ACE — Metamorphose NoEs
SHORT COURSE
Artificial EBG surfaces and metamaterials

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Main topics today

- Some general properties of metamaterials
- Artificial magnetics
- New high-impedance surfaces
- Antennas in metamaterial shells
- Antenna miniaturization
- Near-field control

Energy density

Considering metamaterials with negligible losses (in some frequency ranges):

$$w = \frac{1}{2}\left.\frac{d(\omega\epsilon(\omega))}{d\omega}\right|_{\omega=\omega_0}|E|^2 + \frac{1}{2}\left.\frac{d(\omega\mu(\omega))}{d\omega}\right|_{\omega=\omega_0}|H|^2$$

Assume that ϵ and μ are independent from the frequency (near ω_0):

$$w = \frac{1}{2}\epsilon(\omega_0)|E|^2 + \frac{1}{2}\mu(\omega_0)|H|^2$$

But $w > 0$ in passive media!

Conclusion: It is not possible to neglect dispersion if the material parameters are negative.

Limitations on material parameters

Again for low-loss materials:

$$\frac{d\epsilon(\omega)}{d\omega} > 0, \qquad \frac{d\epsilon(\omega)}{d\omega} > \frac{2(\epsilon_0 - \epsilon)}{\omega}$$

From here:

$$\frac{d(\omega\epsilon(\omega))}{d\omega} > \epsilon_0, \qquad \frac{d(\omega\mu(\omega))}{d\omega} > \mu_0$$

Also,

$$\frac{d(\omega\epsilon(\omega))}{d\omega} > 2\epsilon_0 - \epsilon(\omega)$$

On modelling of magnetics

$$\mu = \mu_0\left(1 + \frac{A\omega^2}{\omega_0^2 - \omega^2}\right)$$

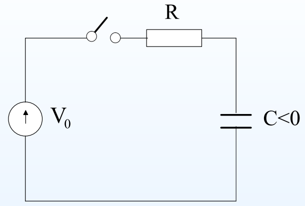
OK at low frequencies [$\mu(\omega) = O(\omega^2)$] and near the resonance, but non-physical at high frequencies!

Condition

$$\frac{d(\omega\mu(\omega))}{d\omega} > \mu_0$$

breaks down at $\omega > \sqrt{3}\omega_0$.

On the non-dispersive model



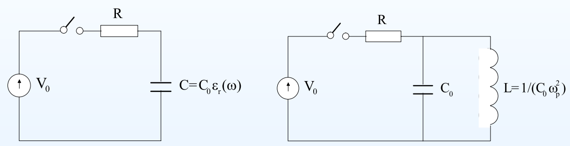
Connect $V = V_0\sin(\omega t)$ at time $t = 0$:

$$i(t) = V_0\omega C\frac{\cos(\omega t) + \omega RC\sin(\omega t) - e^{-\frac{t}{RC}}}{1 + \omega^2R^2C^2}$$

Exponentially grows if $C < 0$!

What really happens?

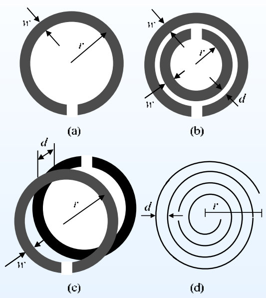
$$\epsilon(\omega) = \epsilon_0\epsilon_r = \epsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right)$$



$$Z = \frac{1}{j\omega C} = \frac{1}{j\omega C_0\left(1 - \frac{\omega_p^2}{\omega^2}\right)}$$

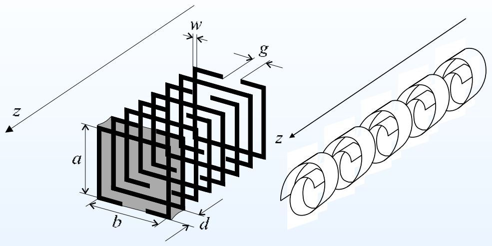
No instabilities! And no exotic response!

Artificial magnetic materials



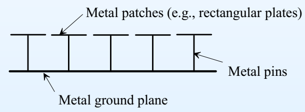
(a) Single split-ring resonator. (b) Split-ring resonator (SRR). (c) Modified split-ring resonator (MSRR) (d) Cross section of the Swiss roll.

