Antenna Arrays in Mobile Communications: Gain, Diversity, and Channel Capacity[1]

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1. Introduction

Are antenna arrays in mobile communications different from arrays in other applications? Yes, sometimes, and it is the purpose of this paper to explain, in a tutorial fashion, when this is the case, and what this means for path loss in link calculations. One thing is the classical gain of an antenna, which we have to understand in a new way. Another thing is the possibility for two arrays, in a scattering environment, to create parallel channels, and thus, in effect, act as many independent antennas at the same time, carrying much more traffic over the same bandwidth [1-3].

Let us review the well-known free-space situation first. Consider two linear arrays of M and N elements, with the assumption that $M > N$. For convenience, it is assumed that the left array of M elements is the transmitting array. The path-loss equation is given by the classical Friis formula,

$$P_r = P_t \frac{G_1 G_2}{(4\pi R)^2} \lambda^2 . \tag{1}$$

Under some qualifying assumptions–like neglect of mutual coupling and element patterns–the two gains are M and N, respectively. A standard spacing of half a wavelength is also assumed. The underlying assumption, here, is that the other antenna looks like a point source as seen from one antenna, and thus a plane wave from one specific direction is radiated (and received). If instead of direct line-of-sight (LOS) there was just one path, like scattering from a dominant scatterer, then the equation would still be valid as far as the antenna gains are concerned. The more usual situation in mobile radio is a wide angular scattering (Figure 1), where the angular spread as seen from the mobile is often large, and where the scattering as seen from the base station depends on the height and the general environment. We have highlighted one path out of many going from one element at the transmitter to one element in the receiver. The distance dependence in Equation (1) is, of course, also changed by the scatterers, but this is not our concern here. The point of view is that the information-carrying signal is scattered in many directions, and the general question is, how should we organize the combining of elements to maximize the power transfer? At each antenna, we assume that we can apply different complex weights at each element, so there is complete freedom at both ends to combine the antenna signals.

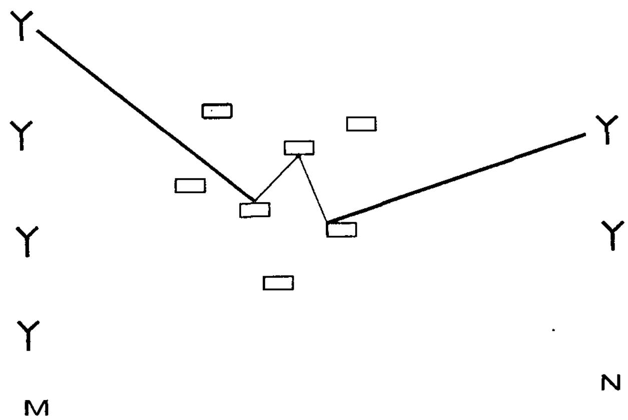

Figure 1. Two linear arrays of M and N elements in a scattering environment.

The notation is $\mathbf{U} = (U_1, U_2, .., U_N)^T$ for the receiver weights, and $\mathbf{V} = (V_1, V_2, .., V_M)^T$ for the transmitter, where T means transpose. Both vectors are normalized to unit length. The narrowband channel connecting all elements may then be described by an M by N complex matrix $\mathbf{H}$, where H_{ij} is the complex transmission coefficient from element j, on the left side, to element i, on the right side.

2. Antenna Gain and Diversity

2.1 Transmitter Weights Fixed

For simplicity, assume that the transmitter weights are fixed, like in a beam mode where all the elements of $\mathbf{V}_0$ are identical in magnitude, with a uniform phase difference. This would be an obvious choice when the transmitter does not "know" the channel.

[1]This article is based on a paper presented at COMMSPHERE 99.

On the receive side, the incident signal is $\mathbf{S} = \mathbf{H} \bullet \mathbf{V}_0$.

As is well known, the receiver maximum-gain weights are

$$\mathbf{U} = \frac{\mathbf{S}^*}{|\mathbf{S}|} = \frac{(\mathbf{H} \bullet \mathbf{V}_0)^*}{|\mathbf{S}|}, \tag{2}$$

and the received power is the sum of the powers from the N elements:

$$P_r = |S_1|^2 + |S_2{}^2| + \cdots + |S_N|^2. \tag{3}$$

For many scatterers and non-LOS, the power at one element will be exponentially distributed (each transmission coefficient is Rayleigh fading), and the distribution of the sum of the powers will depend on the correlation. If the scatterers in Figure 1 shrink to an narrow angular range, then they will appear as a point source, and the fading will be spatially flat. In this case, the signals will be highly correlated:

$$E\{P_r\} = N\, E\{|S_1|^2\}. \tag{4}$$

If we define the array gain as the mean value of the received power relative to one element at each end, then the array gain in this case is clearly N.

