Multiple Antennas

—

Capacity and Basic Transmission Schemes

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Presentation Outline

- Channel capacity — Some fine details and misconceptions
- Basic transmission schemes
  - Single link without CSI@Tx
  - Single link with CSI@Tx
  - Multiple users
- Later talks (not included here!):
  - Space-Time block coding
  - Exploiting partial channel information

(CSI = "Channel State Information")

Channel Capacity

Definition: The highest data rate where the bit error rate can $\to 0$ when the code word length $\to \infty$.

Theorem: Under certain conditions, channel capacity = maximum mutual information between transmitter and receiver.

Theorem: Under certain conditions, the mutual information is maximized when the code words are Gaussian distributed.

Note: Channel statistics are always assumed known (also at the transmitter).

MIMO Capacity, AWGN

Channel assumption: $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$,
$\mathbf{H}$ deterministic fixed, $\mathbf{n} \in CN(0, \mathbf{R}_n)$.

Power constraint: $\mathrm{E}[\|\mathbf{x}\|^2] = \mathrm{Tr}[\mathbf{R}_x] \leq P_{\max}$

Capacity:

$$
\begin{aligned}
C &= \max_{\mathrm{Tr}[\mathbf{R}_x] \leq P_{\max}} \log\det|\mathbf{R}_n + \mathbf{H}\mathbf{R}_x\mathbf{H}^*| - \log\det|\mathbf{R}_n| \\
&= \max_{\mathrm{Tr}[\mathbf{R}_x] \leq P_{\max}} \log\det|\mathbf{I} + \mathbf{H}\mathbf{R}_x\mathbf{H}^*\mathbf{R}_n^{-1}|
\end{aligned}
$$

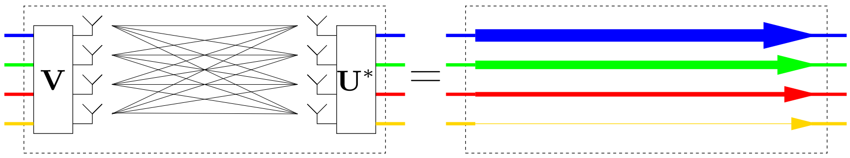
Note: Maximum easily found using "water filling"

Note: It does not make sense to talk about "capacity of an AWGN with no CSI@Tx"!

MIMO Capacity, AWGN, Interpretation

Assume: $\mathbf{R}_n = \sigma_n^2 \mathbf{I}$

Singular Value Decomposition: $\mathbf{H} = \mathbf{USV}^*$



Equivalent channel: $\tilde{\mathbf{y}} = \mathbf{U}^* \mathbf{y} = \mathbf{S}\underbrace{\mathbf{Vx}}_{\tilde{\mathbf{x}}} + \underbrace{\mathbf{Vn}}_{\tilde{\mathbf{n}}} = \mathbf{S}\tilde{\mathbf{x}} + \tilde{\mathbf{n}} \iff$

$\tilde{y}_k = \sigma_k \tilde{x}_k + \tilde{n}_k$, $k = 1, \ldots, \min\{N_{Tx}, N_{Rx}\}$.

$\boxed{\text{Allocate power and rate over orthogonal scalar channels!}}$

MIMO Ergodic Capacity

Definition: The **ergodic capacity** is the channel capacity if

- Channel is fast fading, ergodic stochastic process.
- $M \to \infty$ channel realizations in each transmitted code word.

Note: The transmitted code words are designed only based on the statistics of the channel!

Theorem:

$$C = \max_{\mathrm{Tr}[\mathbf{R}_x] \le P_{\max}} \mathrm{E}\left\{\log\det|\mathbf{R}_n + \mathbf{H}\mathbf{R}_x\mathbf{H}^*| - \log\det|\mathbf{R}_n|\right\}$$
$$= \max_{\mathrm{Tr}[\mathbf{R}_x] \le P_{\max}} \mathrm{E}\left\{\log\det|\mathbf{I} + \mathbf{H}\mathbf{R}_x\mathbf{H}^*\mathbf{R}_n^{-1}|\right\}$$

Theorem: If $\mathbf{H}$ has i.i.d Rayleigh elements and $\mathbf{R}_n = \sigma_n^2\mathbf{I}$, then

$$C = \mathrm{E}\left\{\log\det\left|\mathbf{I} + \frac{P_{\max}}{\sigma_n^2 N_{Tx}}\mathbf{H}\mathbf{H}^*\right|\right\}$$

MIMO Outage Capacity

Scenario:

- Block fading. Length of each fading block $\to \infty$.
- The transmitted code words are designed optimally for a fixed rate, only based on the statistics of the channel!

