}(p)=\overline{E}(O)\sqrt{\frac{Det[Q(p)]}{Det[Q(O)]}}\,e^{-jk\left(p+[\Lambda]^T\frac{Q(p)}{2}[\Lambda]\right)}} \quad [\Lambda]=\begin{bmatrix}x\\y\end{bmatrix} \quad Q(p)=\begin{bmatrix}\frac{1}{\rho_1+S} & 0\\ 0 & \frac{1}{\rho_2+S}\end{bmatrix}$$

- Plane Wave

when $\boxed{\rho_1=\rho_2\rightarrow\infty}$

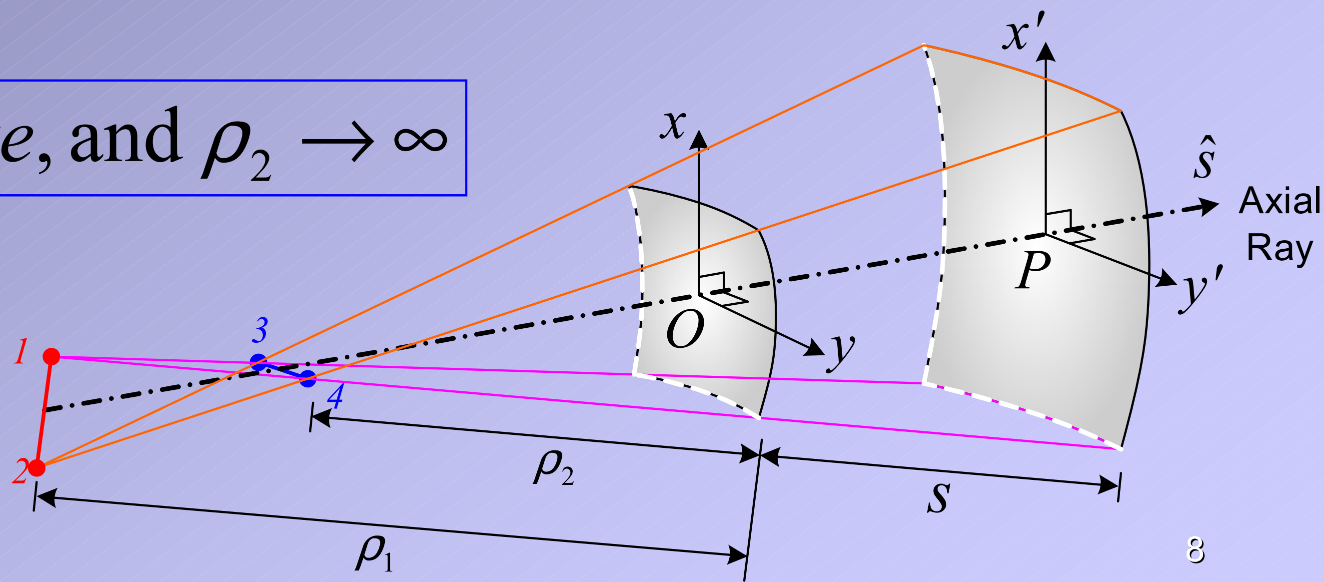
- Cylindrical Wave

when $\boxed{\rho_1 = finite,\text{ and } \rho_2\rightarrow\infty}$

- Spherical Wave

when

$\boxed{\rho_1=\rho_2=finite}$



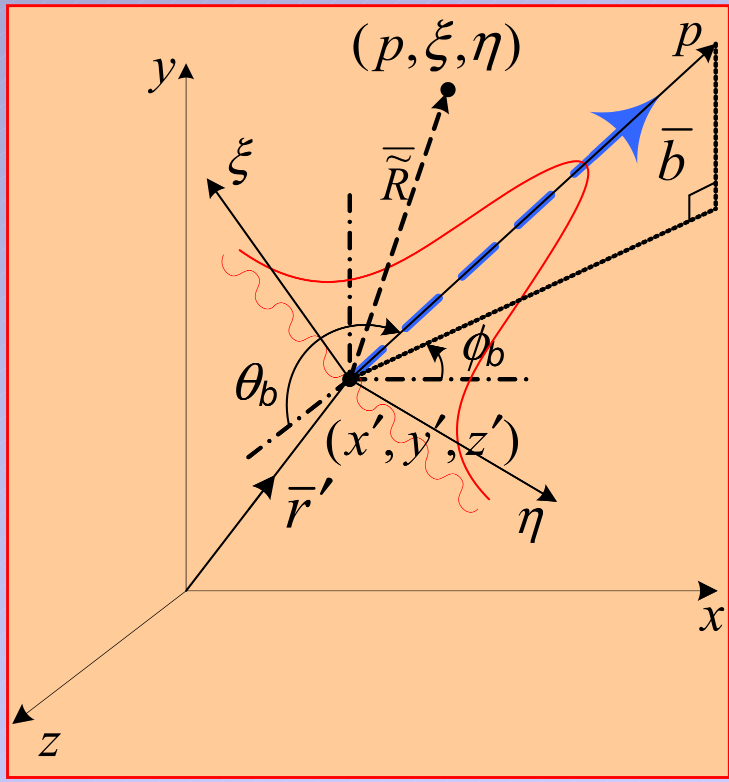
# Astigmatic Gaussian Beam Field

- A Gaussian Beam (GB) Field is described by

$$\boxed{\overline{E}(p,\eta,\xi)=\overline{E}(0)\sqrt{\frac{Det[Q(p)]}{Det[Q(0)]}}\,e^{-jk\left(p+\frac{1}{2}[\Lambda]^{T}Q(p)[\Lambda]\right)}}$$

$$Q(p)=\begin{bmatrix}\dfrac{1}{p+jb_1} & 0\\ 0 & \dfrac{1}{p+jb_2}\end{bmatrix}\qquad [\Lambda]=\begin{bmatrix}\eta\\ \xi\end{bmatrix}$$