In the other extreme, where the scatterers are spread out in all directions, the signals will be uncorrelated but Equation (4) will still be valid, so the array gain is also N in this case. On top of this, we get Nth order diversity gain for the uncorrelated case, where all transmission coefficients are fading independently.

If instead of the maximum gain combining of Equation (1), we had chosen the beam mode for the receiver as well, i.e.

$$\mathbf{U}^T = \frac{(1,1,\cdots,1)}{\sqrt{N}}, \tag{5}$$

then it can be shown that the mean receiver-array gain is 1. This is natural, since a narrow beam does not help when the energy is spread out in all directions. The above may be summarized as in Tables 1a-1c, for the mean link gain and diversity order.

As an example, let us discuss the upper-right corners of Tables 1a-1c, with low angular spread (high correlation) at the transmitter and high angular spread (low correlation) at the receiver. In the beam mode, the transmitter array sees a point source and has gain M, the receiver has gain 1, and the joint link has a gain of M. In the combining mode, at the receiver the mean gain is N (as is the diversity order), so the total gain is MN. The lower-right corners of Tables 1a-1c refer to the joint optimization of the two arrays, which is the subject of the following section.

Table 1a. The mean link gain for the beam mode for both arrays.

	Receiver Low Spread	Receiver High Spread
Transmitter Low Spread	MN	M
Transmitter High Spread	N	1

Table 1b. The mean link gain for maximum-gain combining for both arrays.

	Receiver Low Spread	Receiver High Spread
Transmitter Low Spread	MN	MN
Transmitter High Spread	MN	$\left(\sqrt{M}+\sqrt{N}\right)^2$

Table 1c. The diversity order of the link for maximum-gain combining.

	Receiver Low Spread	Receiver High Spread
Transmitter Low Spread	1	N
Transmitter High Spread	M	MN

2.2 Transmit-Receive Gain for Wide Angular Spreads

The SVD (singular-value decomposition) [4] is an attractive technique for solving the joint optimization of the two sides, the transmit side and the receive side. In the following, it is assumed that the complex matrix (the channel matrix) is known at both the transmitter and receiver. This is not so strange as it sounds, i.e., in a TDD (time-division-duplex) case, where the channel is reciprocal because the frequency is the same in both directions, the channel will be known at the transmitter as well, unless the channel changes too rapidly.

An SVD expansion is a description of $\mathbf{H}$ as given by

$$\mathbf{H} = \mathbf{U} \bullet \mathbf{D} \bullet \mathbf{V}', \tag{6}$$

where $\mathbf{D}$ is a diagonal matrix of real, non-negative singular values, the square roots of the eigenvalues of $\mathbf{G} = \mathbf{H}'\mathbf{H}$, a Hermitian matrix. The columns of the orthogonal matrices $\mathbf{U}$ and $\mathbf{V}$ are the corresponding singular vectors. Since $\mathbf{G}$ may be written as

$$\mathbf{G} = \mathbf{H}' \bullet \mathbf{H} = \mathbf{V} \bullet \mathbf{D}' \bullet \mathbf{D} \bullet \mathbf{V}', \tag{7}$$

it follows that the columns of $\mathbf{V}$ are eigenvectors of $\mathbf{G}$.

The SVD is particularly useful for interpretation in the antenna context. Writing Equation (7) differently,

$$\mathbf{H} \bullet \mathbf{V}_1 = \sqrt{\lambda_1}\, \mathbf{U}_1 \tag{8}$$

for one particular eigenvalue, it is noted that $\mathbf{V}_1$ is the transmit weight factor for excitation of the singular value $\sqrt{\lambda_1}$. A receive

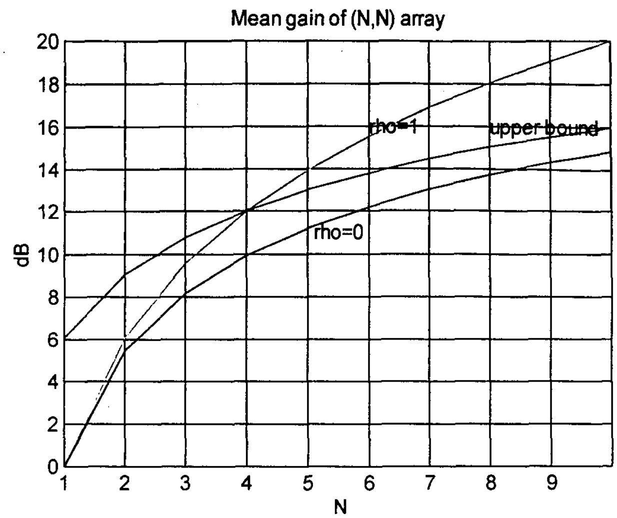

Figure 2. The gain relative to one element of (N,N) arrays in a correlated situation ($\rho=0$), and in an uncorrelated case ($\rho=1$). The upper bound equals $4N$, and is the asymptotic upper bound for the gain for N tending to infinity.