Definition: The outage probability is the probability that transmission within a fading block can be done with bit error rate $\to 0$.

Theorem: The outage probability at rate R is given by

$$p_{\text{out}}(R) = \min_{\text{Tr}[\mathbf{R}_x] \le P_{\max}} P\left\{\log\det|\mathbf{I} + \mathbf{H}\mathbf{R}_x\mathbf{H}^*\mathbf{R}_n^{-1}| < R\right\}$$

Theorem: If $\mathbf{H}$ has i.i.d Rayleigh elements and $\mathbf{R}_n = \sigma_n^2\mathbf{I}$, the outage probability at rate R is given by

$$p_{\text{out}}(R) = P\left\{\log\det|\mathbf{I} + \frac{P_{\max}}{\sigma_n^2 N_{Tx}}\mathbf{H}\mathbf{H}^*| < R\right\}$$

MIMO Single Link – Basic Options

- Increased data rate – Spatial Multiplexing
- Increased robustness to fading – Diversity

Trade-off Multiplexing $\leftrightarrow$ diversity

Asymptotic multiplexing gain r: $R = r\log\text{SNR}$, when $\text{SNR} \to \infty$.

Diversity gain d: $p_{out}(\text{SNR}) \approx \text{SNR}^{-d}$, when $\text{SNR} \to \infty$.

Max multiplexing gain: $r_{\max} = \min\{N_{Tx}, N_{Rx}\}$

Max Diversity gain: $d_{\max} = N_{Tx}N_{Rx}$

Fundamental trade-off: Optimal diversity gain at multiplexing gain r: $d(r) = (N_{Tx} - r)(N_{Rx} - r)$

MIMO Single Link – no CSI@Tx

Multiplexing Schemes: Transmit different data streams on different "antennas". Use multi-user receive techniques to separate them. Requires non-linear processing. V-BLAST, D-BLAST, ...

Diversity Schemes: Introduce redundancy over space and time. Alamouti, Space-Time block codes, Space-time trellis codes, ...

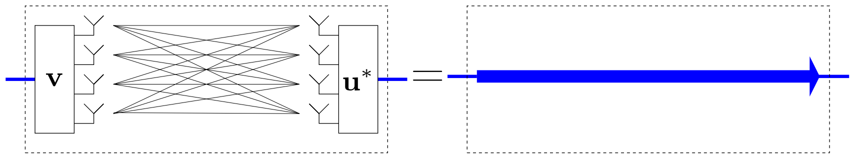
(**Note:** Additional structure from diversity schemes may also be used for interference suppression.)

MIMO Single Link – CSI@Tx

Multiplexing Schemes: Create spatial subchannels, transmit different data streams over the subchannels. Only linear processing required.

Diversity: Obtained by adapting the transmitted signal to the channel. Beamforming, ...

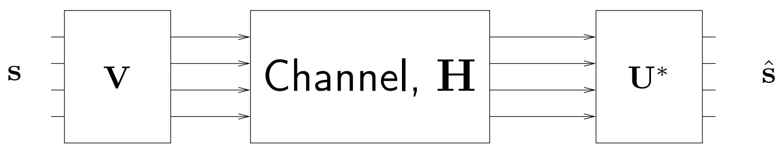
Single Link CSI@Tx, Beamforming



$$\hat{s}(t) = \mathbf{u}^*(\mathbf{H}\mathbf{v}s(t) + \mathbf{n}(t))$$

- Single data stream!
- Optimal beamformers: left and right hand singular vectors with highest singular value.
- Diversity order $N_{Tx}N_{Rx}$ for i.i.d. Rayleigh fading channel.
- Note: Full channel state information needed at the transmitter (or feedback of Tx beamformer $\mathbf{v}$).
- Note: Same diversity order can be obtained without any CSI@Tx!