$$\begin{bmatrix}p\\ \xi\\ \eta\end{bmatrix}=\begin{bmatrix}\sin\theta_b\cos\phi_b & \sin\theta_b\sin\phi_b & \cos\theta_b\\ \cos\theta_b\cos\phi_b & \cos\theta_b\sin\phi_b & -\sin\theta_b\\ -\sin\phi_b & \cos\phi_b & 0\end{bmatrix}\cdot\begin{bmatrix}x-x'\\ y-y'\\ z-z'\end{bmatrix}$$




which is an analytic continuation of a GO field to a GB field.

# Astigmatic Gaussian Beam Field
(CONTD.)

- Alternatively,

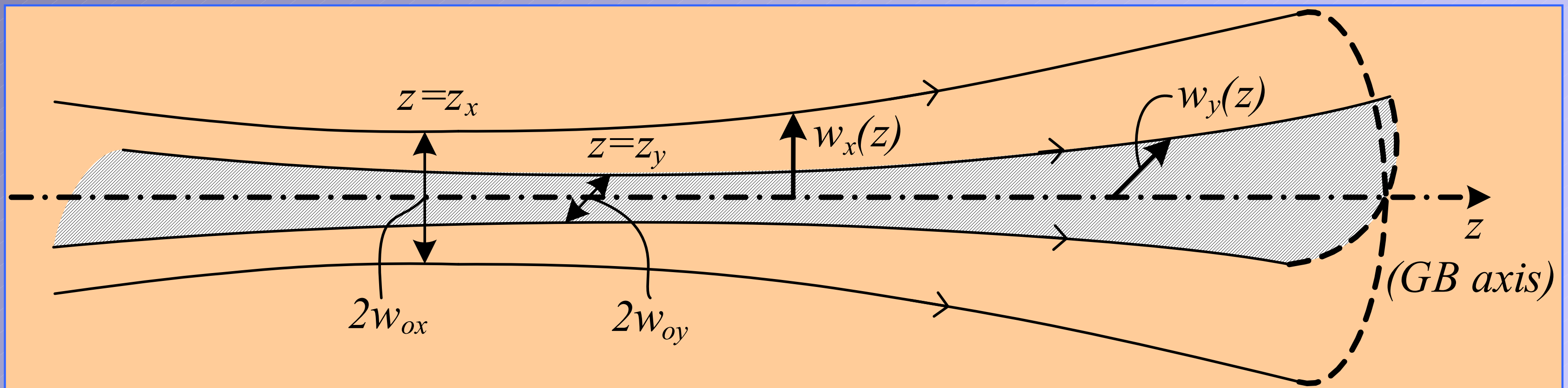
$$\boxed{\overline{E} = \hat{u}\sqrt{\frac{w_{ox}\cdot w_{oy}}{w_x(z)\cdot w_y(z)}}\,e^{-j[kz-\zeta(z)]}e^{-j\frac{kx^2}{2q_x}}e^{-j\frac{ky^2}{2q_y}}}$$

with

$$\zeta(z) \equiv \frac{1}{2}\tan^{-1}\frac{2(z-z_x)}{kw_{ox}^2} + \frac{1}{2}\tan^{-1}\frac{2(z-z_y)}{kw_{oy}^2}$$

and

$$\frac{jk}{2q_x} \equiv \frac{1}{w_x^2(z)} + \frac{jk}{2R_x(z)} \qquad \frac{jk}{2q_y} \equiv \frac{1}{w_y^2(z)} + \frac{jk}{2R_y(z)}$$