weight factor of $\mathbf{U}_1'$, a conjugate match, gives the receive voltage, and the square of that gives the received power:

$$V_1 = \mathbf{U}_1' \bullet \mathbf{U}_1 \sqrt{\lambda_1},$$
$$P_r = |V_1|^2 = \lambda_1, \tag{9}$$

Thus, the eigenvalues correspond to the power gains, and all we need to do is to extract the largest eigenvalue with corresponding $\mathbf{V}$ and $\mathbf{U}$ vectors, and maximum gain is achieved for that particular channel matrix.

Recent results [5], concerning the distribution of the eigenvalues of a random Hermitian matrix, can give some insight into the maximum gain and how it varies with M and N. In the asymptotic limit when M and N are large, it may be shown that the largest eigenvalue is bounded above by

$$G_{\text{joint}} = \lambda_{\max} < \left(\sqrt{M} + \sqrt{N}\right)^2. \tag{10}$$

This indicates that the joint link gain can no longer be separated into a transmitter-antenna gain and a receiver-antenna gain. For $M = N$, the gain equals $4N$, which is much less than the N^2 available when the spreading is small. As an explanation, compare the (M,N) case with the $(1,N)$ case. In the latter case, we have N degrees of freedom (elements of the weight vector) and N different signals, which matches. In the former case, we have $M+N$ degrees of freedom but MN different signals, a clear mismatch for large M and N, and we must accept a reduced gain. Although the gain is reduced, it is still greater than M and N, and the diversity order is truly large, namely MN. This follows from the fact that there are MN different signals, and the probability of having all paths fading at the same time is vanishing small. Thus, the fading has practically disappeared for reasonable values of M and N.

The mean gains for $\rho = 0$ (uncorrelated signals) are shown in Figure 2, together with the upper bound and the gain for the correlated, free-space case, N^2. For $N = 10$, the true mean gain is just 1 dB below the upper bound. Thus, the price to pay for the random scattering is a diminishment of the gain from N^2 to $4N$ for N large. For a partly correlated case, we can expect the gain to lie between the $\rho = 0$ and the $\rho = 1$ cases.

2.3 Implications for Path Loss

Equation (1) implies that the received power decreases as the square of the carrier frequency for frequency-independent gains like dipole, or other small handset, antennas. If instead the antenna apertures are introduced in the free-space case (or in the case of small angular spreads), the situation reverses, as is well known. Let $A_1 = M\dfrac{\lambda^2}{4\pi}$, $A_2 = N\dfrac{\lambda^2}{4\pi}$; then, the path-loss equation for the correlated case reads

$$P_r = P_t \frac{A_1 A_2}{\lambda^2 R^2}, \tag{11}$$

and for the uncorrelated case,

$$P_r = P_t \frac{\left(\sqrt{A_1} + \sqrt{A_2}\right)^2}{4\pi R^2}. \tag{12}$$

This has the interesting result that the frequency dependence has disappeared for the uncorrelated case. The assumptions behind Equation (12) are the same as in Equation (10).

The equations imply that we can only gain from going to higher carrier frequencies for given areas of antenna arrays, when atmospheric absorption and diffraction are ignored. When the spreading is small, the joint gain may be very high, and when the spreading is large, the worst situation is a constant power: it does not decrease as in Equation (1). The true benefit would be that there is more bandwidth available at the higher microwave frequencies.

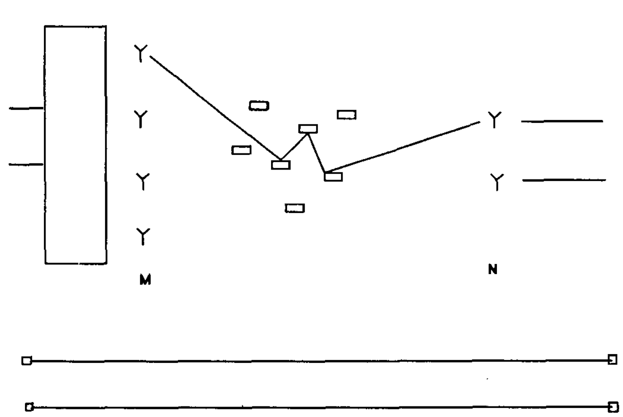

Figure 3. N different signals are distributed over the M antennas, with the resulting N parallel channels.