Single Link CSI@Tx, Linear Processing



$$\hat{\mathbf{s}} = \mathbf{U}^*(\mathbf{H}\mathbf{V}\mathbf{s} + \mathbf{n})$$

$\mathbf{U}$: Linear transmit processing

$\mathbf{V}$: Linear receive processing

Strategies for $\mathbf{U}$ and $\mathbf{V}$:

Zero-forcing: $\mathbf{U}^*\mathbf{H}\mathbf{V} = \mathbf{I} \Longrightarrow \hat{\mathbf{s}} = \mathbf{s} + \mathbf{U}^*\mathbf{n}$. Risk for noise amplification!

MMSE: $\min_{\mathbf{U},\mathbf{V}} \mathrm{E}\|\hat{\mathbf{s}} - \mathbf{s}\|^2$

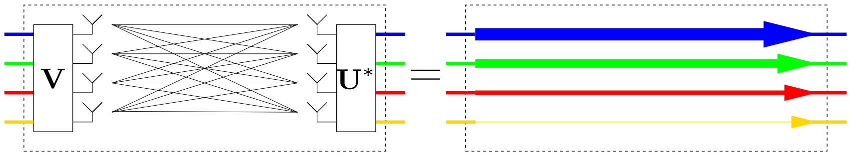
Minimum Bit Error Rate: ...

...

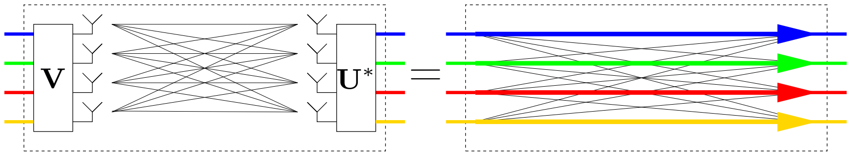
Single Link CSI@Tx, Unified Analysis

Two main transmit solutions!

Shur-Concave Criteria: MMSE, Max mutual information, mean SINR across subchannels, ...



Shur-Convex Criteria: Max MSE across subchannels, mean&max BER across subchannels, ...



MMSE Receiver is always optimum.

Receiver Structures in General

Maximum Likelihood

Approximate Maximum Likelihood, examples:

- Sphere decoding
- Semidefinite relaxations
- Lattice-Reduction-Aided Detectors

Other non-linear techniques, examples:

- Successive interference cancellation
- Parallel interference cancellation
- Iterative "Turbo decoding" techniques

Linear techniques, examples:

- MMSE
- Zero-Forcing
- Maximum Ratio Combining (conventional beamforming)

Different Power Constraints

Total Employed Transmit Power: Most common choice!

$$P_{\text{tot}} = \text{Tr}[\mathbf{V}\mathbf{V}^*] \leq P_{\max}$$

Maximum Element Power:

$$P_{\text{element}} = \max_{l=1,\ldots,m} \|\mathbf{V}_{l,:}\|^2 \leq P_{\max}$$

Equivalent isotropic radiated power (EIRP):

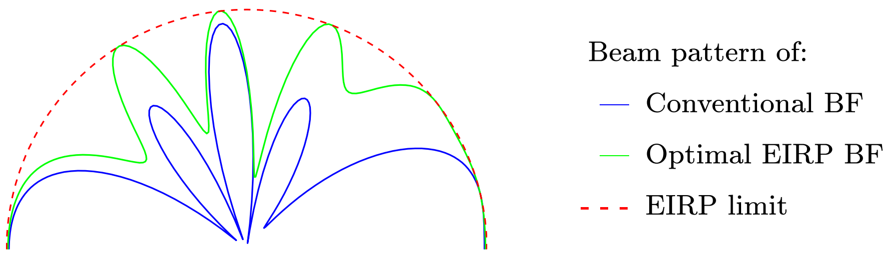
$$EIRP = \max_{\theta} \mathbf{a}(\theta)^*\mathbf{V}\mathbf{V}^*\mathbf{a}(\theta) \leq P_{\max}$$

(where $a(\theta)$ is array response vector in free space)

Note: Different constraints $\Longrightarrow$ different transmit schemes are optimal.