Metasolenoid

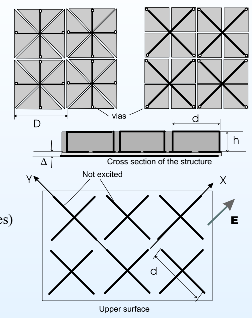


Two examples of alternative geometries of the metasolenoid.

Artificial impedance surfaces



“Mushroom layer”

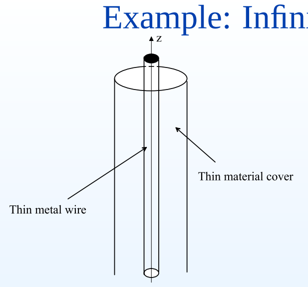


New HIS — artificial magnetic layer

On metamaterial coverings

Question: How thin (meta)material coverings influence antenna performance? (Is it possible to reduce the size and increase the bandwidth?)

Example: Infinite current line



Thin infinite metal cylinder, radius r_0, with a given time-harmonic current I.
The source is covered by a material cylinder of radius $a \ll \lambda$ ($r_0 \ll a$).

Electric and magnetic fields

The electric field:

$$\begin{cases} E_z = AH_0^{(2)}(kr) + BH_0^{(1)}(kr), & r_0 \le r \le a, \\ E_z = CH_0^{(2)}(k_0 r), & r \ge a \end{cases}$$

The magnetic field:

$$H_\varphi = -\frac{j}{\omega\mu}\frac{\partial E_z}{\partial r} = \begin{cases} \frac{jk}{\omega\mu}\left(AH_1^{(2)}(kr) + BH_1^{(1)}(kr)\right), & r_0 \le r \le a \\ \frac{jk_0}{\omega\mu_0} CH_1^{(2)}(k_0 r), & r \ge a \end{cases}$$

Boundary conditions

Continuity conditions for E_z and H_φ:

$$AH_0^{(2)}(ka) + BH_0^{(1)}(ka) = CH_0^{(2)}(k_0 a)$$
$$\frac{\mu_0 k}{\mu k_0}\left(AH_1^{(2)}(ka) + BH_1^{(1)}(ka)\right) = CH_1^{(2)}(k_0 a)$$

Relation between the wire current I and the magnetic field H_φ at the wire surface $r = r_0$: $2\pi r_0 H_\varphi = I$, thus

$$\frac{2\pi j\, kr_0}{\omega\mu}\left(AH_1^{(2)}(kr_0) + BH_1^{(1)}(kr_0)\right) = I$$

Assumptions and solution

Assumptions: $k_0 a \ll 1$, $|k|a \ll 1$, $r_0 \ll a \Rightarrow$ the solution is

$$A = -\frac{\omega I}{8}\left\{\mu_0 + \mu + \frac{2j}{\pi}\left[\mu \log\frac{\gamma k a}{2} - \mu_0 \log\frac{\gamma k_0 a}{2}\right]\right\}$$
$$B = -\frac{\omega I}{8}\left\{\mu_0 - \mu + \frac{2j}{\pi}\left[\mu \log\frac{\gamma k a}{2} - \mu_0 \log\frac{\gamma k_0 a}{2}\right]\right\}$$
$$C = -\frac{\omega \mu_0 I}{4}$$

($\gamma \approx 1.781$ is the Euler constant)

Impedance per unit length

The quality factor we define as usually:

$$Q = \frac{\omega W}{P}$$

where W is the average reactive energy stored in the resonator and P is the total dissipated power.

To evaluate this, we introduce the wire impedance per unit length

$$Z = -\frac{E_z(r_0)}{I}$$

$$Z = \frac{\omega\mu_0}{4} + j\frac{\omega}{2\pi}\left[\mu_0 \log\frac{4}{\gamma k_0 r_0} + (\mu - \mu_0)\log\frac{a}{r_0}\right]$$

Lossless covering material

Consider exponentially growing (or decaying) amplitude of harmonic oscillations. Complex frequency $\Omega = \omega - j\alpha$, $\alpha \ll \omega$:

$$\frac{\partial W}{\partial t} \approx 2\alpha W$$

$$Z(\Omega) = R(\Omega) + jX(\Omega) \approx R(\omega) + \alpha\frac{\partial X(\omega)}{\partial \omega} + j\left[X(\omega) - \alpha\frac{\partial R(\omega)}{\partial \omega}\right]$$