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# A Complex Source Point (CSP) can generate a GB field

- Keller and Deschamps independently showed that a CSP generates a GB field
  - J. B. Keller and W. Streifer, "Complex Rays with an Application to Gaussian Beam, " J. Opt. Soc. Amer., Vol. 61, pp. 40-43, 1971.
  - G. A. Deschamps, "The Guassian beam as a bundle of complex rays," Electronics Letters, 1971.
- Felsen has written extensively about the propagation and diffraction of GBs via the CSP. One of his earliest but most comprehensive papers on CSP model for GB is
  - L. B. Felsen, "Complex Source Point Solution of the Field Equations and Their Relation to the Propagation and Scattering of Gaussian Beams, " Symposia Mathematica, Vol. 18, pp. 39-56, 1975.

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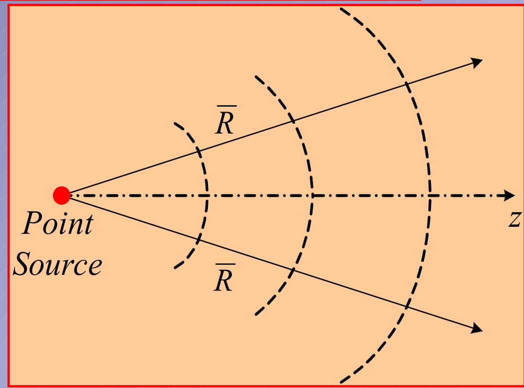
# How to get GB from CSP Field?

- Real Source Point (RSP) case

$$\overline{E} = \hat{u}E_o \frac{e^{-jkR}}{R}$$




- Complex Source Point (CSP) case

$$\overline{E} = \hat{u}E_o \frac{e^{-jk\tilde{R}}}{\tilde{R}}$$

(Beam Field) with $(\tilde{x}', \tilde{y}', \tilde{z}') = (0,0,-jb)\,,\ b > 0$

In paraxial region, CSP field ⇒ <u>rotationally symmetric</u> GB Field

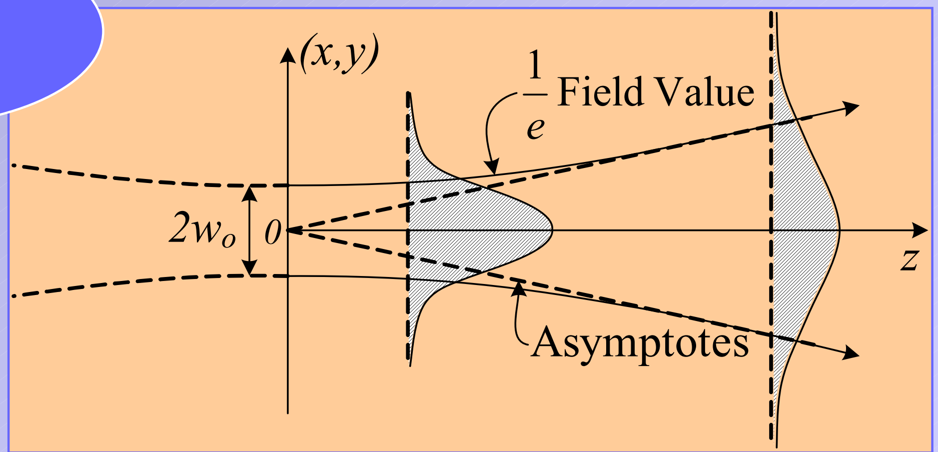
$$\tilde{R} = \sqrt{x^2 + y^2 + (z+jb)^2} \Rightarrow \tilde{R} \approx z + jb + \frac{x^2+y^2}{2(z+jb)}$$

Paraxial Region

$$|z+jb|^2 >> x^2 + y^2$$

GB Field

$$\overline{E} = \hat{u}E_o \frac{e^{kb}}{z+jb} e^{-jkz\left[1+\frac{1}{2}\frac{(x^2+y^2)}{(z^2+b^2)}\right]} e^{\frac{-kb}{2}\frac{(x^2+y^2)}{(z^2+b^2)}}$$