Example, EIRP Beamforming, MIMO

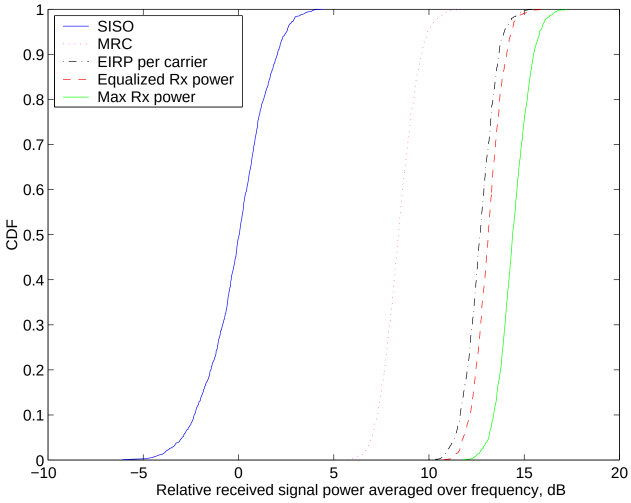
- Use beamforming at transmitter and receiver $\Longrightarrow$ one spatial channel per MIMO link.
- Constraints on Equivalent Isotropic Radiated Power (EIRP) at the transmitter.
- Maximize the received power.



EIRP Beamforming, Properties

- Fixed transmit beamformer $\Longrightarrow$ standard MRC receive beamformer optimum.
- Fixed receive beamformer $\Longrightarrow$ optimal transmit beamformer found using convex optimization.
- Trick: iterate!
- Easily generalized to OFDM.
- Can provide antenna gain of 10-15dB compared to single antenna transmitter.

EIRP Beamforming, Example MIMO+OFDM



Multicarrier 4×4 MIMO beamforming with EIRP constraints.

Exploiting the Spatial Dimension for Spectral Efficiency – System Capacity

- Interference rejection capabilities

  Tighter frequency reuse
- Multiple users sharing same resource (time, frequency, code)

  Spatial division multiple access
- Link capacity can be increased

  TX and RX diversity in MIMO systems

Linear Receivers in Multi-user Systems

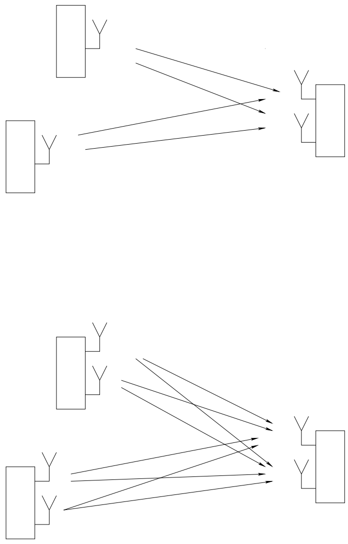
General Strategy:

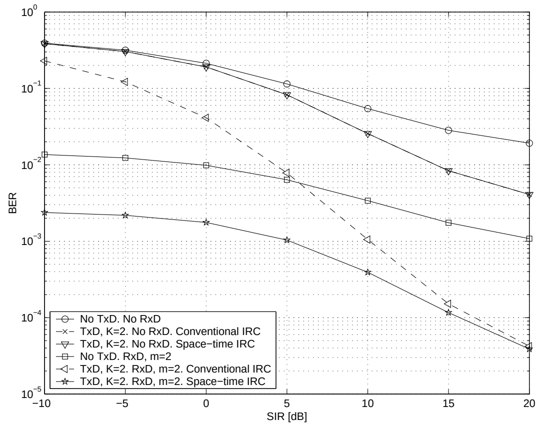
- Model interference plus noise as (spatially and/or temporally) colored noise. Characterized by:
  - Covariance matrix or
  - Vector valued AR-model or ...
- Use linear MMSE receiver.

Exploit structure in desired signal and/or interference!

Example: Alamouti coded interference has structure that can be exploited!

MIMO Multi-User Alamouti Example

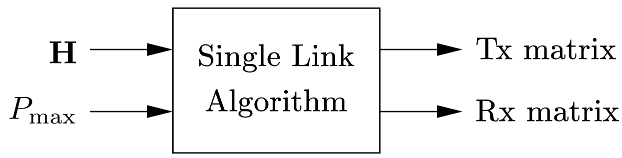




Two users, with and without TX diversity and RX diversity.