On the other hand,

$$\frac{\partial W}{\partial t} = (\mathrm{Re}[Z(\Omega)] - R_{\mathrm{rad}})\,|I|^2 = \alpha\frac{\partial X(\omega)}{\partial \omega}|I|^2$$

Antenna quality factor

$$W = \frac{1}{2}\frac{\partial X(\omega)}{\partial \omega}|I|^2$$

$$Q = \frac{\omega W}{P} = \frac{\omega}{2R_{\mathrm{rad}}}\frac{\partial X(\omega)}{\partial \omega}$$

$$Q = \frac{1}{\pi}\left\{\frac{\partial(\omega\mu_r)}{\partial \omega}\log\frac{a}{r_0} + \log\frac{4}{\gamma k_0 a} - 1\right\}$$

But we know that

$$\frac{\partial(\omega\mu_r)}{\partial \omega} \ge 1$$

!

Lossy and dispersive covers

The quality factor can be found *only* if we know the internal structure of the system.

Assume a magnetic modelled by

$$\mu = \mu_0 \left(1 + \frac{A\omega^2}{\omega_0^2 - \omega^2 + j\omega\Gamma}\right)$$

Then

$$Z = Z_{\rm wire} + Z_{\rm medium}$$

$$Z_{\rm wire} = \frac{\omega\mu_0}{4} + j\frac{\omega\mu_0}{2\pi}\log\frac{4}{\gamma k_0 r_0}, \qquad Z_{\rm medium} = j\frac{\omega}{2\pi}(\mu - \mu_0)\log\frac{a}{r_0}$$

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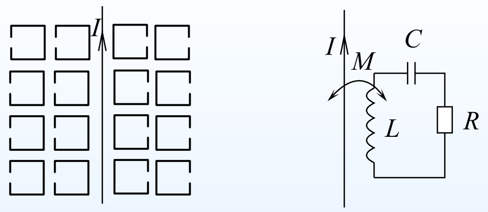
Equivalent circuit



$$Z_{\rm material} = \frac{j\omega^3(M^2/L)}{\omega_0^2 - \omega^2 + j\omega(R/L)} \qquad \text{with} \quad \omega_0 = 1/\sqrt{LC}$$

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Stored energy and loss power

$$W_{\rm medium} = L\frac{|I_L|^2}{2} + C\frac{|U_C|^2}{2} = \frac{\omega^2 M^2 C(1+\omega^2 LC)}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}\frac{|I|^2}{2}$$

At the resonant frequency

$$W_{\rm medium}(\omega_0) = \frac{M^2|I|^2}{R^2 C}$$

$$P_{\rm loss} = R|I_L|^2 = \frac{\omega^4 M^2 C^2 R|I|^2}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}$$

$$P_{\rm loss}(\omega_0) = \frac{\omega_0^2 M^2 |I|^2}{R}$$

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Efficiency and quality factor

$$\eta = \frac{P_{\rm rad}}{P_{\rm rad} + P_{\rm loss}} = \frac{1}{1 + \dfrac{4\omega_0 M^2}{\mu_0 R}}$$

$$Q_{\rm total} = \frac{\omega(W_{\rm wire} + W_{\rm medium})}{P_{\rm rad} + P_{\rm loss}}$$

$$= \frac{1}{1 + \dfrac{4\omega_0 M^2}{\mu_0 R}}\left[\frac{1}{\pi}\left(\log\frac{4}{\gamma k_0 r_0} - 1\right) + \frac{4M^2}{\mu_0 R^2 C}\right]$$

$$Q_{\rm total} = \frac{1}{1 + \dfrac{2\omega_0 A}{\pi\Gamma}\log\dfrac{a}{r_0}}\left[\frac{1}{\pi}\left(\log\frac{4}{\gamma k_0 r_0} - 1\right) + \frac{2\omega_0^2 A}{\pi\Gamma^2}\log\frac{a}{r_0}\right]$$

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Limiting cases

1. Small losses ($R \to 0$):

$$Q_{\rm total} = \frac{1}{\omega_0 RC}$$

(most of the energy is stored in the medium layer and almost all the power goes into heat)

2. Fixed efficiency (R), the resonant frequency and the coupling. To decrease $Q_{\rm total}$ we increase $C \to \infty$. This means $L \to 0$. The total impedance of a medium particle is simply resistive:

$$Z_{\rm medium} = \frac{\omega_0^2 M^2}{R}, \qquad \text{and} \quad Q_{\rm total} = \eta Q_{\rm wire}$$