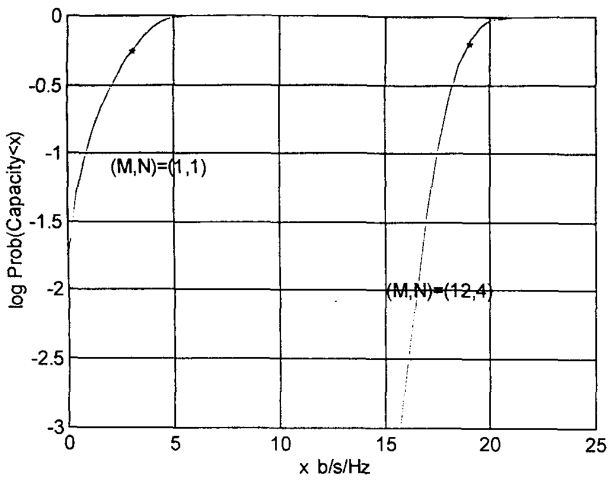

Figure 4. The cumulative distribution of capacity for a single channel with $\mathrm{SNR} = 10\,\mathrm{dB}$, and for an optimized four-channel case with 12 transmit antennas. The mean values are indicated with an asterisk.

3. Spectral Efficiency of Parallel Channels

The joint gain of the link corresponded to the largest eigenvalue of **G**. The first question is, how many eigenvalues are there? For the completely correlated case, there is only one, but for the uncorrelated case, there are $\min(M,N)$ distinct eigenvalues with corresponding pairs of **V** and **U** vectors. Since we have been assuming $M > N$, we have N eigenvalues. This tells us that it is possible to send multiple sets of data over the same physical channel for the same bandwidth, since the weight vectors corresponding to separate eigenvalues are orthogonal [1-3, 6-7]. In this way, the spectral efficiency can be greatly increased. Physically, the many different paths in the environment create the possibility of multiple channels, as illustrated in Figure 3: All we have to do is to diagonalize the channel matrix, as was done in the previous section.

The situation is easily described by Shannon's information measure:

$$C = \log_2(1 + P/\sigma) \text{ b/s/Hz}, \qquad (13)$$

where P/σ is the signal-to-noise ratio, SNR, for one channel. For N parallel channels, the capacities add:

$$C = \sum_{1}^{N} \log_2 \left(1 + \lambda_i P_i/\sigma\right), \qquad (14)$$

where P_i is the power put into channel i, and λ_i is the gain of that channel. The total power, P, as the sum of all the separate powers, is assumed constant, to make comparisons fair. The problem of assigning powers to the individual channels may be solved by the "water-filling scheme" [8], which makes sure that the channels with the highest gains get most of the power. If P is very small, only the largest gain gets any power, and we are in the situation of the one-eigenvalue case of the previous section.

In accordance with the simplifying assumptions of large N and M, it may be shown that all the eigenvalues are bounded by

$$\left(\sqrt{M} - \sqrt{N}\right)^2 < \lambda_i < \left(\sqrt{M} + \sqrt{N}\right)^2. \qquad (15)$$

Thus if M is larger than N, the eigenvalues tend to cluster around M: they will approximately be allocated the same power, and Equation (14) reduces to

$$C = N \log_2\left(1 + MP/N\sigma\right). \qquad (16)$$

An illustrative case is shown in Figure 4, for a basic mean SNR of 10 dB for one antenna. The spectral efficiency has increased in the mean from 2.9 to 18.8 b/s/Hz, when using 12 transmit antennas and four receive antennas, compared with one antenna at each end: an impressive improvement. Also, in this case, the diversity effect is important: the (12,4) case shows an almost constant capacity. The approximate upper bound of Equation (15) gives a value of 19.6 b/s/Hz.

4. Discussion

The seemingly complicated, random environment in mobile and personal communications gives rise to some new possibilities in the antenna area. The received fields come from many different directions, and for a noise-limited system, it is important to absorb all the energy. For those situations where the channel transfers from all elements to all elements are known, it is possible to maximize the total transfer power by jointly adjusting the antenna weights in the arrays. The resulting joint antenna gain depends on the angular spread of the environment as seen from the two antennas: In one extreme of high correlation, we get the usual free-space gain, while in the other extreme, we get a smaller gain, due to the lack of degrees of freedom. If we introduce antenna apertures in the link budget instead of directivities, it is interesting to observe that we obtain a link gain that is independent of carrier frequency. This seems to indicate that it should be worthwhile to go to higher frequencies to get the higher bandwidths available.

The other possibility in a wide-scattering situation is to apply different information signals to the various antennas, and to utilize the inherently high spectral efficiencies of the channel. This may be done by effectively creating a number of parallel orthogonal channels, which all have high gain and high-order diversity, especially when the number of transmit antennas is higher than the number of receive antennas.

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