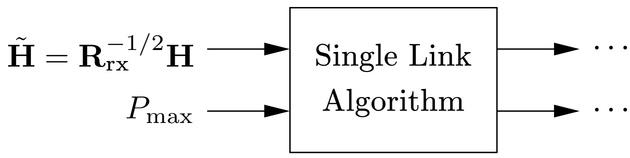
Interference Suppression using Prewhitening

Assume given: Algorithms for a single link in white noise. E.g. optimize MMSE or SINR or mutual information or BER or ...



Interference suppression at the receivers:

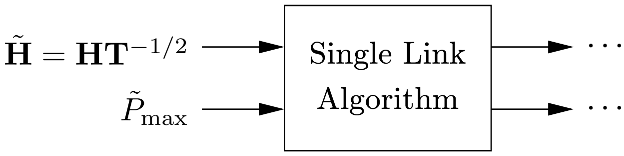
Noise prewhitening at the receivers (well-known).



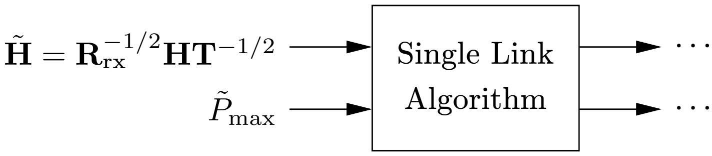
Interference Suppression, cont.

Interference suppression at the transmitters:

"Noise prewhitening" at the transmitters.



Combine:



Iterate! Circular dependence,

$\mathbf{R}_{\text{rx}}$ and $\mathbf{T} \iff$ Tx and Rx parameters.

Solve iteratively for each link! Hope for convergence!

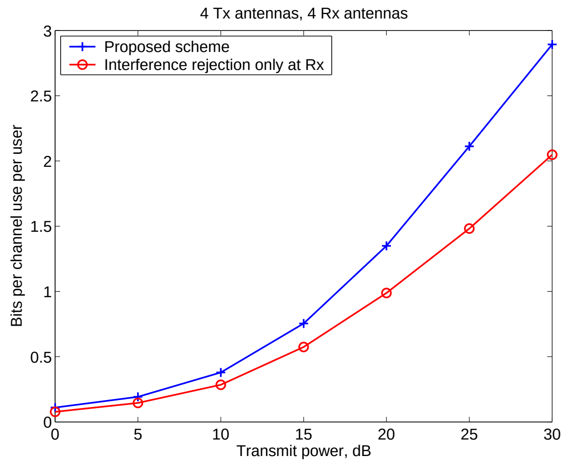
Interference Suppression, cont.

Choice of Transmit Prewhitening Matrix $\mathbf{T}$?

- Theoretical result: Some choice of $\mathbf{T}$ gives optimal system performance.
- Practical proposal, for systems with linear Tx, Rx matrices:
  - Each receiver sends pilot data using its receiver matrix.
  - Each transmitter collects the pilot data and estimates the resulting covariance matrix. Use as $\mathbf{T}$!

Interference Suppression, Example

- Max mutual information per link.
- Narrowband.
- 3GPP suburban channel model, $\pm 120^\circ$ sectors.
- 20 mobiles placed randomly in 7×3 cells.



What Information Theory Says on Multi-user MIMO

- Very few results on general interference channel (multiple cells).
- Special cases where capacity region is known:
  - Multiple Access Channel "MAC" (single cell uplink)
  - Broadcast Channel "BC" (single cell downlink)
- Approaching these limits requires non-linear processing ("dirty-paper" coding, . . .)

Observations for Dual Array Structures

- Capacity is not limited by frequency bandwidth.
- Multiple RX/TX at terminal feasible.
- High performance requires "rich scattering" environment.
- Systems must operate (and be designed for) operation with partial/limited channel knowledge.
- Efficient exploitation of limited channel knowledge essential for efficient designs, intricate problem for multi-user MIMO systems.
- Can trade complexity (power) between access point and terminal.

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References

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