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Discussion I

No passive media in thermodynamic equilibrium can store negative reactive energy

Field instabilities take place in non-equilibrium systems that can be described by negative energy density

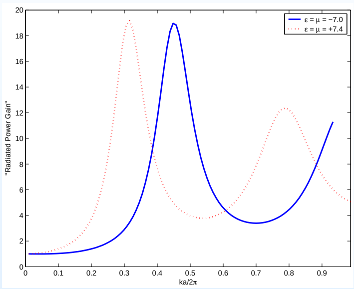
Discussion II

The use of (low-loss) Veselago materials does not allow to realize *negative capacitance* or *negative inductance*.

Such "negative capacitor" made of a wire medium is equivalent to a parallel connection of a normal capacitor and a normal inductor.

Discussion III

Veselago material shells around an antenna "increase radiated power" (R. Ziolkowski) as well as shells of usual materials



What can work?

- *Radiating* inclusions, instead of *material* coverings. Increase of the stored reactive energy can be overcompensated by an increase of the radiated power.
- Shells of resonant dimensions. New materials can offer more possibilities in optimizing resonant antennas.
- *Non-uniform* coverings or material inclusions. This can modify the current distribution, possibly leading to increased bandwidth.
- *Active* materials. If the passivity requirement is dropped, it is in principle possible to overcome the limitation

$$\frac{\partial(\omega\mu_r)}{\partial\omega} \geq 1$$

without introducing heavy losses. The bandwidth can be very large for very small antennas.

Antenna miniaturization

For resonant antennas like microstrip antennas and PIFAs, the main idea is to fill them with a magnetic:

$$F_r \sim \frac{1}{\sqrt{\varepsilon_r\mu_r}}, \qquad BW \sim \frac{\sqrt{\mu_r}}{\sqrt{\varepsilon_r}}$$

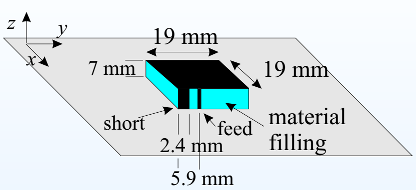
Figure of merit?

$$Q > \frac{1}{(k_0a)^3} \qquad (k_0 a \ll 1)$$

a) QV should be minimized

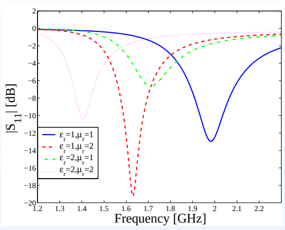
or b) QF^3_{res} should be minimized

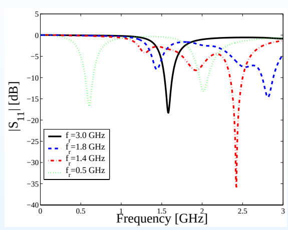
Numerical model



$$\mu(\omega) = \mu_0\left(1 + \frac{\Delta\mu\omega_{0m}^2}{\omega_{0m}^2 - \omega^2 + j\delta_m\omega}\right)$$

Numerical results



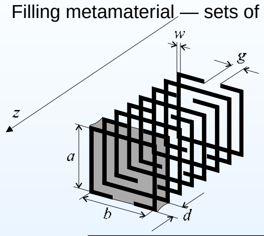


Non-dispersive filling

Dispersive isotropic material filling

Experiments: Filled PIFA

Filling metamaterial — sets of metasolenoids

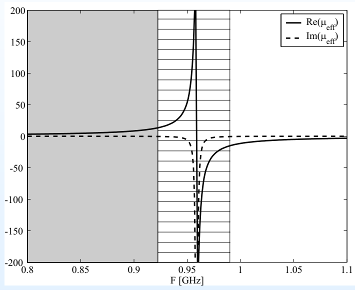


a	b	w	g	d	ϵ_r
mm	mm	mm	mm	mm	
3.5	3.5	0.4	1.0	0.127	2.20 - j0.002 †

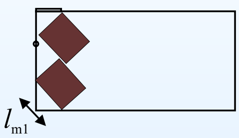
† ϵ_r is the permittivity of the host substrate.