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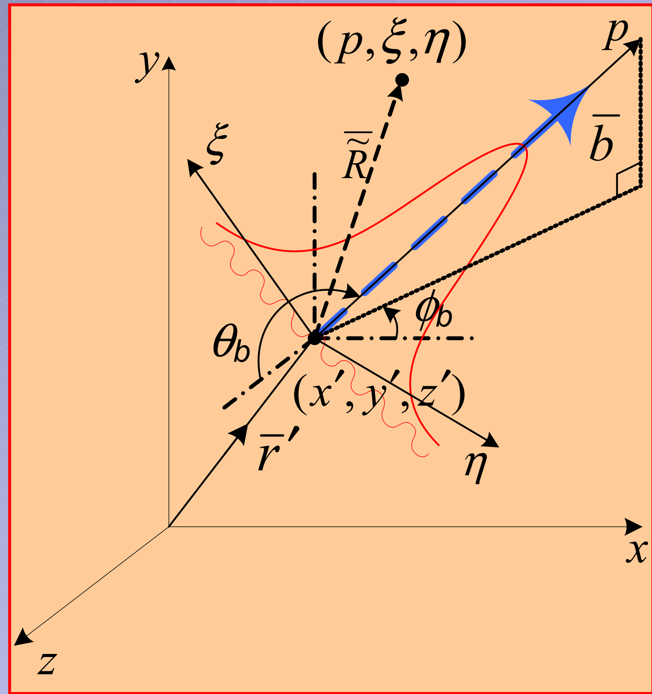
# CSP for Arbitrary GB Direction

- Complex source point location

$$\overline{E} = \hat{u}E_o \frac{e^{kb}}{p+jb} e^{-jkp\left[1+\frac{1}{2}\frac{(\xi^2+\eta^2)}{(p^2+b^2)}\right]} e^{\frac{-kb}{2}\frac{(\xi^2+\eta^2)}{(p^2+b^2)}}$$




$$\tilde{R} = \sqrt{\xi^2+\eta^2+(p+jb)^2} \quad \text{with} \quad (\tilde{x}',\tilde{y}',\tilde{z}') = (0,0,-jb)$$

$$\tilde{\overline{R}} = \left|\overline{r} - \tilde{\overline{r}}'\right| \qquad \tilde{\overline{r}}' = \overline{r}' - j\overline{b},\ b > 0$$

$$\overline{b} = b(\sin\theta_b\cos\phi_b\hat{x} + \sin\theta_b\sin\phi_b\hat{y} + \cos\theta_b\hat{z});\ b > 0$$

- $\tilde{R}$ has branch point singularities when $\tilde{R} = 0$

$$\boxed{\mathrm{Re}\{\tilde{R}\} \geq 0} \Rightarrow \text{Radiation Condition}$$

$$\tilde{R} = \sqrt{(x-\tilde{x}')^2+(y-\tilde{y}')^2+(z-\tilde{z}'^2)}$$

# CSP-GB Reflection and Diffraction

- Some previous works:
  - J. W. Ra, H. L. Berton, and L. B. Felsen, “Reflection and Transmission of Beams at a Dielectric Interface,” SIAM J., May 1973.
  - F. J. V. Hasselmann, and L. B. Felsen, “ Asymptotic Analysis of Parabolic Reflector Antennas,” IEEE Trans. AP, July 1982.
  - A. C. Green, H. L. Bertoni, and L. B. Felsen, “Properties of the Shadow Cast by a Half-Screen When Illuminated by a Gaussian Beam,” J. Soc. Amer., Vol. 69, Nov. 1979.
  - E. Heyman and R. Ianconescu, “Pulsed Beam Diffraction by a Perfectly Conducting Wedge: Local Scattering Models,” IEEE Trans. AP, May 1995.
  - G. A. Suedan, and E. V. Jull, “Three-dimensional Scalar Beam Diffraction by a Half Plane,” Comp. Phys. Comm., vol. 68, 1991.
  - H. D. Cheung and E. V. Jull, “Beam Scattering by a Right-Angled Impedance Wedge, ” IEEE Trans. AP, vol. 52, Feb 2004.
  - J. J. Maciel, and L. B. Felsen, “Systematic Study of Fields due to Extended Apertures by Gaussian Beam Discretization,” IEEE Trans. AP, vol.37, July 1989.

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