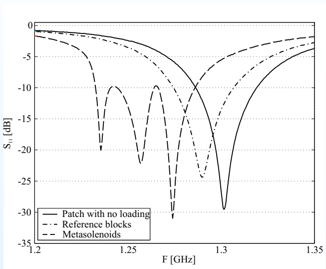
Experiments: Filled PIFA

Two possible regimes: Resonant inclusions or non-resonant magnetic filling

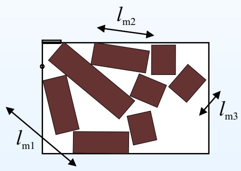


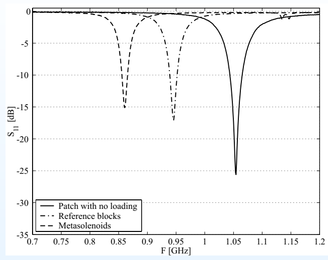
Resonant inclusions





Metamaterial filling





Measured parameters

Resonant inclusions

Loading	F_{res}	$BW\vert_{-6dB}$	$BW\vert_{-9.5dB}$	η	Q_0	$Q_0F^3_{res}$
	GHz	percent	percent	percent		
Air	1.30	4.5	2.7	94	27.2	59.8
Reference blocks	1.29	4.4	2.6	93	27.6	59.3
Metasolenoids	1.26	5.4	4.3	90, 90, 88 †	22.1	45.8

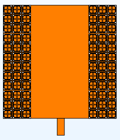
Measured parameters

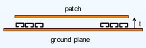
Metamaterial filling

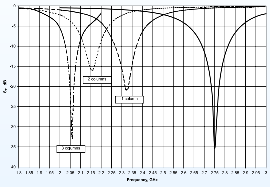
Loading	F_{res}	$BW\vert_{-6dB}$	$BW\vert_{-9.5dB}$	η	Q_0	$Q_0F_{res}{}^3$
	GHz	percent	percent	percent		
Air	1.05	3.6	2.2	84	34.3	40.2
Reference blocks	0.95	2.5	1.5	74	51.0	43.2
Metasolenoids	0.86	2.2	1.3	70	59.4	37.8

Microstrip antennas

Calculated results:

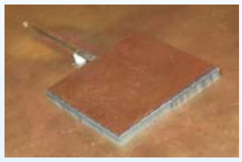


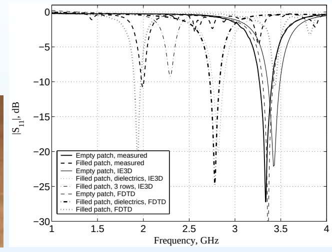




Microstrip antennas

Measured results:

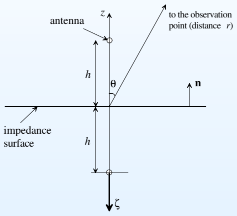




Near field control

$$\mathbf{E}_t = Z_s \mathbf{n}\times\mathbf{H} = \eta \overline{Z_s}\mathbf{n}\times\mathbf{H} = \frac{\eta}{\overline{Y_s}}\mathbf{n}\times\mathbf{H}$$

$$\overline{Z_s} = Z_s/\eta, \qquad \overline{Y_s} = Y_s\eta, \qquad \eta = \sqrt{\mu_0/\epsilon_0}$$



Exact image theory

$$I_i = \delta_+(\zeta)I - 2jk\overline{Y_s}e^{-j\overline{Y_s}k\zeta}U_+(\zeta)I$$

The total field in the far zone:

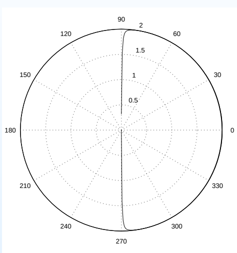
$$E = -\frac{\eta k}{\sqrt{8\pi}}e^{j\pi/4}\left\{\frac{e^{-jk(r-h\cos\theta)}}{\sqrt{k(r-h\cos\theta)}} + \frac{e^{-jk(r+h\cos\theta)}}{\sqrt{k(r+h\cos\theta)}}\right.$$
$$\left. -2jk\overline{Y_s}\int_0^\infty \frac{e^{-jk[\overline{Y_s}\zeta + r + (h+\zeta)\cos\theta]}}{\sqrt{k[r+(h+\zeta)\cos\theta]}}d\zeta\right\} I$$

The integral converges if

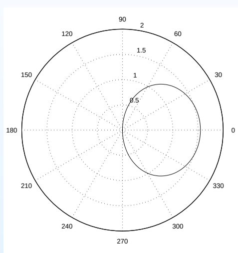
$\mathrm{Im}\{\overline{Y_s}\} < 0$ or $\mathrm{Im}\{\overline{Z_s}\} > 0$ (positive reactance)

Far-field pattern

$$F(\theta) = \left|1 + \frac{\cos\theta - \overline{Y_s}}{\cos\theta + \overline{Y_s}}e^{-2jkh\cos\theta}\right|$$



$\overline{Z_s} = 100j$



$\overline{Z_s} = j$

Near-field pattern

$$E = -\frac{\eta k}{4}\left[H_0^{(2)}\left(k\sqrt{r^2 - 2rh\cos\theta + h^2}\right)\right.$$
$$\left. + H_0^{(2)}\left(k\sqrt{r^2 + 2rh\cos\theta + h^2}\right) - 2j\overline{Y_s}G\right] I$$

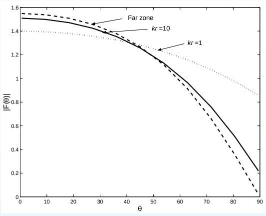
where

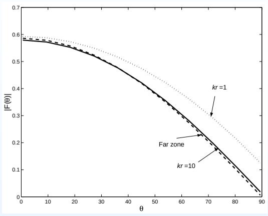
$$G = \int_0^\infty e^{-j\overline{Y_s}\zeta}H_0^{(2)}\left(\sqrt{(kr\cos\theta + kh + \zeta)^2 + (kr\sin\theta)^2}\right)d\zeta$$

The field pattern:

$$F(kr,\theta) = 1 + \frac{H_0^{(2)}\left(\sqrt{(kr)^2 + 2k^2rh\cos\theta + (kh)^2}\right) - 2j\overline{Y_s}G}{H_0^{(2)}\left(\sqrt{(kr)^2 - 2k^2rh\cos\theta + (kh)^2}\right)}$$

Numerical example

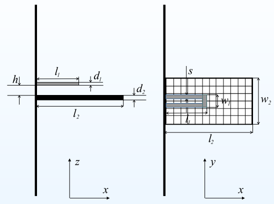


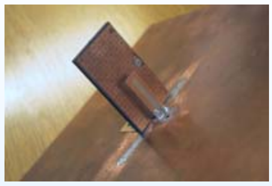


$\overline{Z_s} = j$ $\overline{Z_s} = 0.2j$

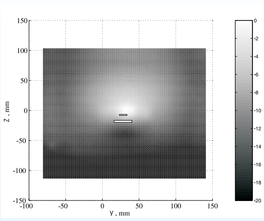
Compromise: Moderate inductive surface impedance

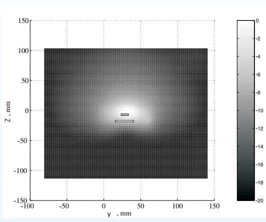
Experimental set-up





Near field distributions





Electric field Magnetic field

Local screening effect (LSE)

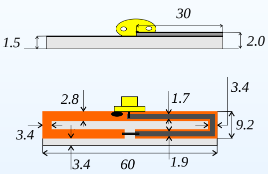
LSE = the ratio (in dB) between the field amplitudes at two points located in front ($z > 5$ mm) and behind ($z < -45$ mm) the antenna structure

The averaged screening effect (ASE) = the averaged value of LSE over these two planes

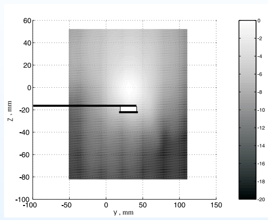
For the mushroom structure ASE is approximately equal to 15 dB for electric fields and 20 dB for magnetic fields.

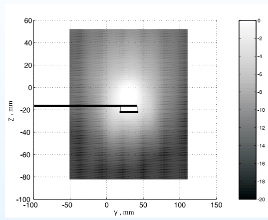
Full-sized folded dipole

over a mushroom layer with a moderate inductive surface impedance



Near-field distibutions





Electric field Magnetic field

Main antenna parameters

	Metal plate	Mushrooms	Jerusalem crosses
Central frequency, GHz	1.81	1.77	1.83
Radiation efficiency, %	76	73	88
Total thickness H, mm	9.35	8.3	12.3
Bandwidth, %	–	5.5 (-9.5 dB)	4 (-10 dB)
Electric field ASE, dB	15	13	15
Magnetic field ASE, dB	15–17	12–13	13–15